Group Velocity Measurement

$$U = \frac{X(p)}{T(p)} \qquad (8.3)$$

- U: Group Velocity
- $X(p)$: Distance Travelled
- $T(p)$: Time It Took

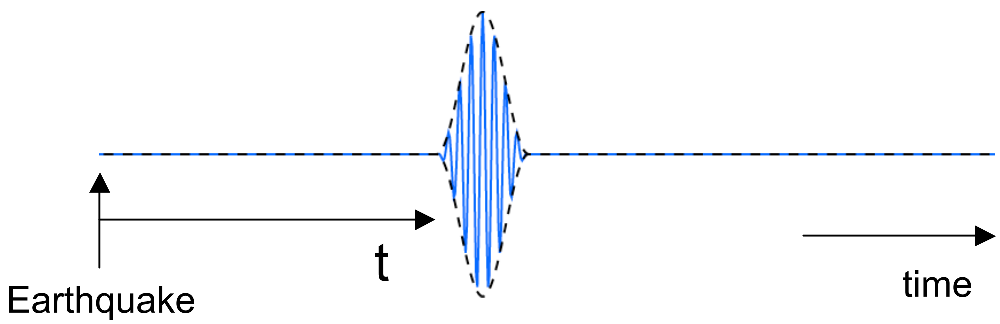
Group Velocity Measurement

Distance Travelled

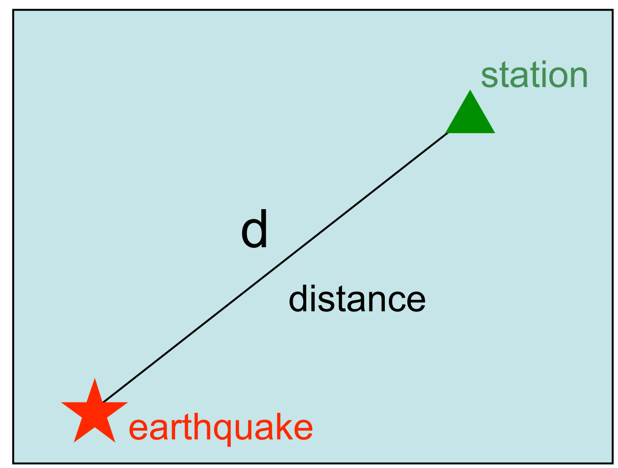
$$U = \frac{X(p)}{T(p)} \qquad (8.3)$$

Group Velocity

Time It Took



$$U = \frac{d}{t}$$



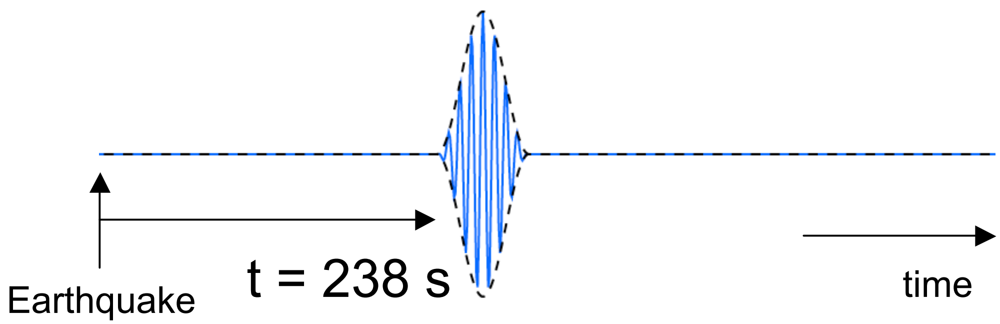
Group Velocity Measurement

Distance Travelled

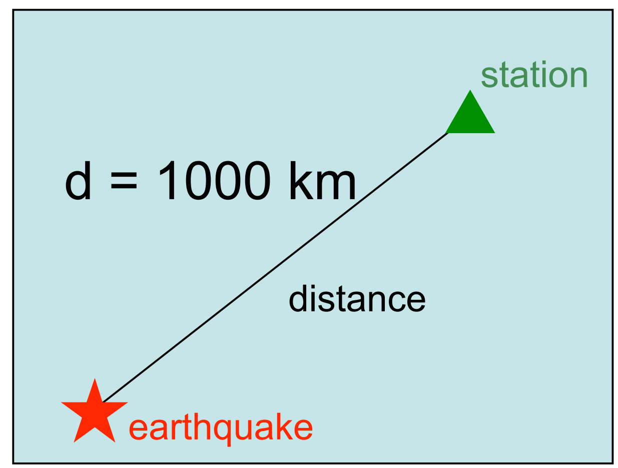
$$U = \frac{X(p)}{T(p)} \qquad (8.3)$$

Group Velocity

Time It Took



$$U = \frac{d}{t} = \frac{1000\ km}{238\ s} = 4.2\ km/s$$



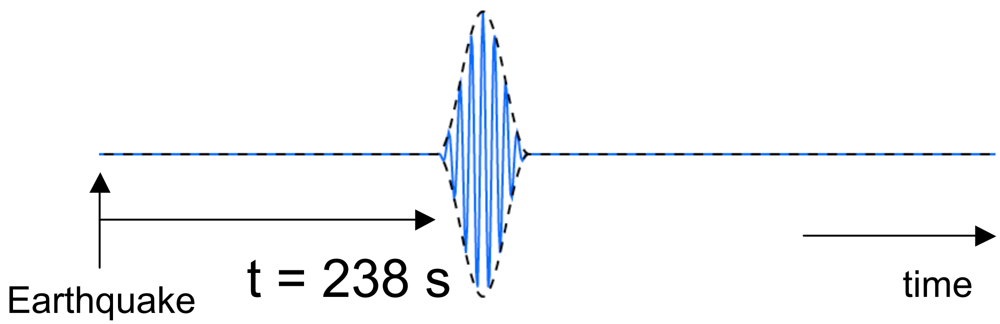
Group Velocity Measurement

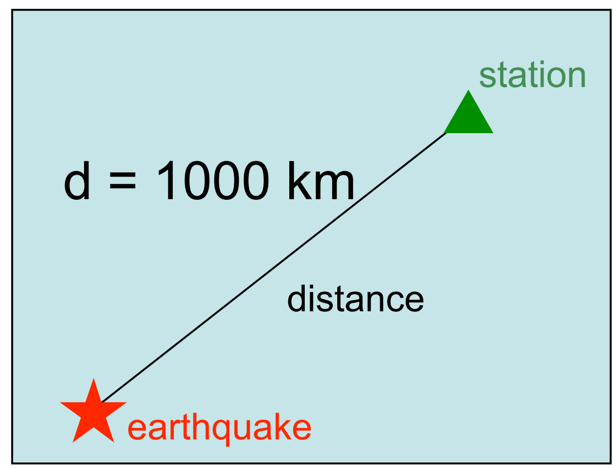
Distance Travelled

$$U = \frac{X(p)}{T(p)} \qquad (8.3)$$

Group Velocity

Time It Took





$$U = \frac{d}{t} = \frac{1000\ km}{238\ s} = 4.2\ km/s$$

Need to know exact location and timing of an earthquake

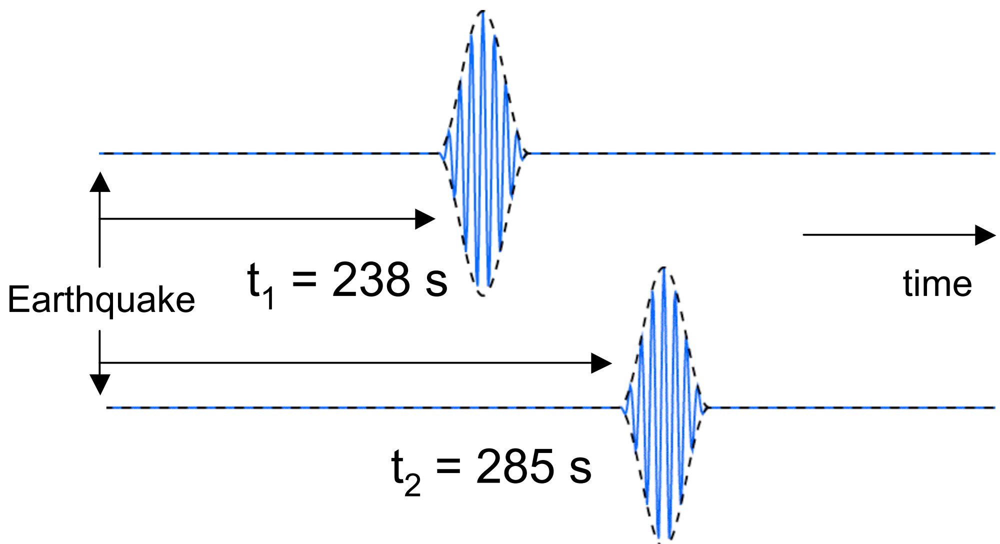
Group Velocity Measurement

$$U = \frac{X(p)}{T(p)} \qquad (8.3)$$

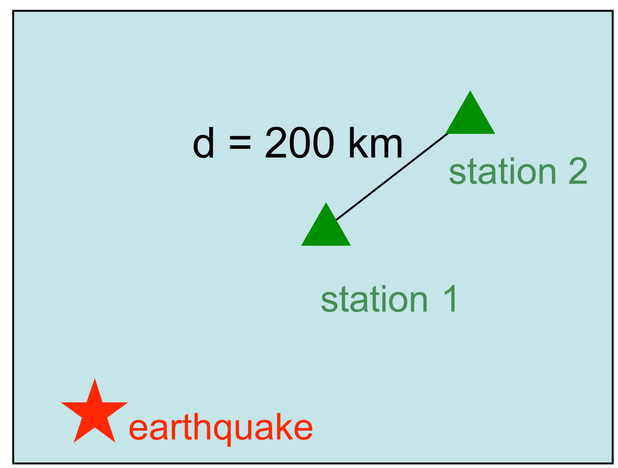
Distance Travelled

Group Velocity

Time It Took



Location of stations are well known



$$U = \frac{d}{t_2 - t_1} = \frac{200\ km}{285\ s - 238\ s} = \frac{200\ km}{47\ s} = 4.2\ km/s$$

Group Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Filter seismograms ➔ Measure group velocity ➔ Plot group velocity vs. frequency

Group Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Filter seismograms ➔ Measure group velocity ➔ Plot group velocity vs. frequency

Use single, well-dispersed surface wave arrival

Group Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Filter seismograms ➔ Measure group velocity ➔ Plot group velocity vs. frequency

more accurate

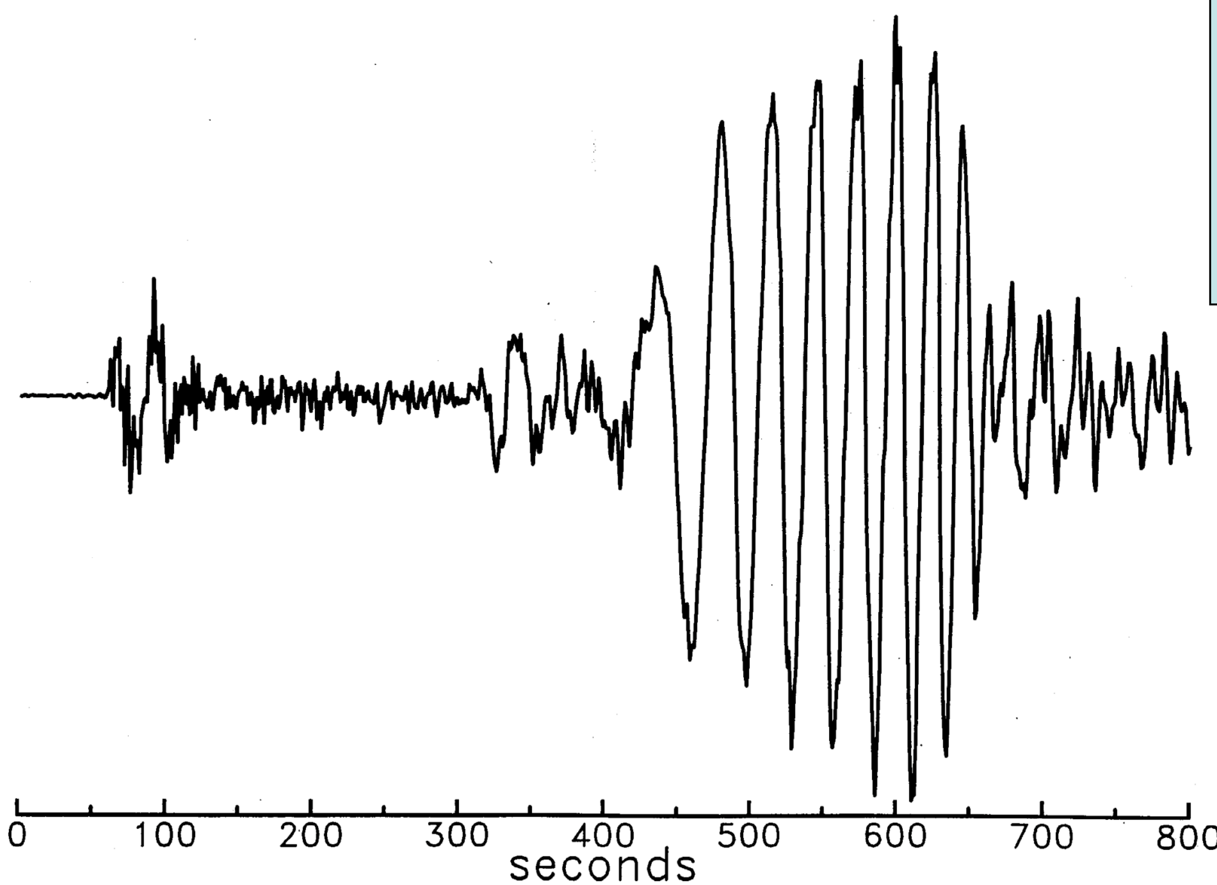
Use single, well-dispersed surface wave arrival

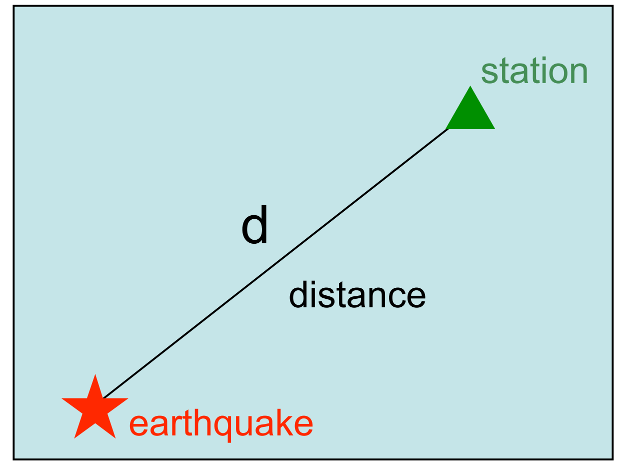
less accurate

Group Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Use single, well-dispersed surface wave arrival



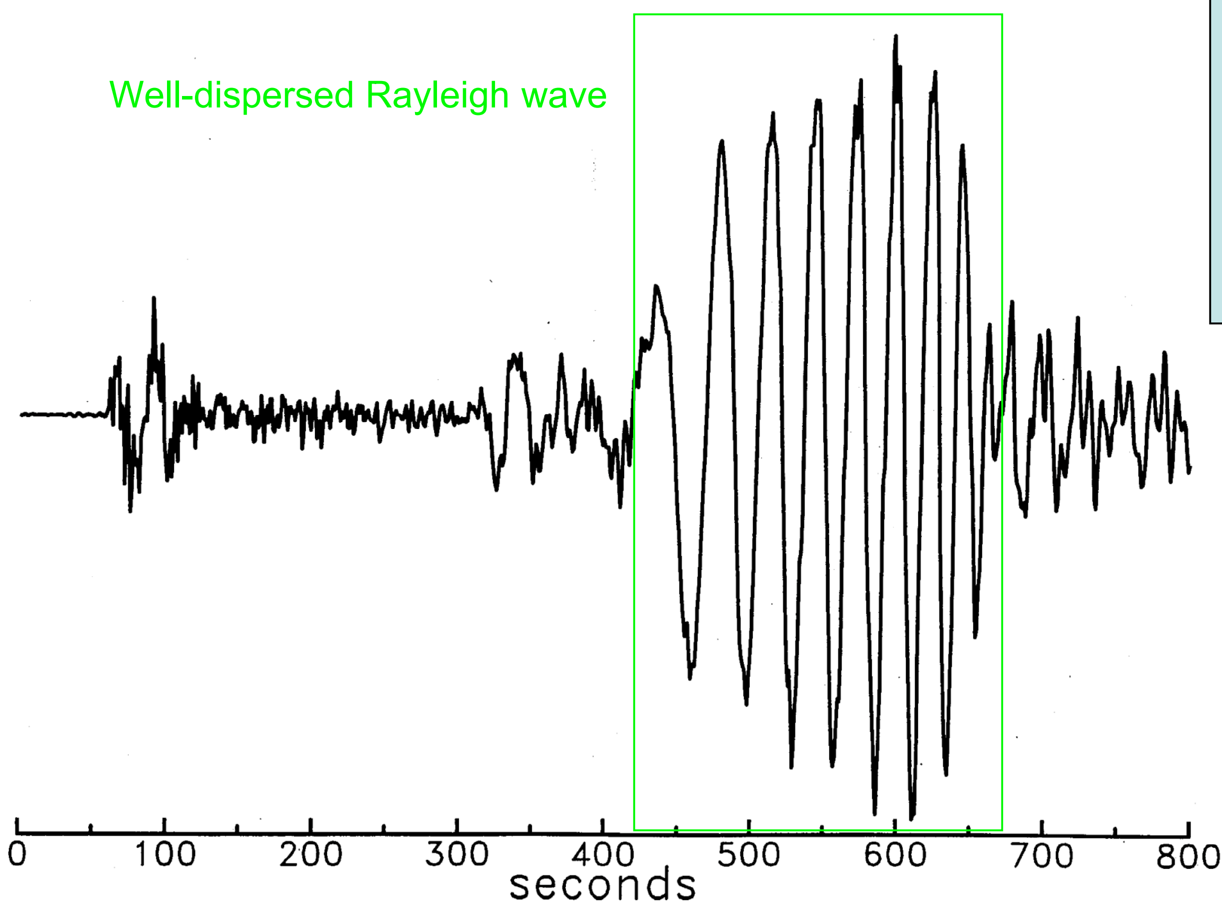


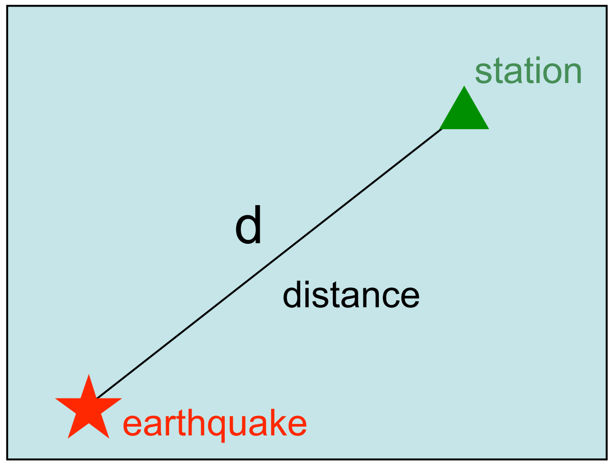
Earthquake: Mexico
Station: CCM, Cathedral Cave, Missouri
Distance: 22.4 degrees
Component: Vertical

Group Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Use single, well-dispersed surface wave arrival





Earthquake: Mexico
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Group Velocity Dispersion

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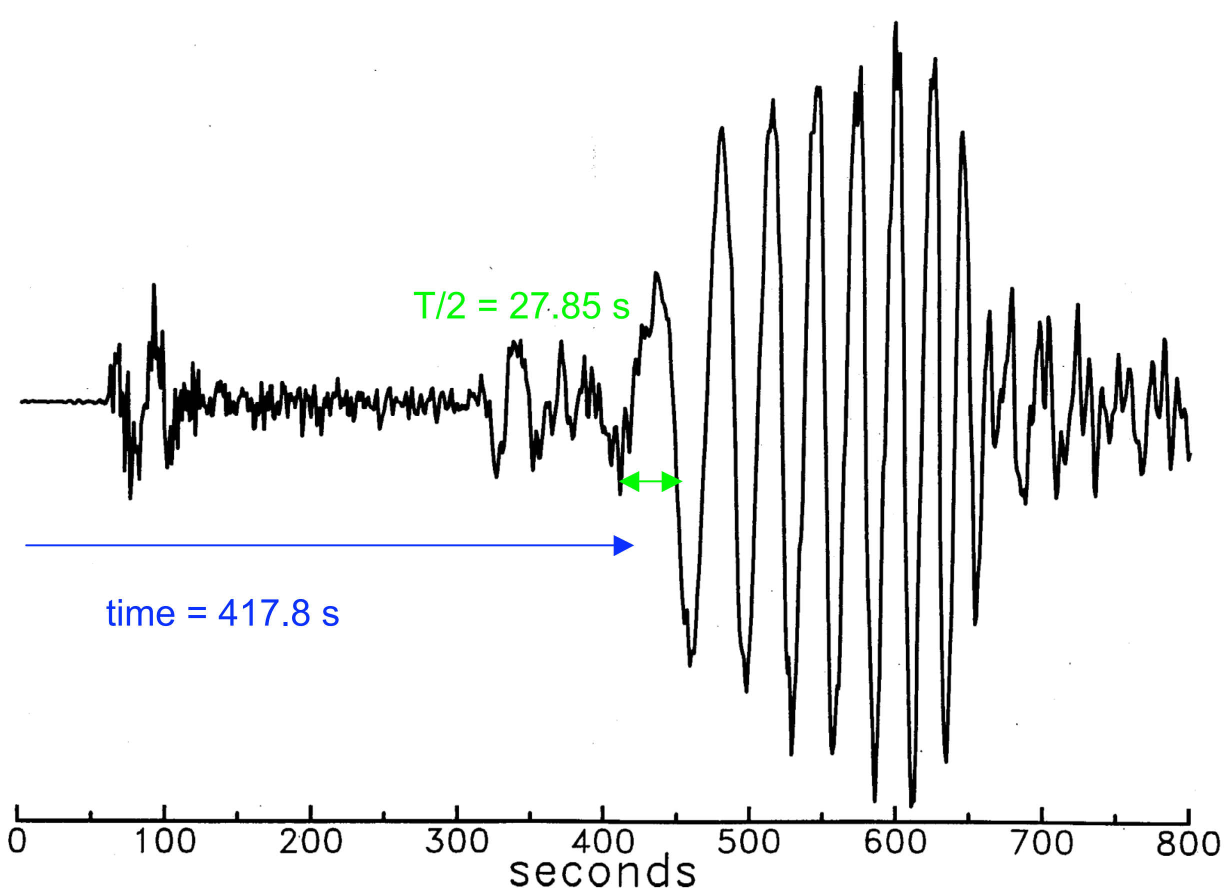


Earthquake: Mexico
Station: CCM, Cathedral Cave, Missouri
Distance: 22.4 degrees
Component: vertical

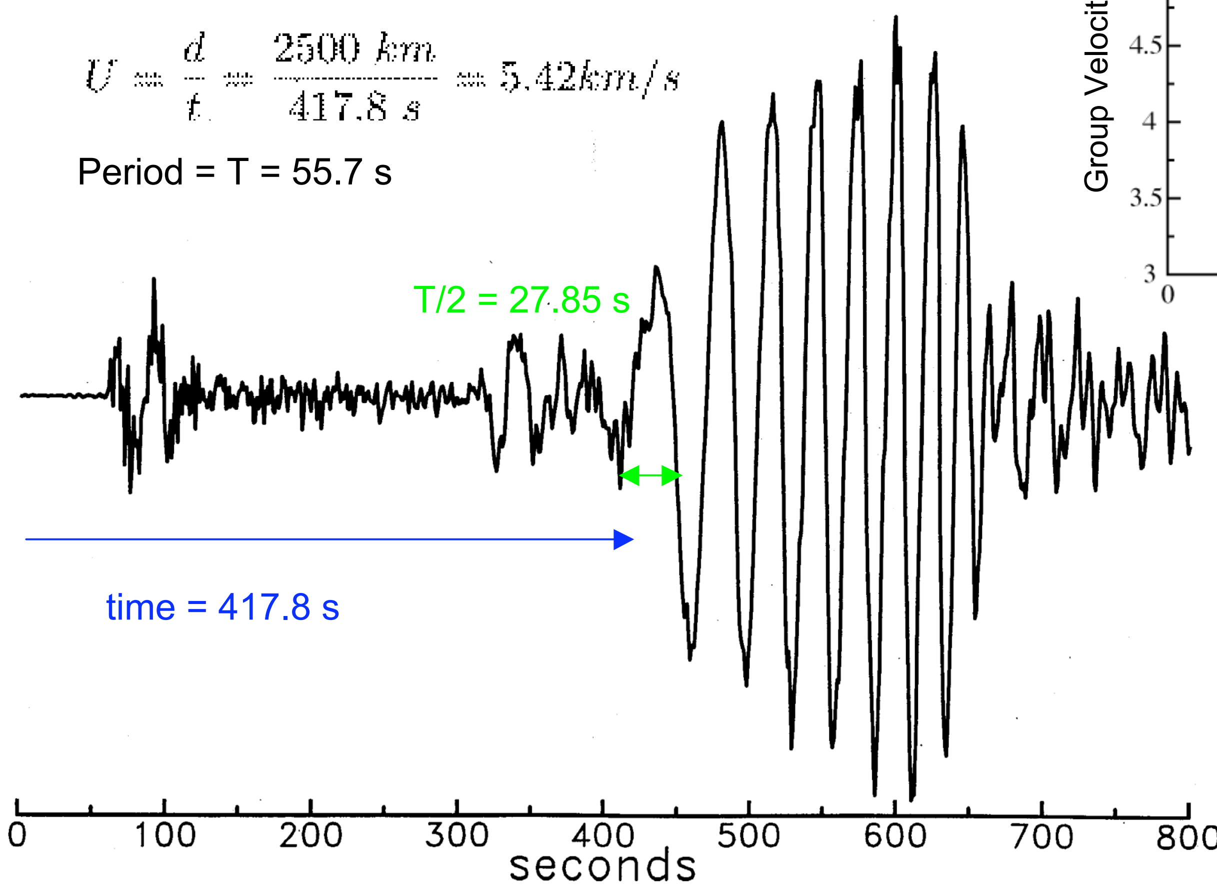
Group Velocity Dispersion

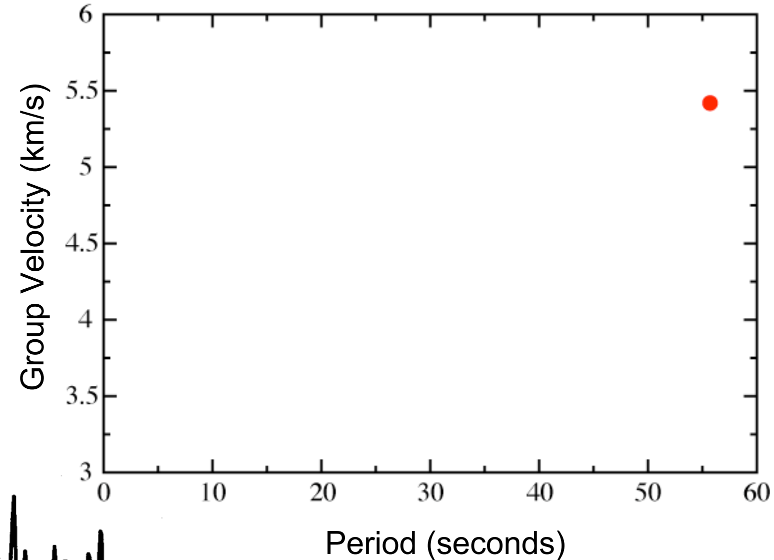
Dispersion: dependence of wave speed on frequency

Use single, well-dispersed surface wave arrival

$$U = \frac{d}{t} = \frac{2500\ km}{417.8\ s} = 5.42 km/s$$

Period = T = 55.7 s





Earthquake: Mexico
Station: CCM, Cathedral Cave, Missouri
Distance: 22.4 degrees
Component: vertical

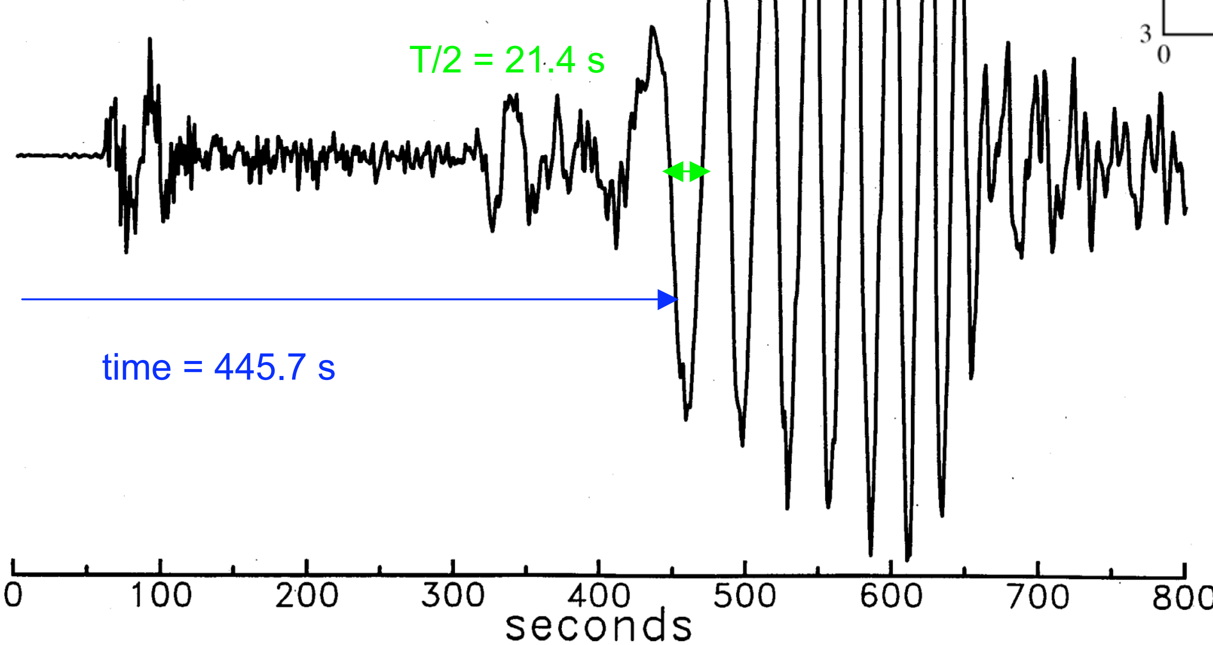
Group Velocity Dispersion

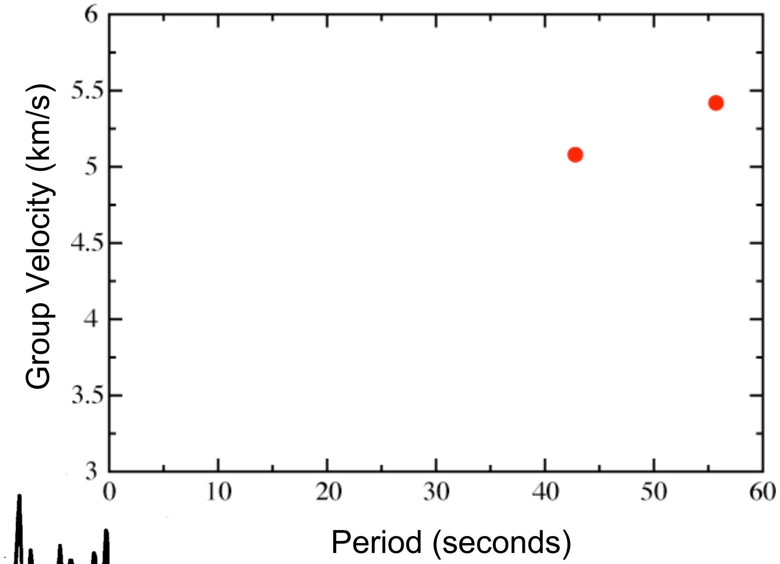
Dispersion: dependence of wave speed on frequency

Use single, well-dispersed surface wave arrival

$$U = \frac{d}{t} = \frac{2500\ km}{445.7\ s} = 5.08 km/s$$

Period = T = 42.8 s





Earthquake: Mexico
Station: CCM, Cathedral Cave, Missouri
Distance: 22.4 degrees
Component: vertical

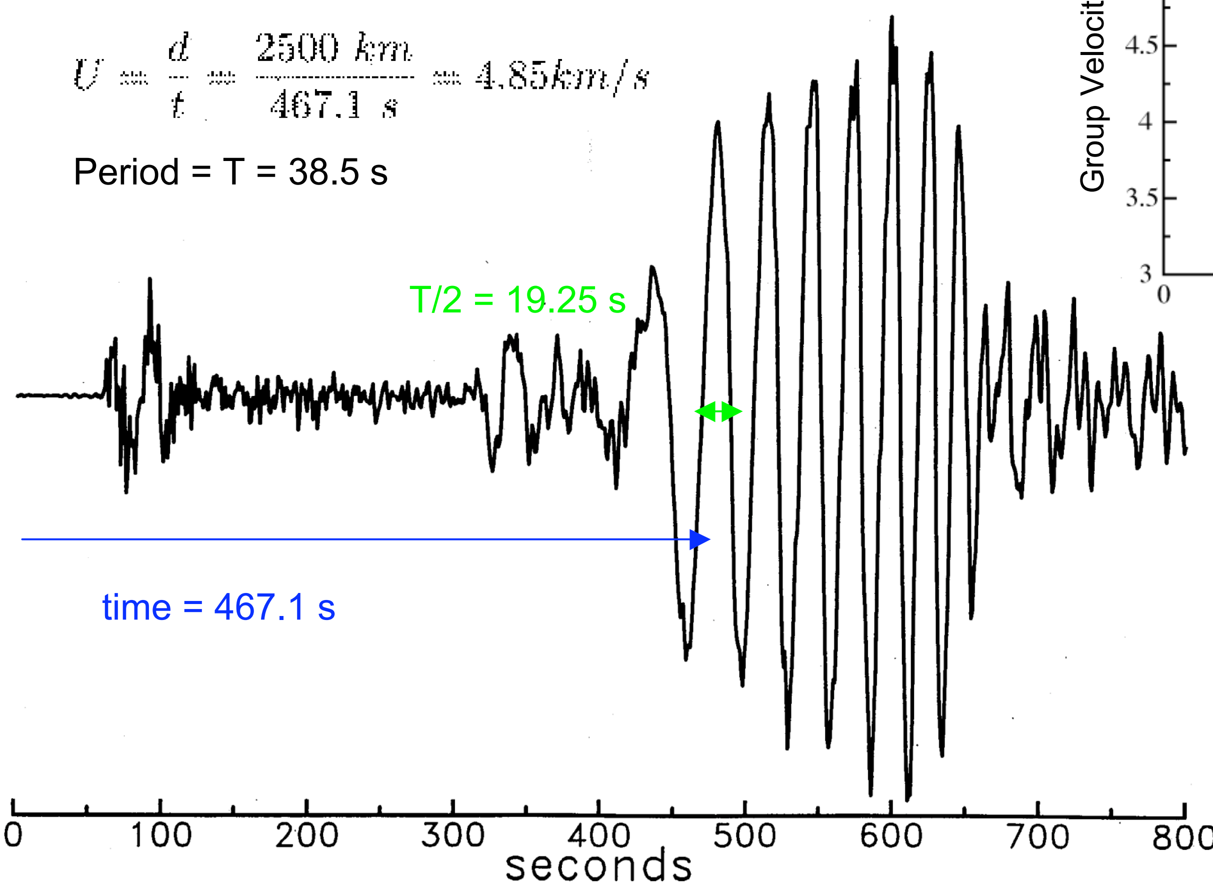
Group Velocity Dispersion

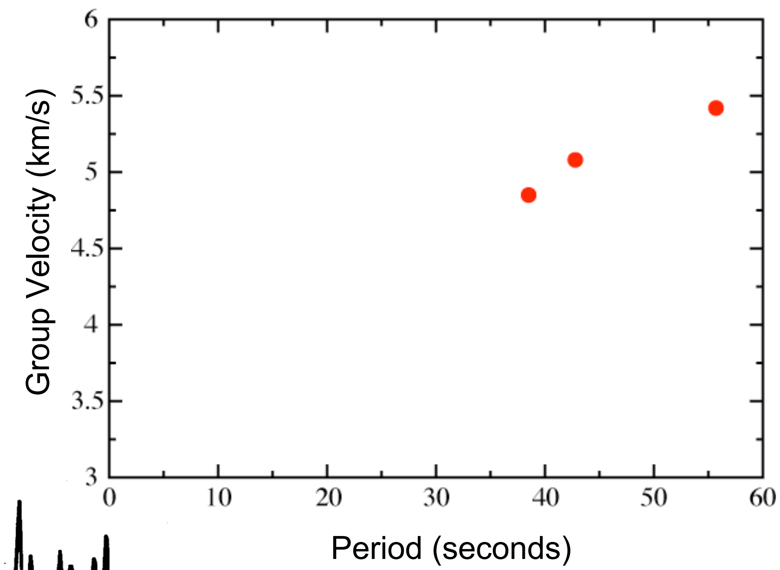
Dispersion: dependence of wave speed on frequency

Use single, well-dispersed surface wave arrival

$$U = \frac{d}{t} = \frac{2500\ km}{467.1\ s} = 4.85 km/s$$

Period = T = 38.5 s



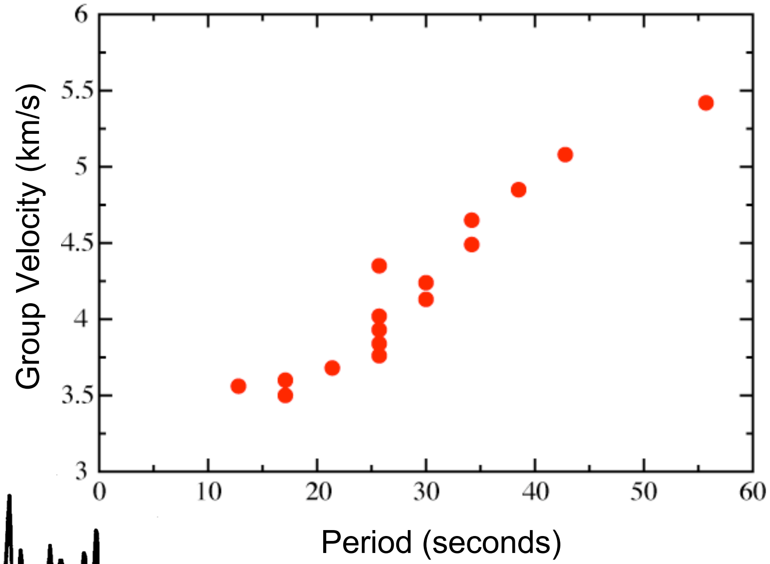


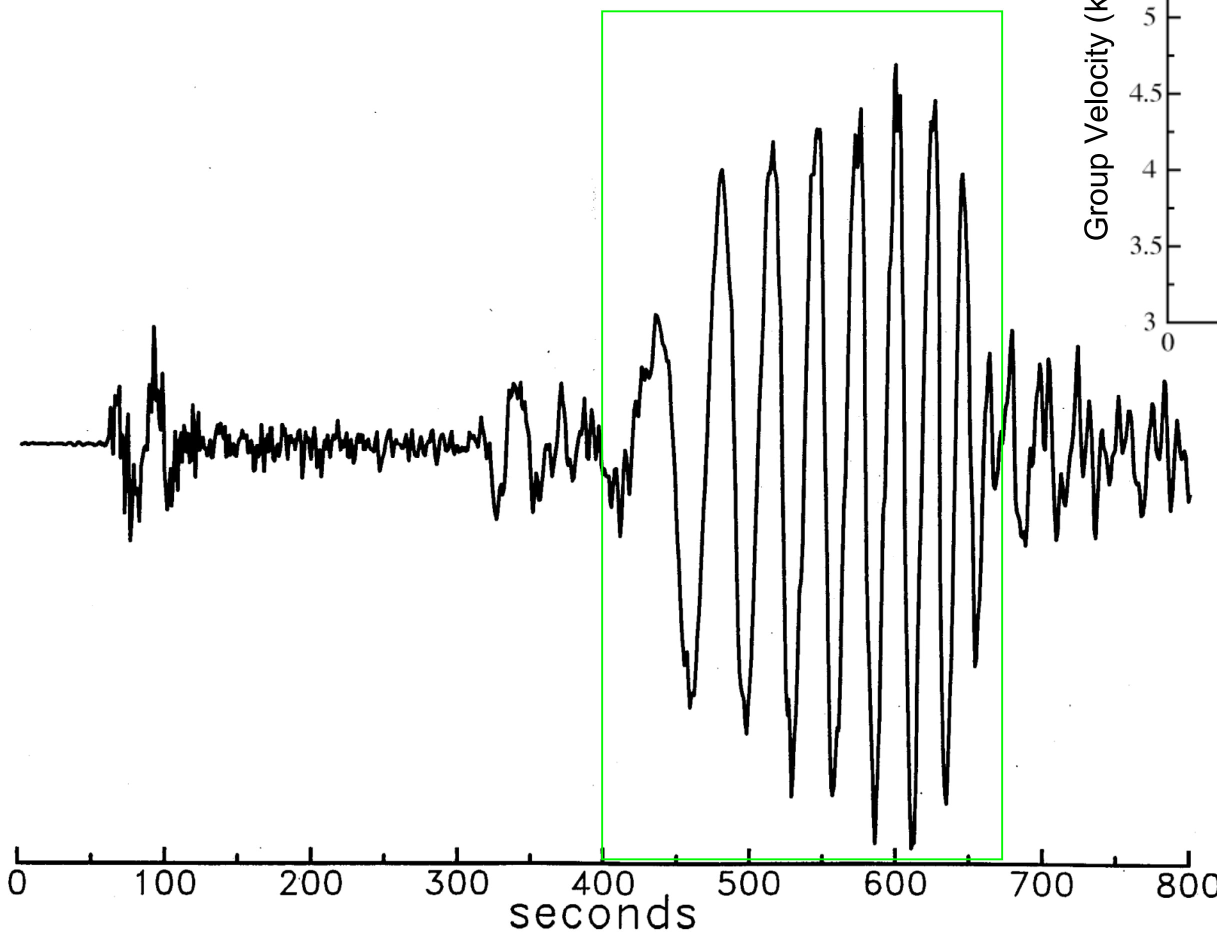
Earthquake: Mexico
Station: CCM, Cathedral Cave, Missouri
Distance: 22.4 degrees
Component: vertical

Group Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Use single, well-dispersed surface wave arrival





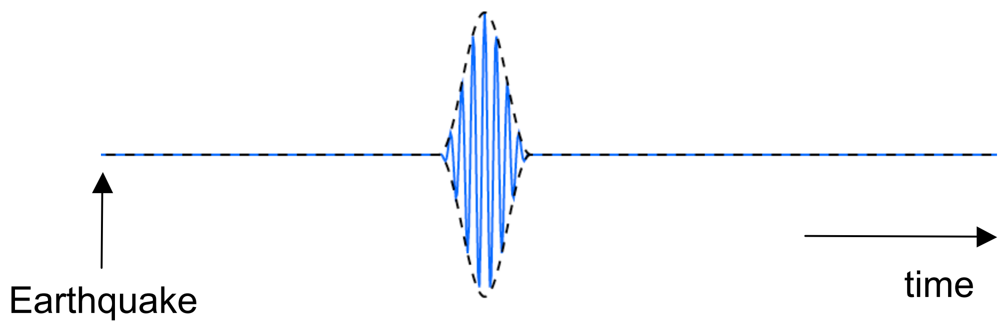
Earthquake: Mexico
Station: CCM, Cathedral Cave, Missouri
Distance: 22.4 degrees
Component: vertical

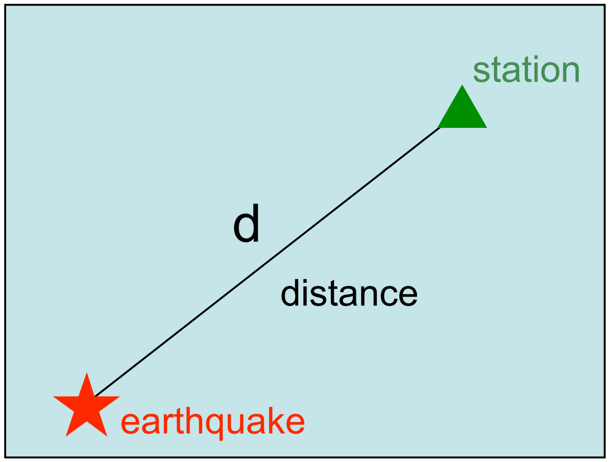
Group vs. Phase Velocities

Illustration from
http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/sines/GroupVelocity.html

Phase Velocity Measurement

Phase Velocity: speed at which phase travels

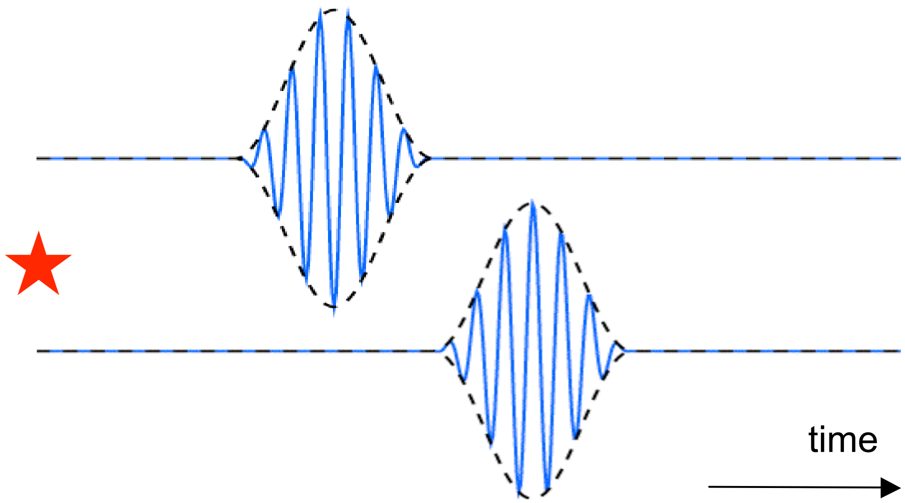


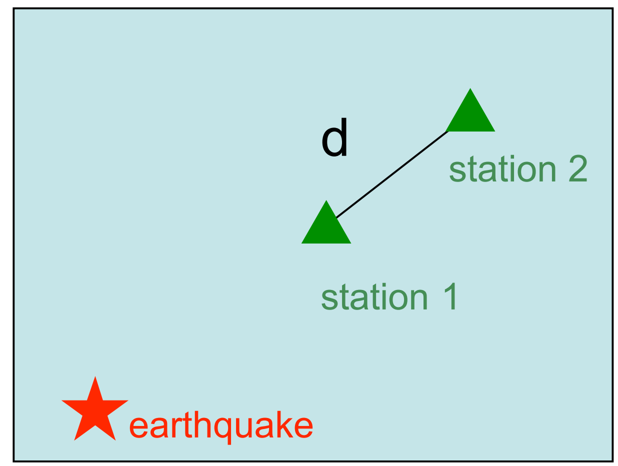


We need to know the initial phase of the wave generated by the earthquake
➔ single station method not reliable

Phase Velocity Measurement

Phase Velocity: speed at which phase travels

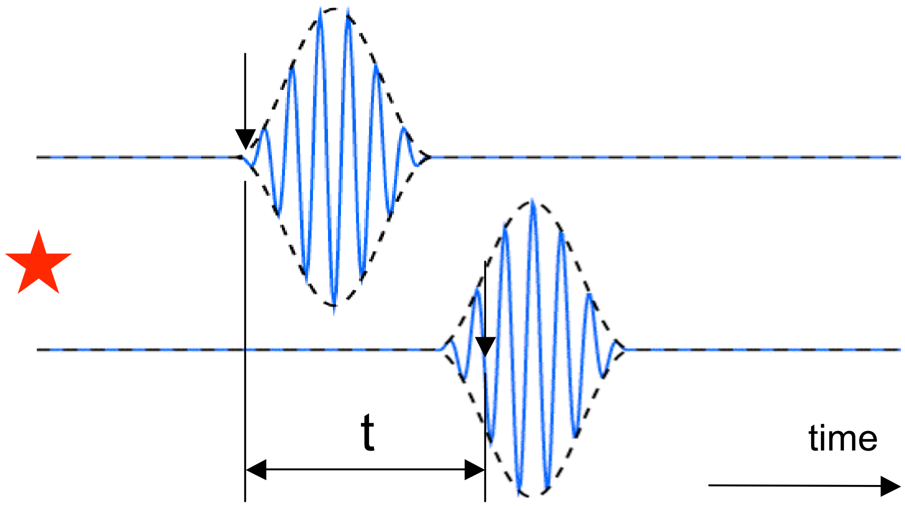


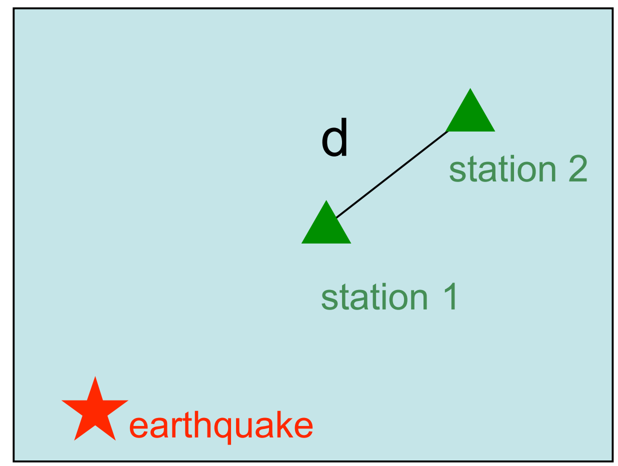


Cycle ambiguity

Phase Velocity Measurement

Phase Velocity: speed at which phase travels





$$c = \frac{d}{t}$$

Phase Velocity Dispersion

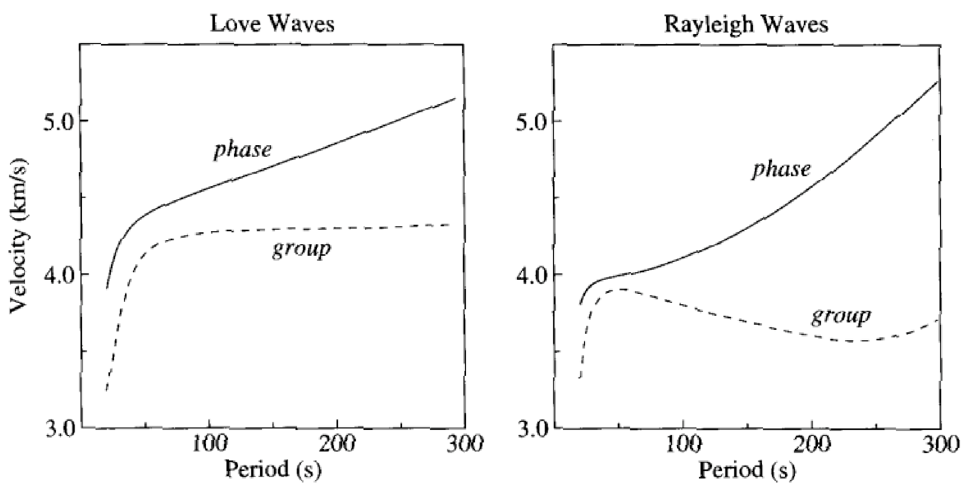
Dispersion: dependence of wave speed on frequency

Filter seismograms ➔ Measure phase velocity ➔ Plot phase velocity vs. frequency

Phase Velocity Dispersion

Dispersion: dependence of wave speed on frequency

Filter seismograms ➔ Measure phase velocity ➔ Plot phase velocity vs. frequency

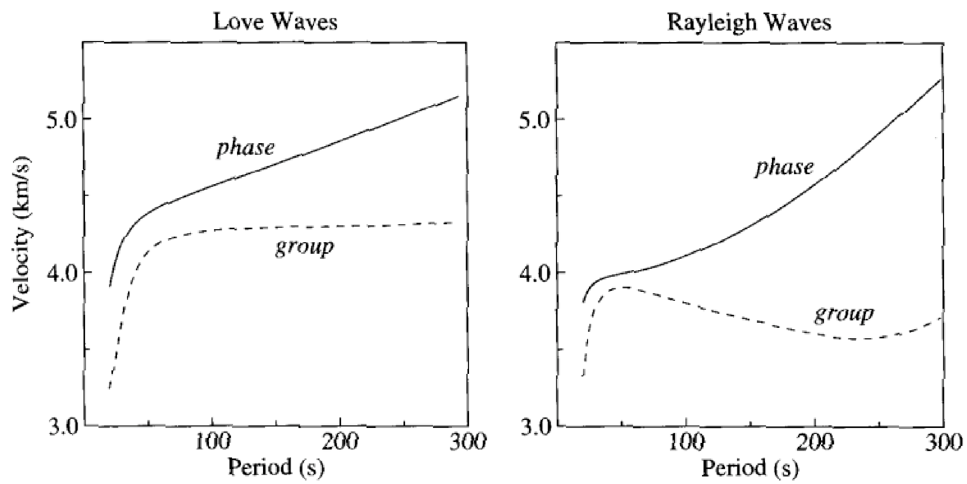


Shearer (1999)

Phase Velocity Dispersion

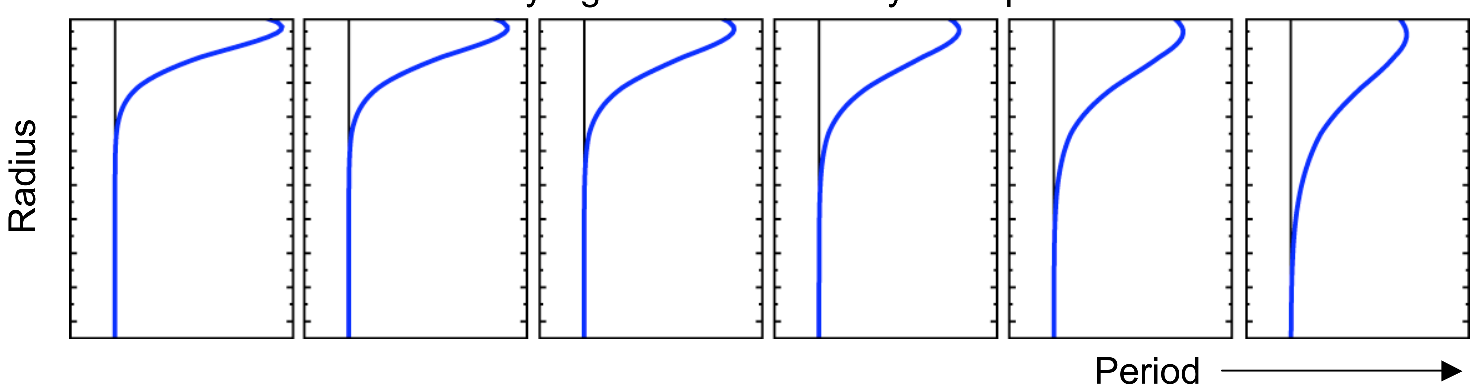
Dispersion: dependence of wave speed on frequency

Filter seismograms ➔ Measure phase velocity ➔ Plot phase velocity vs. frequency



Shearer (1999)

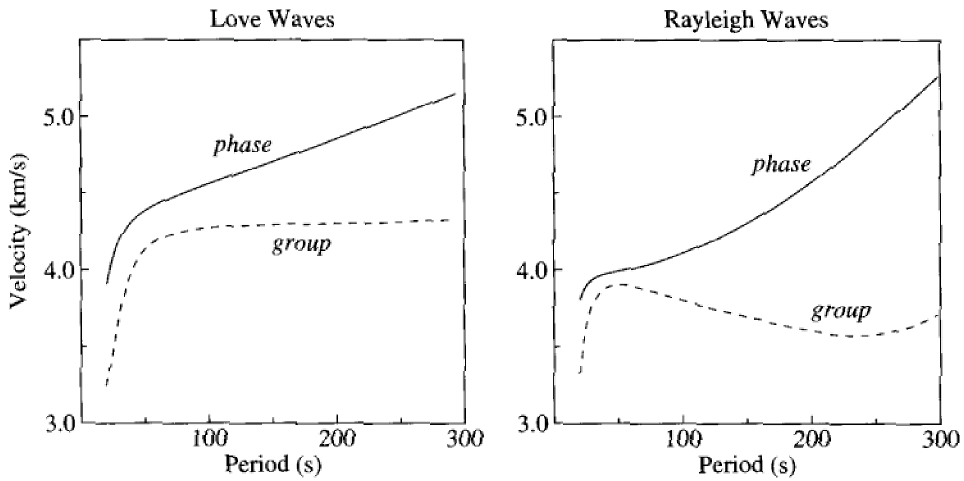
Rayleigh Wave Sensitivity to Depth



Phase Velocity Dispersion

Dispersion: dependence of wave speed on frequency

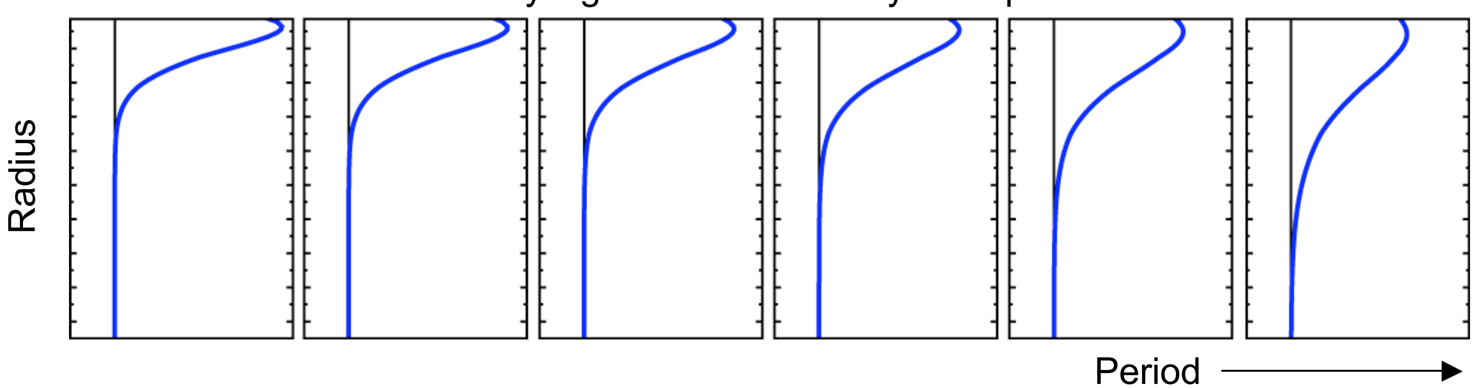
Filter seismograms ➔ Measure phase velocity ➔ Plot phase velocity vs. frequency



Shearer (1999)

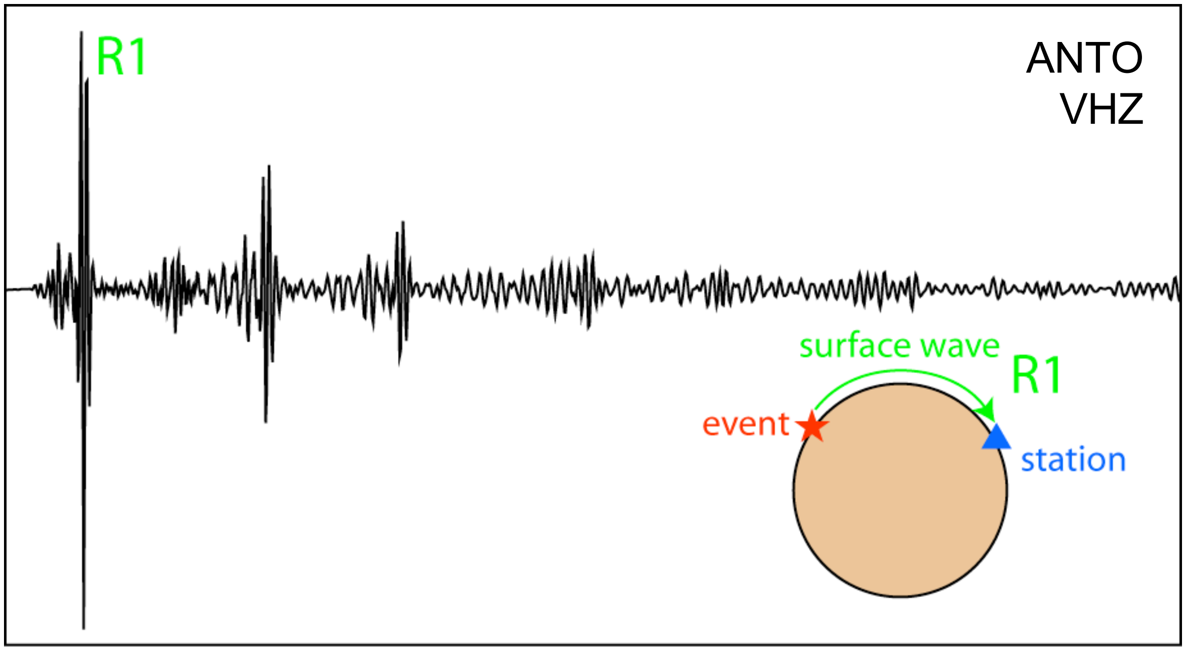
➔ Implication for mantle velocity structure?

Rayleigh Wave Sensitivity to Depth



Global Surface Waves

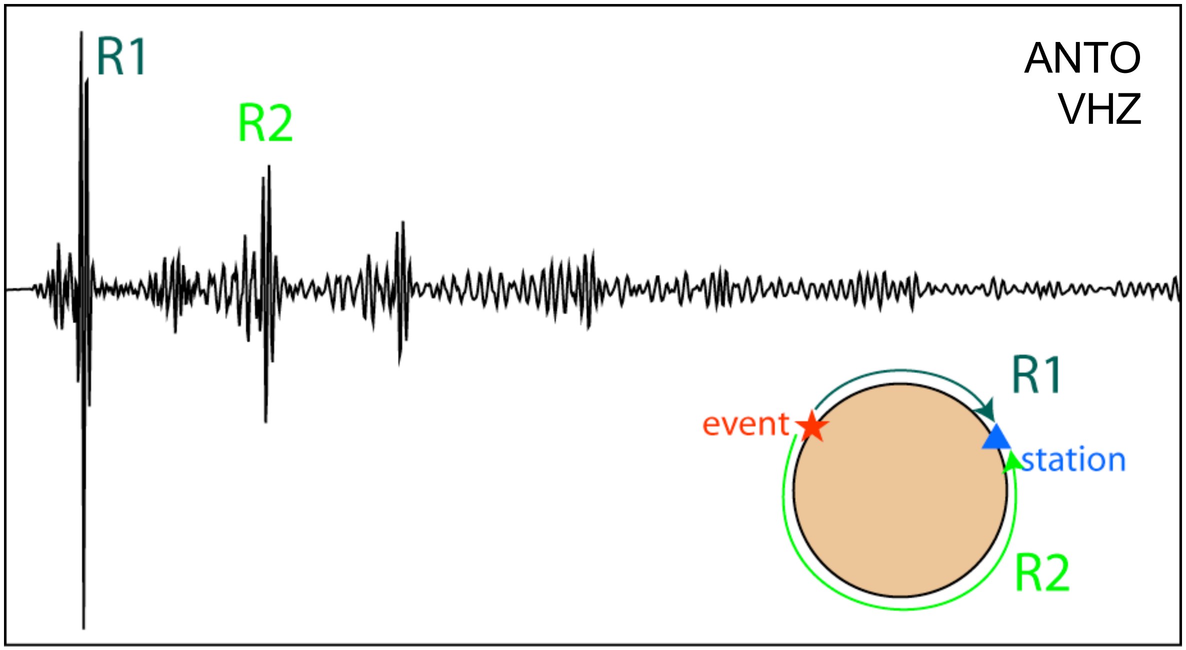
November 3, 2002 Denali, Alaska Earthquake



Filtered between 3 and 8 mHz

Global Surface Waves

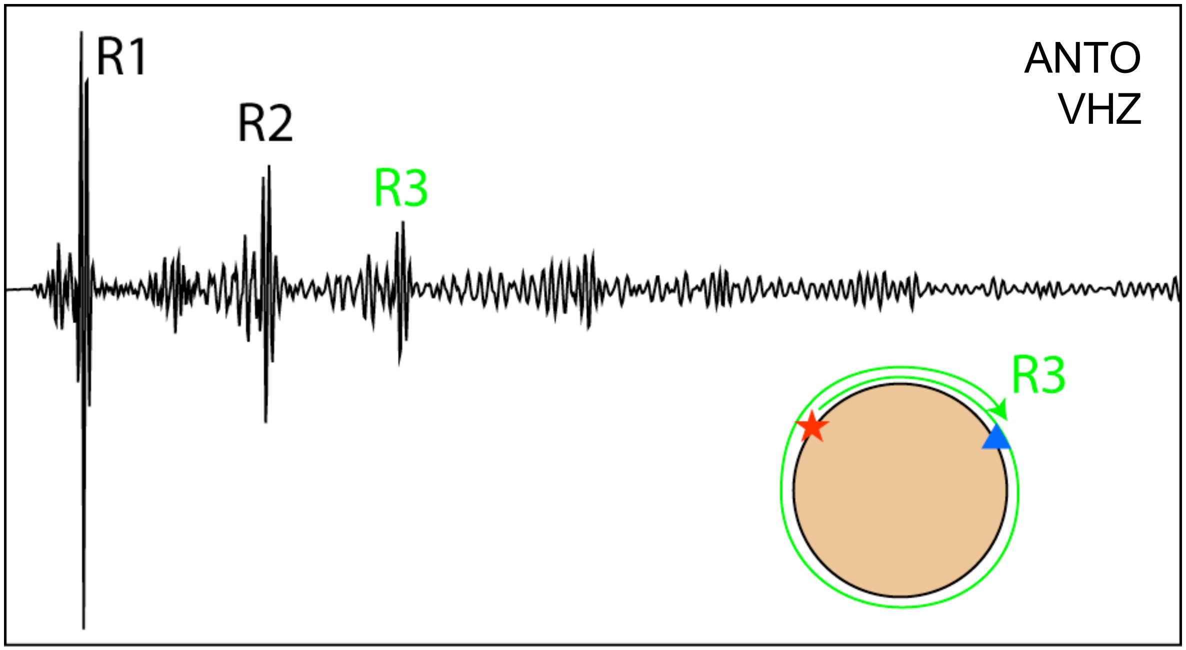
November 3, 2002 Denali, Alaska Earthquake



Filtered between 3 and 8 mHz

Global Surface Waves

November 3, 2002 Denali, Alaska Earthquake



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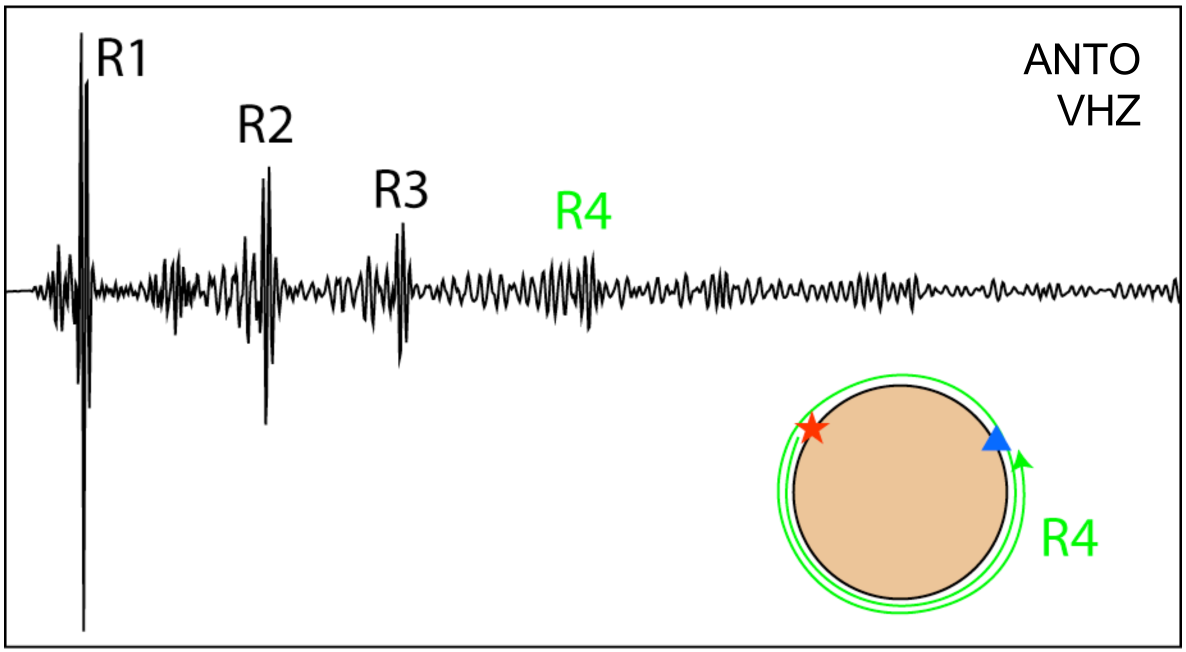
Global Surface Waves

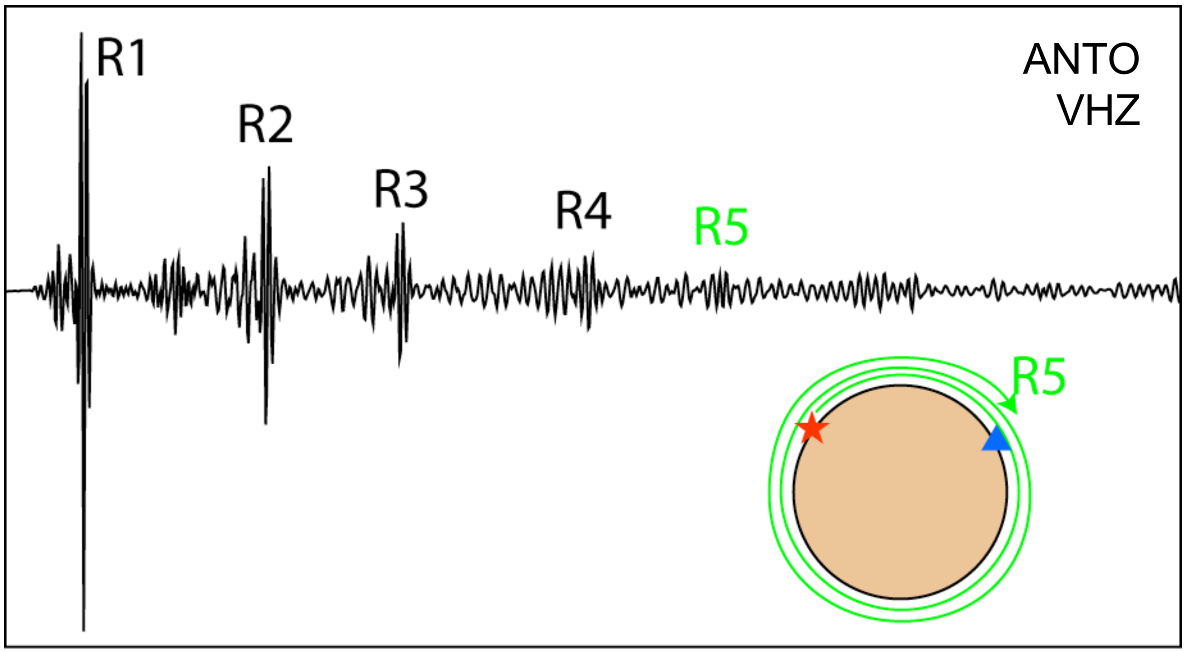
November 3, 2002 Denali, Alaska Earthquake



Filtered between 3 and 8 mHz

Global Surface Waves

November 3, 2002 Denali, Alaska Earthquake



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Global Surface Waves

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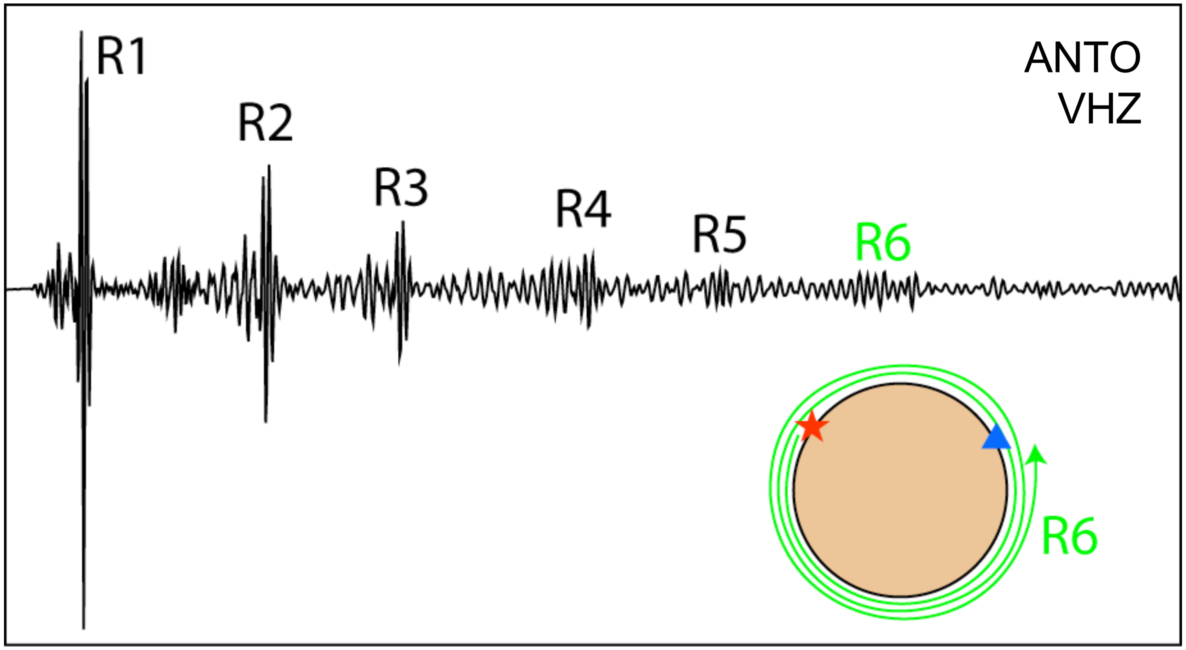
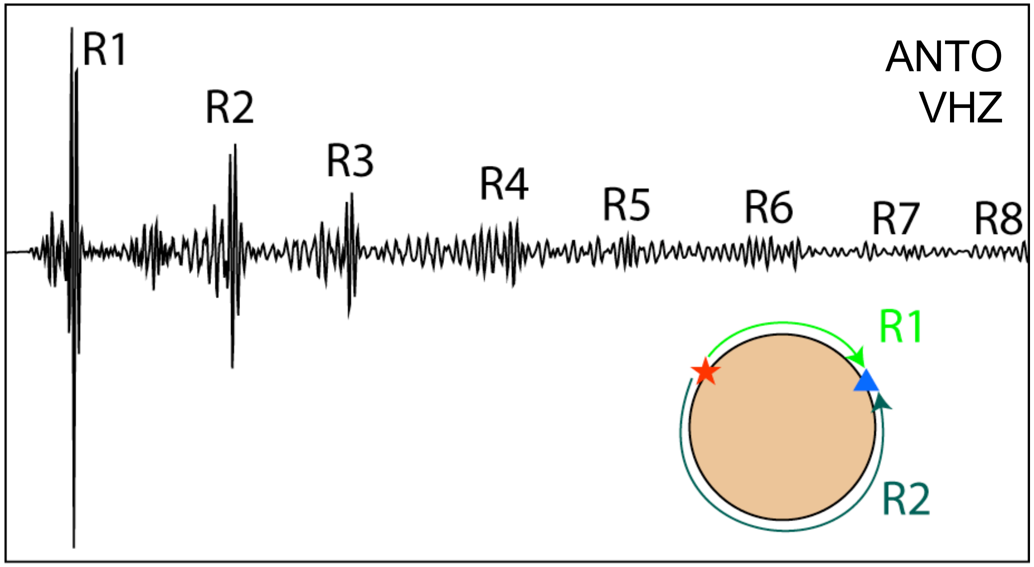
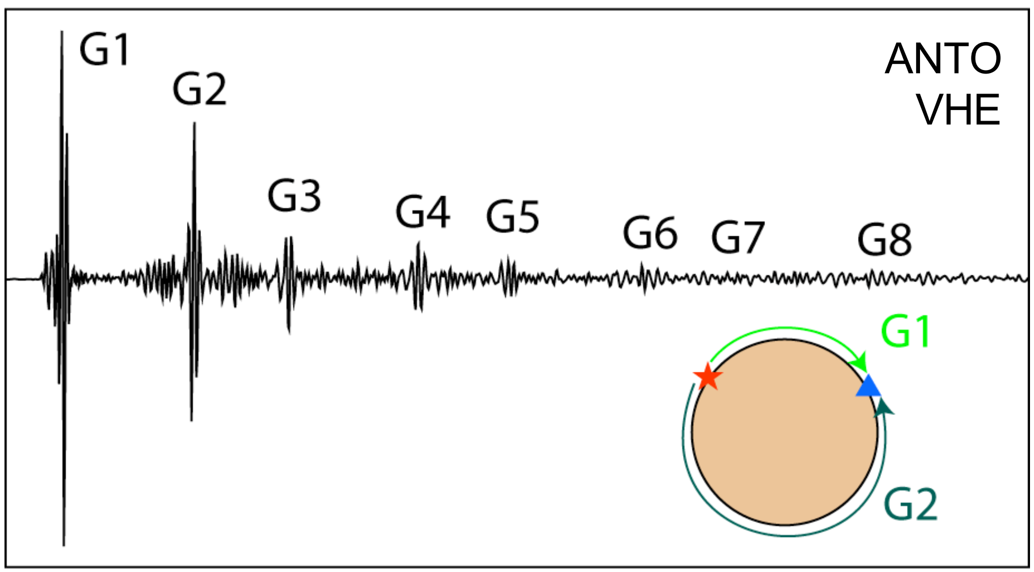


Filtered between 3 and 8 mHz

Global Surface Waves

November 3, 2002 Denali, Alaska Earthquake

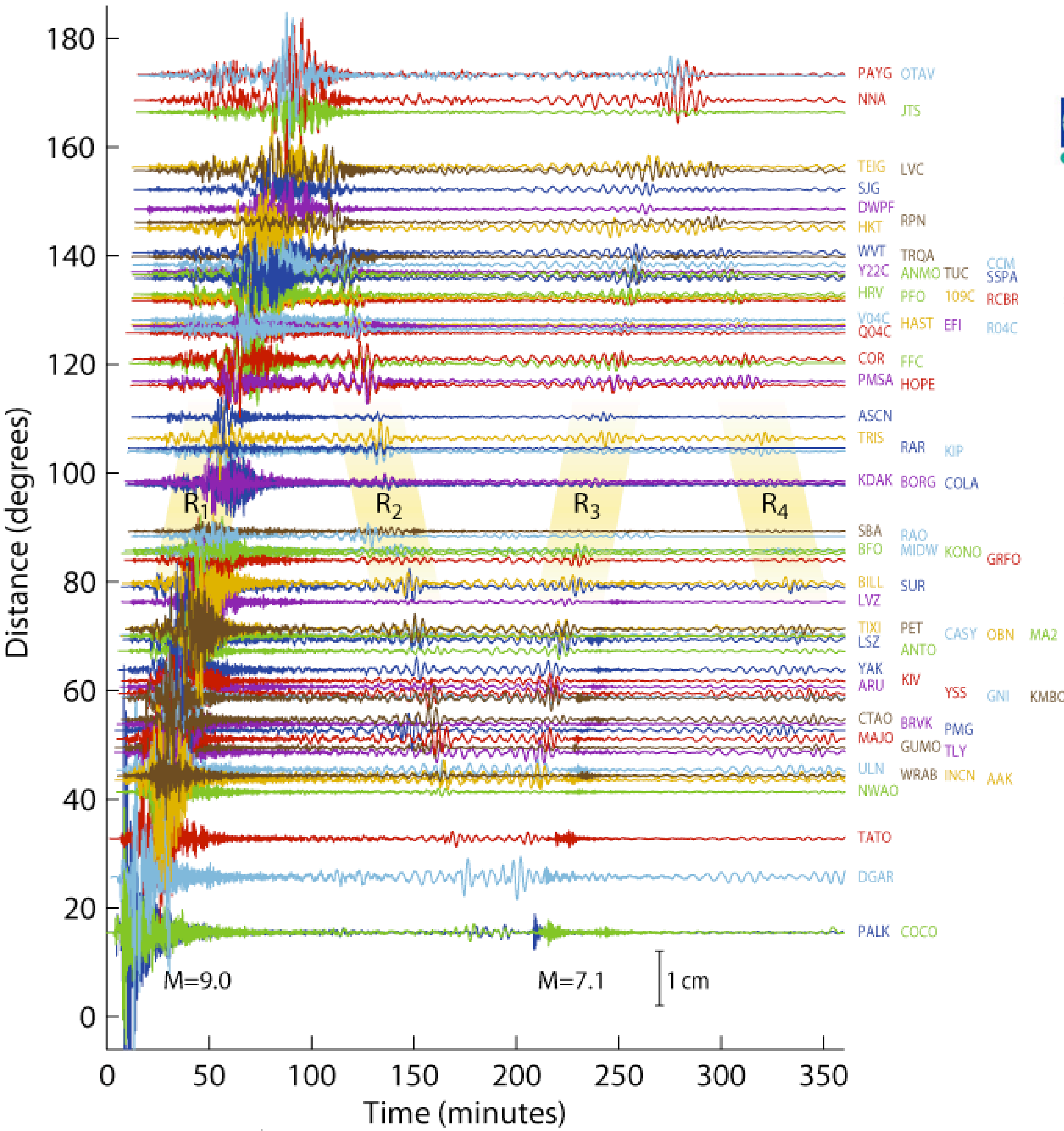


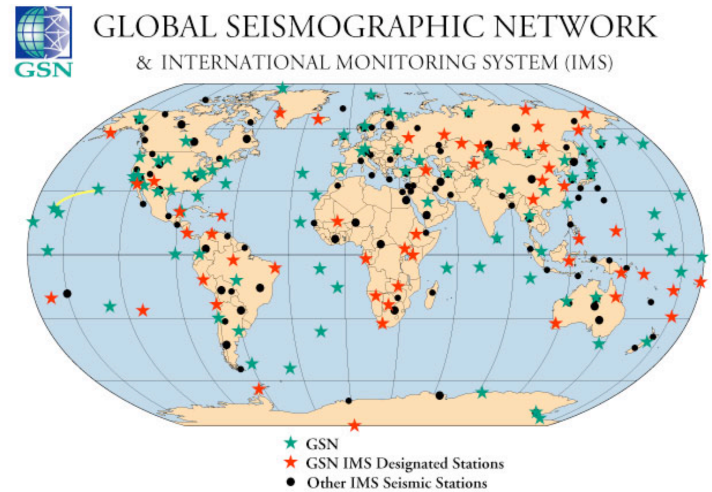


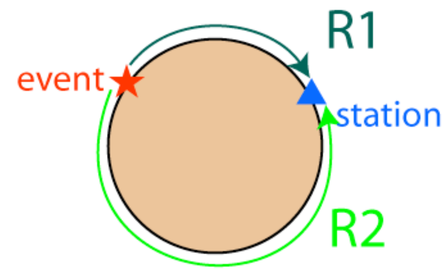
Filtered between 3 and 8 mHz

Sumatra - Andaman Islands Earthquake (M_W=9.0)

Global Displacement Wavefield from the Global Seismographic Network

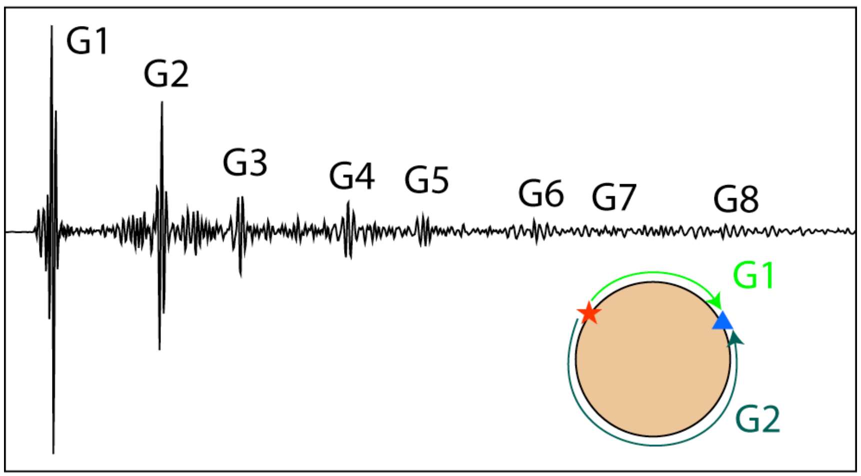






Standing Waves

Standing Wave: Stationary wave generated by constructive/destructive interference of two waves travelling in opposite directions

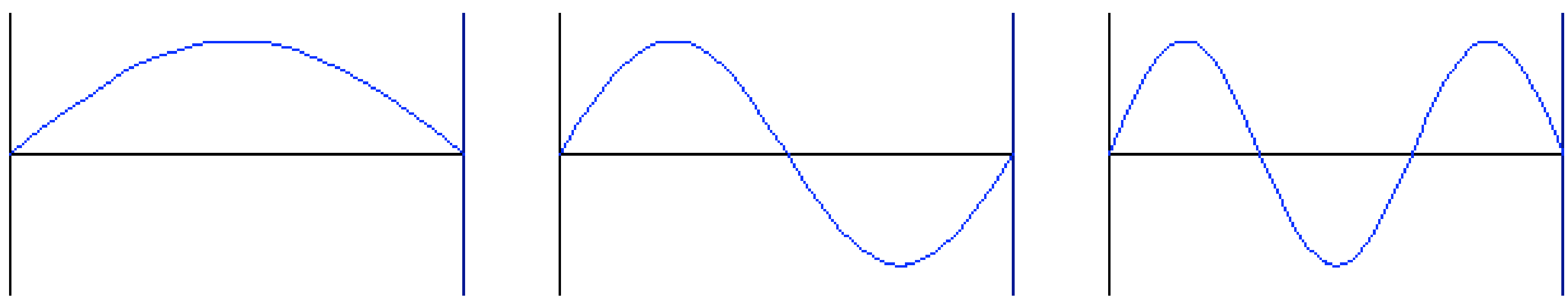


➔ Standing waves or normal modes of the Earth

Standing Waves

Standing Wave: Stationary wave generated by constructive/destructive interference of two waves travelling in opposite directions

1-D, fixed ends



m = 0 m = 1 m = 2

Standing waves constitute basis functions.

Sines and Cosines

Fourier Transform: combination of sines and cosines ➔ describe any 1-D function

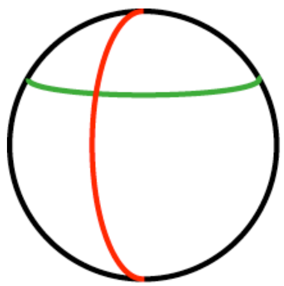
Index gives the number of nodes.

Basis Functions

1-D Sines and Cosines

Fourier Transform: combination of sines and cosines ➔ describe any 1-D function

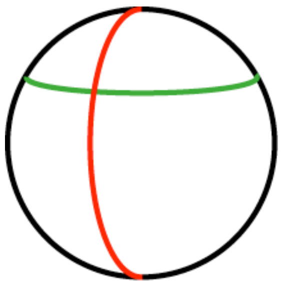
2-D
(Spherical Surface)



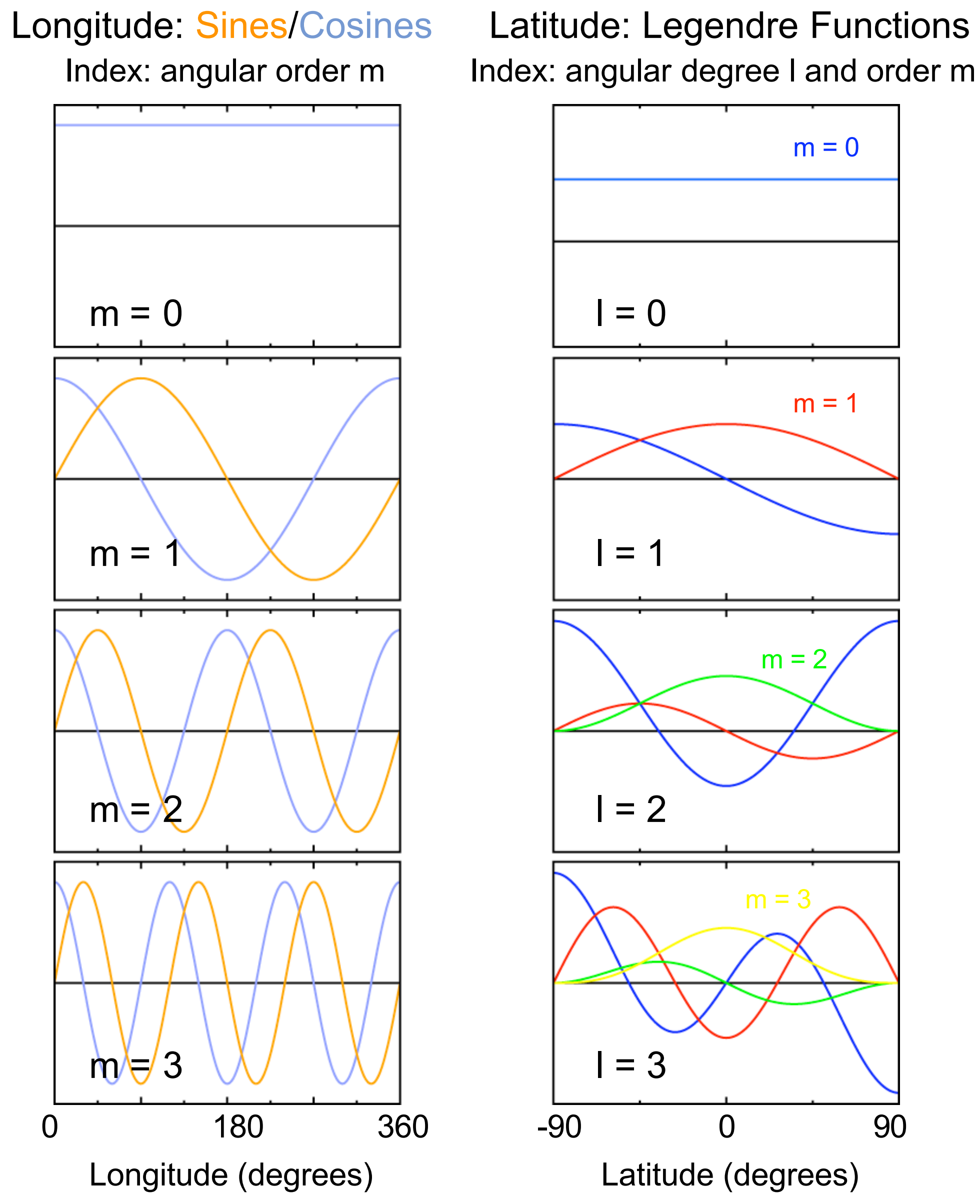
What are the standing waves or basis functions we can use to describe any 2-D functions on a sphere?

Basis Functions

2-D
(Spherical Surface)

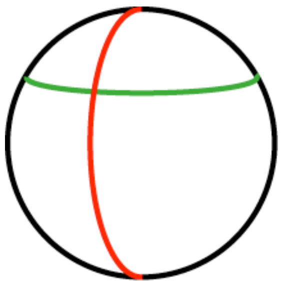


Rule: $-l \leq m \leq l$



Basis Functions

2-D
(Spherical Surface)



Rule: -l ≤ m ≤ l

$$Y_l^m$$

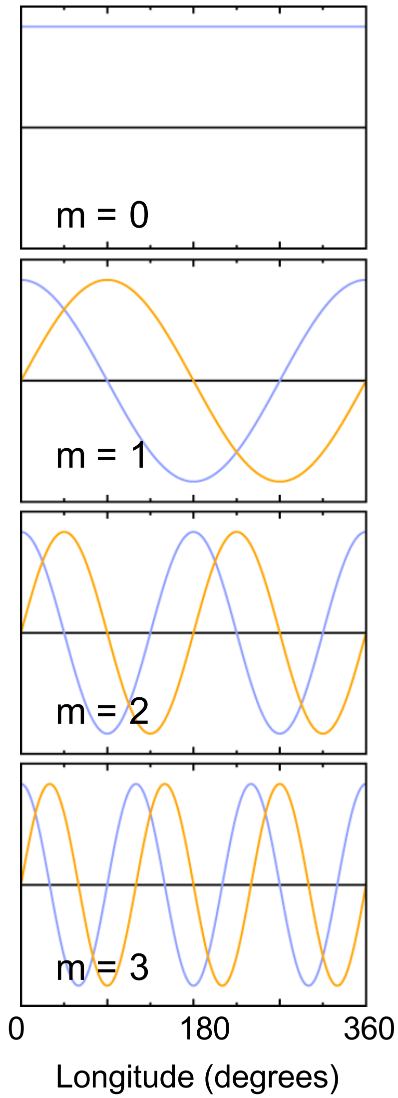
$$Y_0^0$$

$$Y_1^{-1} \quad Y_1^0 \quad Y_1^1$$

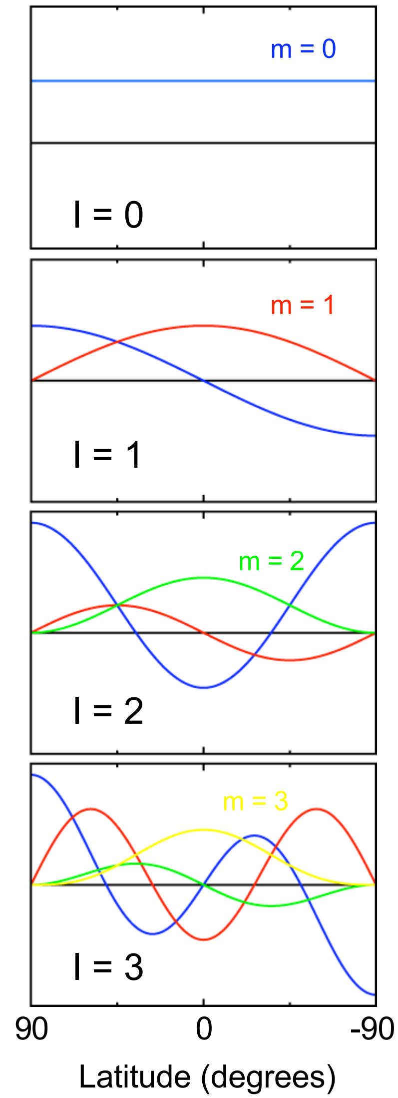
$$Y_2^{-2} \quad Y_2^{-1} \quad Y_2^0 \quad Y_2^1 \quad Y_2^2$$

$$Y_3^{-3} \quad Y_3^{-2} \quad Y_3^{-1} \quad Y_3^0 \quad Y_3^1 \quad Y_3^2 \quad Y_3^3$$

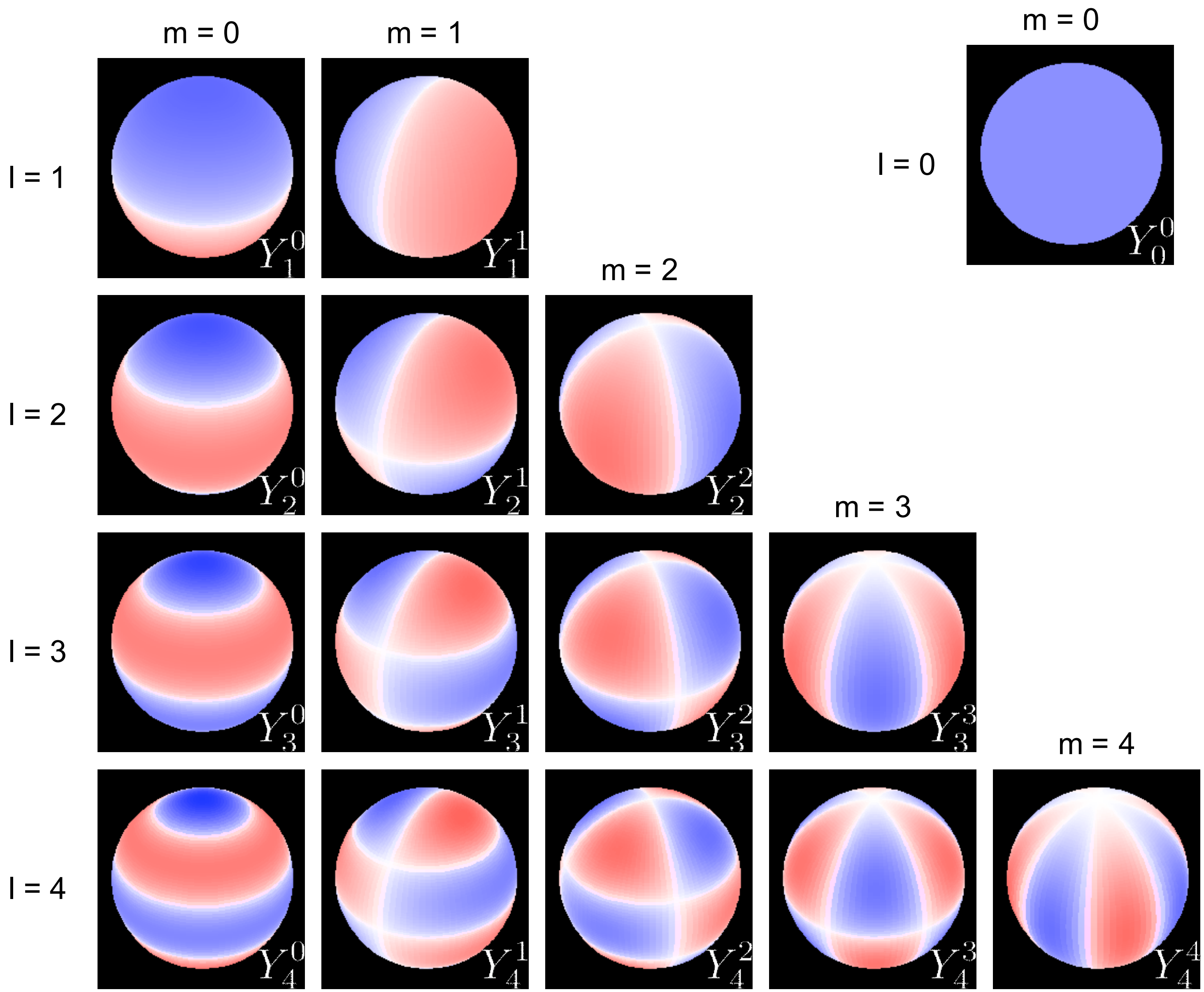
Longitude: Sines/Cosines
Index: angular order m



Latitude: Legendre Functions
Index: angular degree l and order m

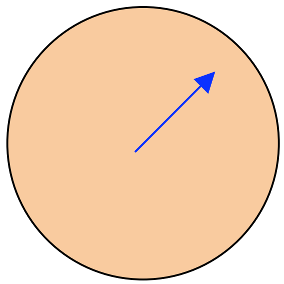


Spherical Harmonics



Basis Functions

3-D
(Sphere)

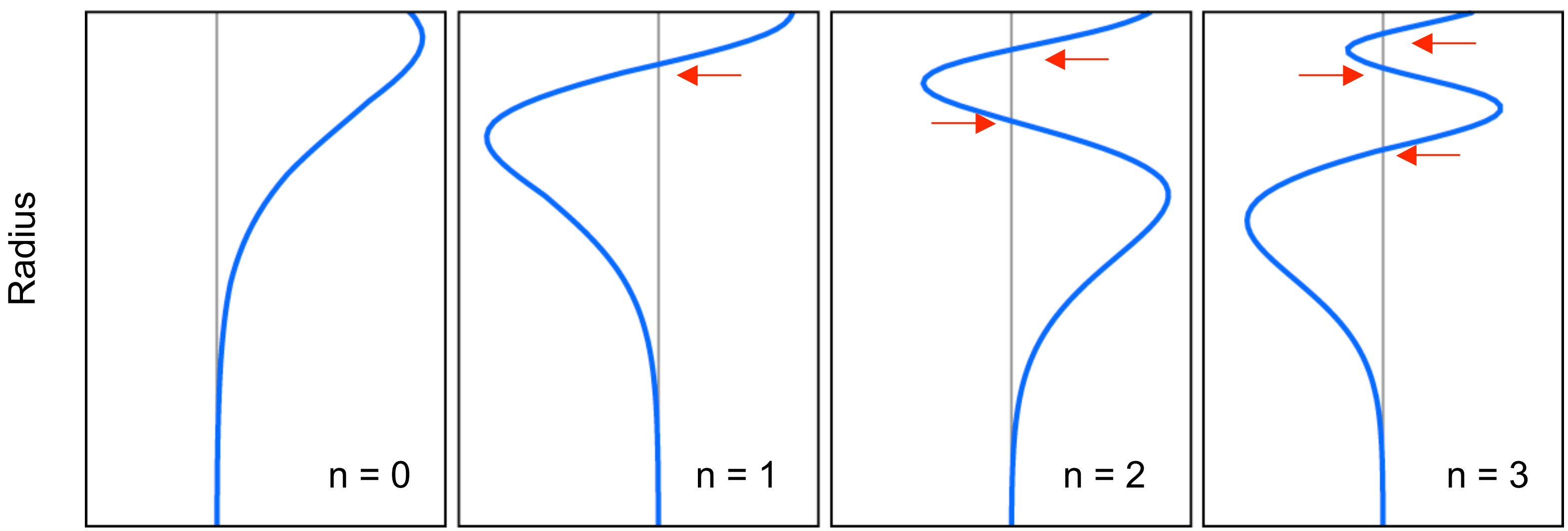


Spherical Surface: Spherical Harmonics

Indices: angular degree l and order m

Radius: Bessel Functions (homogeneous sphere)

Index: number of zero crossings n

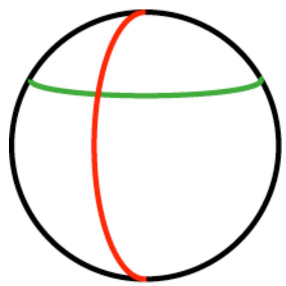


Basis Functions

1-D —— Sines and Cosines

Fourier Transform: combination of sines and cosines ➔ describe any 1-D function

2-D
(Spherical Surface)



Longitude: Sines and Cosines

Index: angular order m

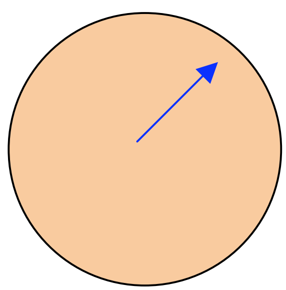
Rule: $-l \leq m \leq l$

Latitude: Legendre Polynomials

Index: angular degree l and order m

Spherical Harmonic Transform: combination of spherical harmonics
➔ describe any function on a spherical surface

3-D
(Sphere)



Spherical Surface: Spherical Harmonics

Indices: angular degree l and order m

Radius: Bessel Functions (homogenous sphere)

Index: number of zero crossings n

Normal Modes: combination of the Earth’s normal modes
➔ describe any motion

Normal Mode Nomenclature

Earth: Sphere

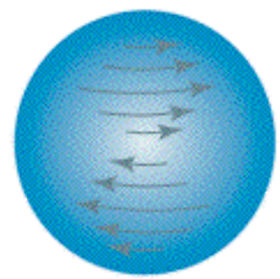
➔ Spherical Harmonics and radial function to describe standing waves

➔ Need three indices: n = radius; l = latitude; m = longitude

Type of Motion

Toroidal ➔ pure shear, denote by T

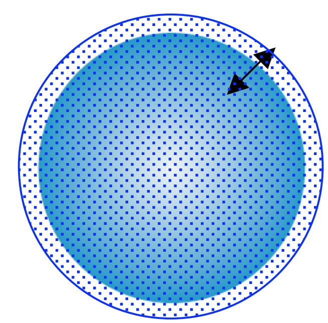
←→ SH waves, Love waves



$${}_nT_l^m$$

Spheroidal ➔ Combination of shear and change in shape
denote by S

←→ SV waves, Rayleigh waves



$${}_nS_l^m$$

Characteristic Frequency

Each mode has its characteristic frequency and decay constant.

Degeneracy

If Earth is

- Spherically symmetric
- Isotropic
- Non-rotating
- Laterally homogeneous

then $_n\omega_l^m = {_n\omega_l^{m'}}$

i.e., modes with same overtone number n, and angular degree l

➔ same characteristic frequency regardless of angular order m

➔ Mode names are often denoted $_nS_l$ and $_nT_l$

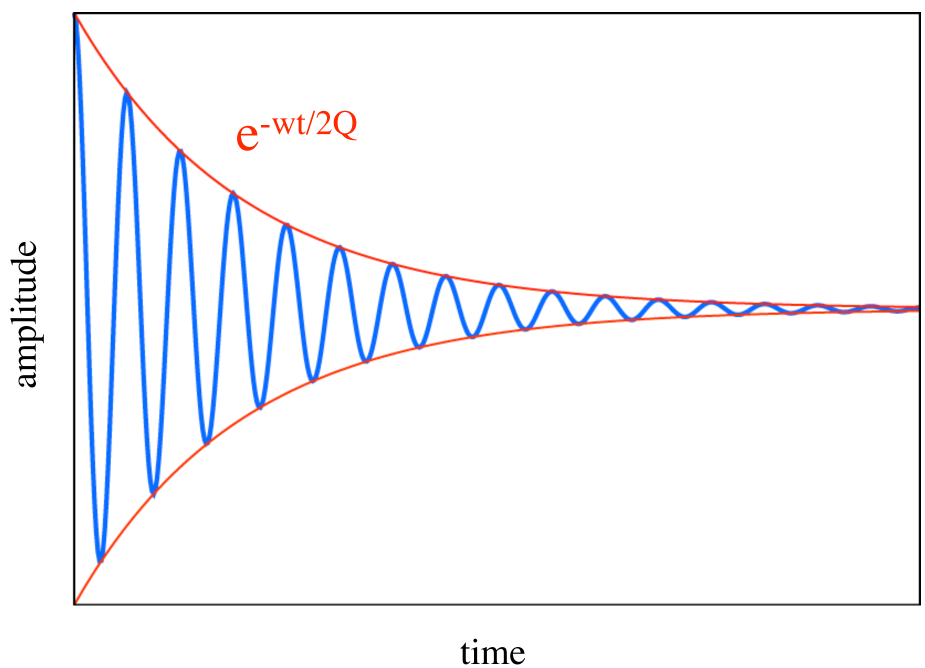
Attenuation

amount of energy dissipated

$$Q^{-1} \propto \frac{\Delta E}{E}$$

quality factor Q

initial energy



Seismograms

Combination of normal modes can describe ANY motion on spherical Earth

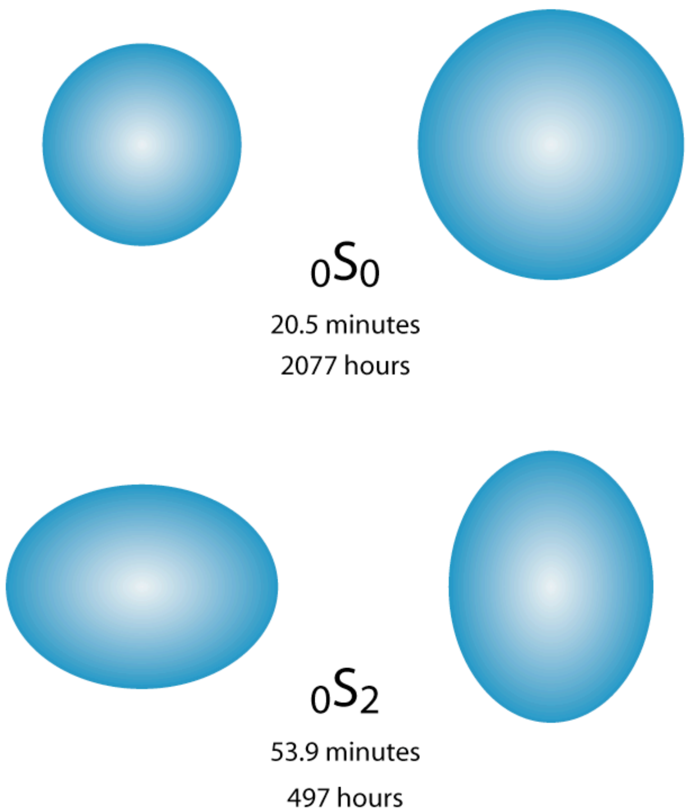
Seismogram observed at a station from a given earthquake

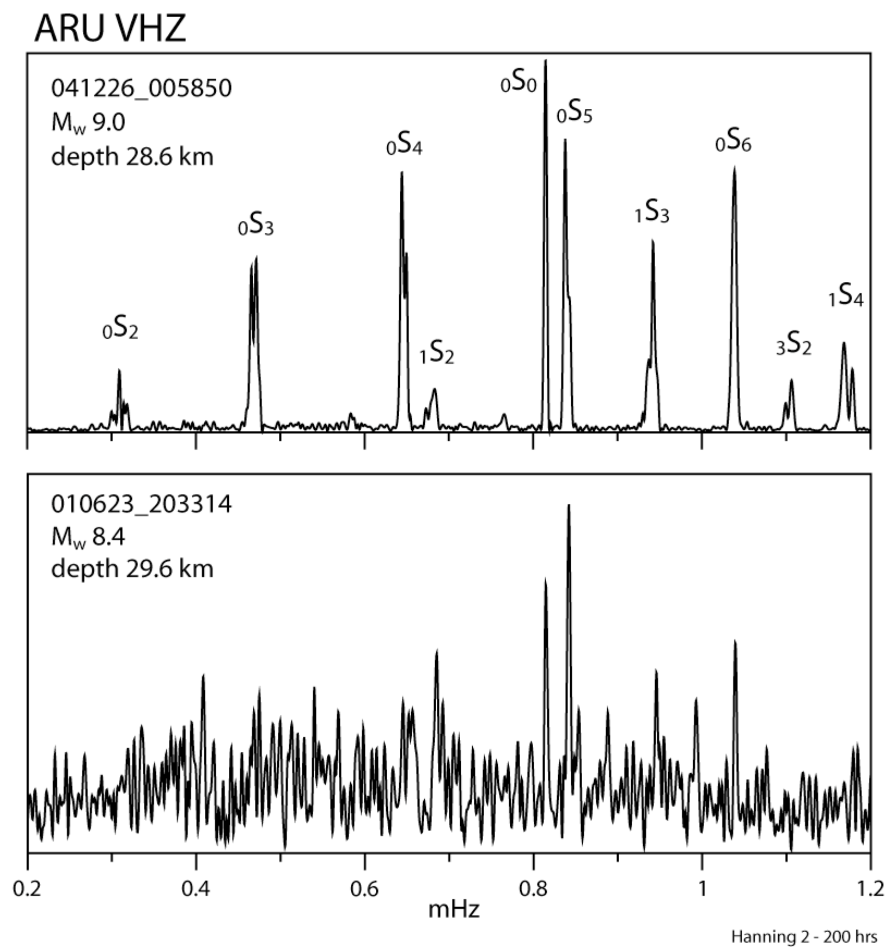
$$u(t) = \sum_{modes} \left[{}_nA_l^m \sin({}_n\omega_l\, t) + {}_nB_l^m \cos({}_n\omega_l\, t) \right]$$

Constants determined by earthquake source mechanism, and station/hypocentral locations.

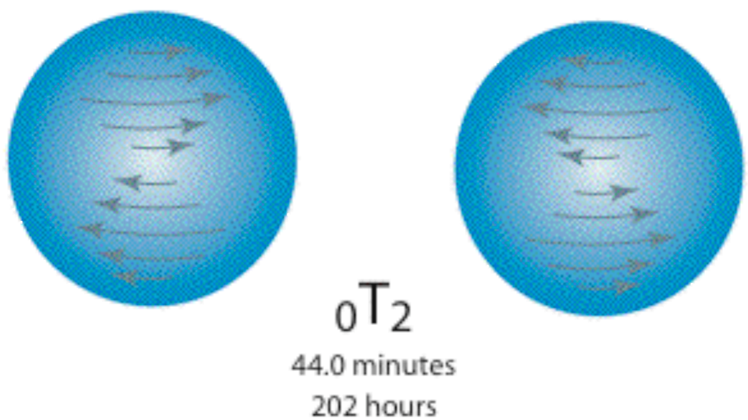
➔ Fourier Transform

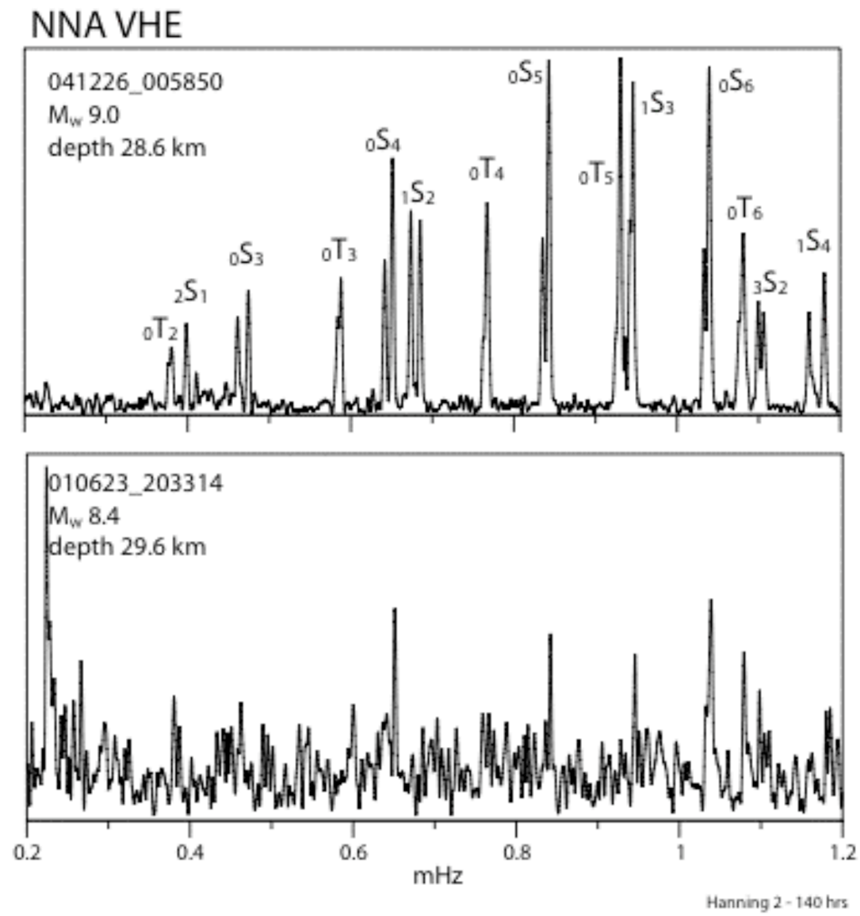
Earth's Free Oscillations (Spheroidal Mode)



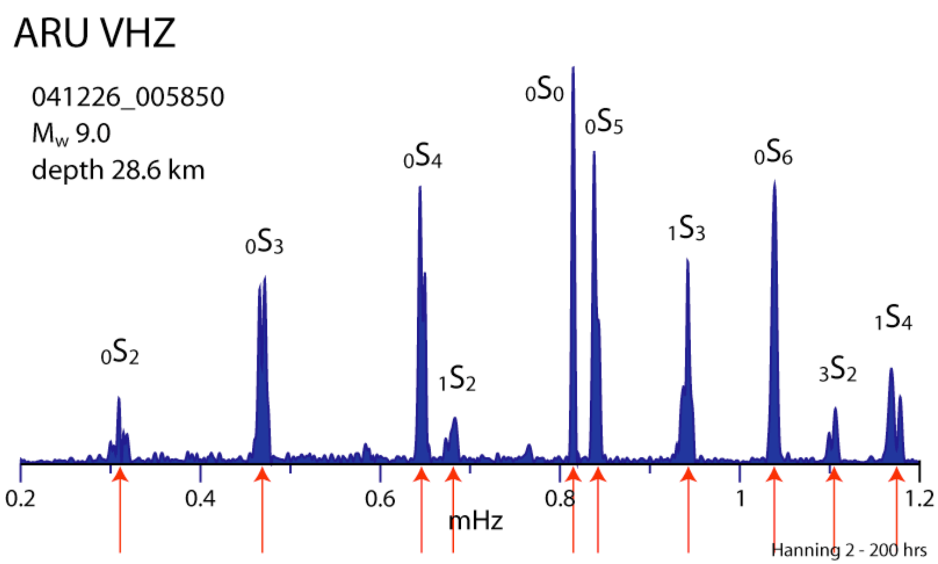


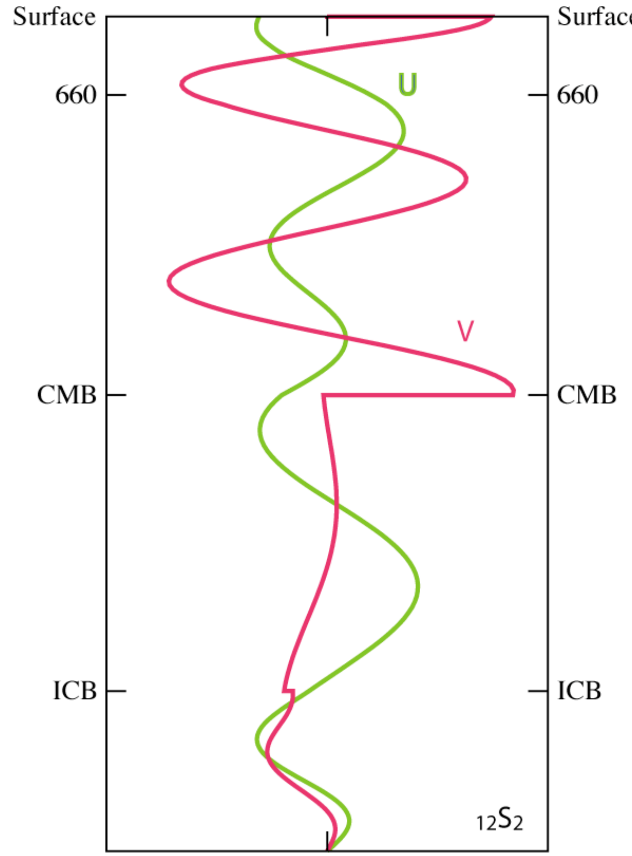
Earth's Free Oscillations (Toroidal Mode)



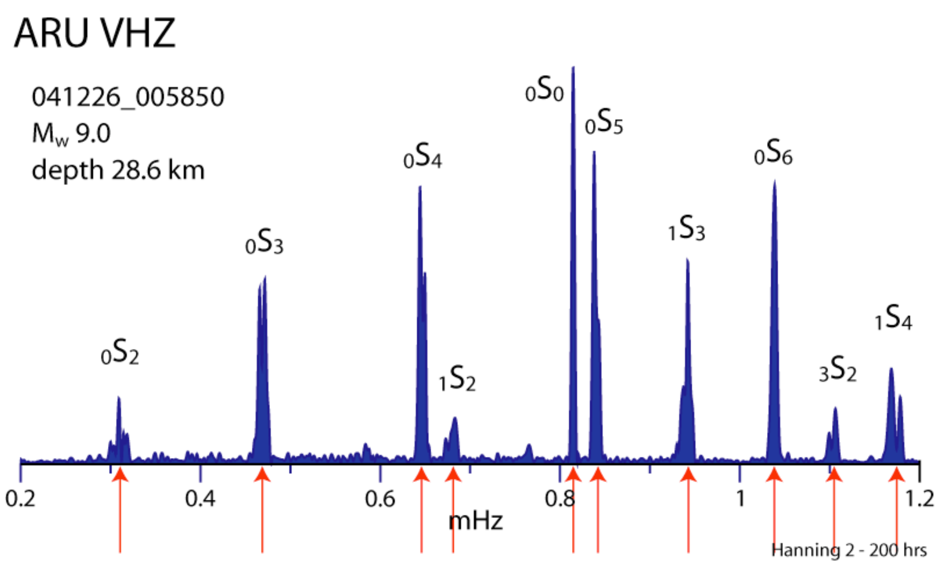


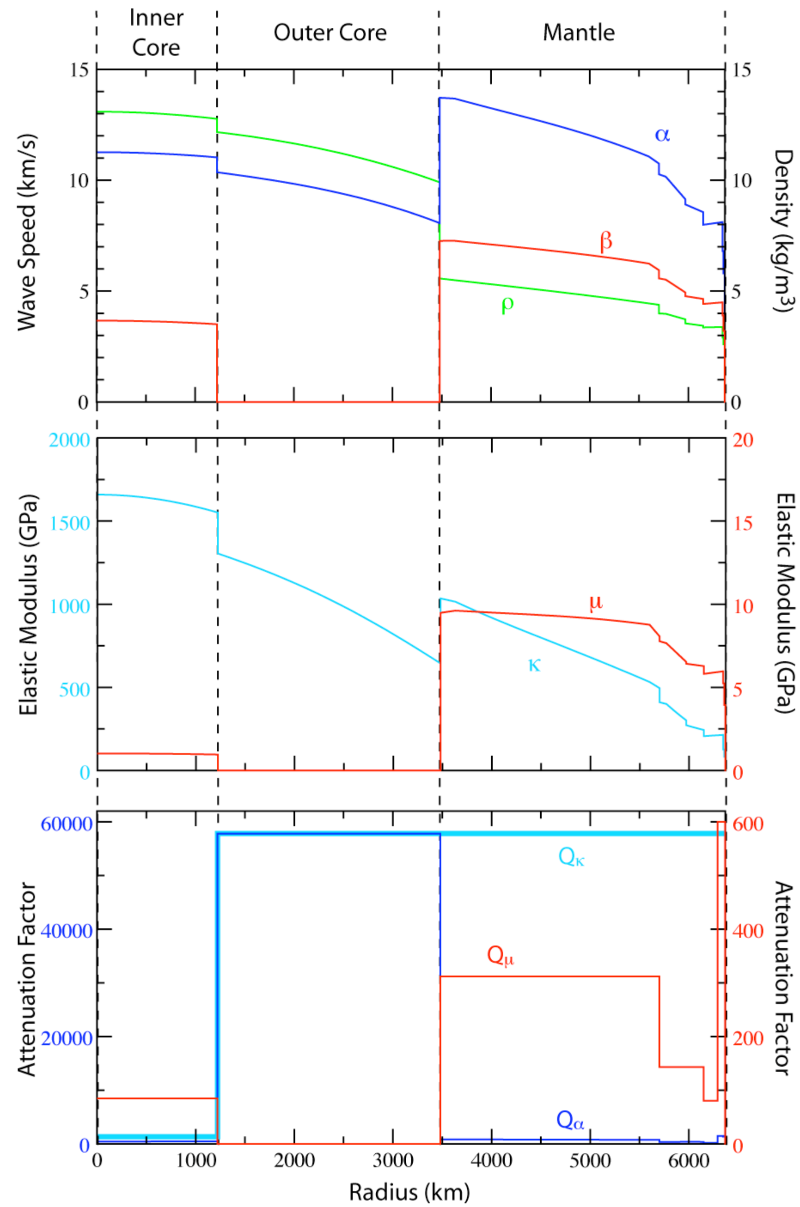
Normal-Mode Central Frequency





Normal-Mode Central Frequency





Synthetic Seismograms

Combination of normal modes can describe ANY motion on spherical Earth

➔ Generate synthetic seismograms

Seismogram observed at a station from a given earthquake

$$u(t) = \sum_{modes} \left[{}_nA_l^m \sin({}_n\omega_l\, t) + {}_n B_l^m \cos({}_n\omega_l\, t) \right]$$

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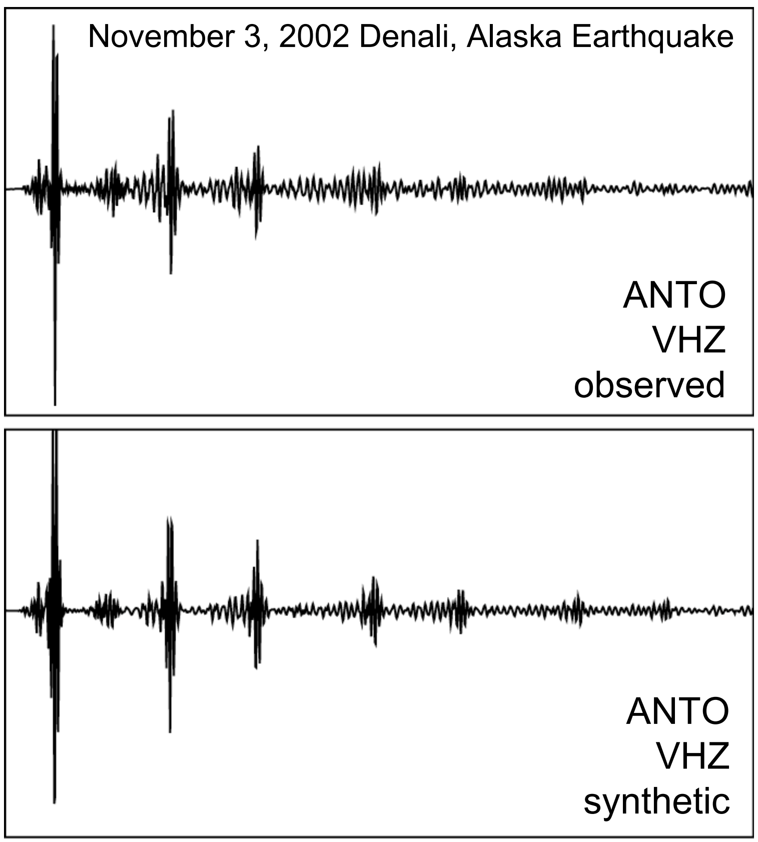
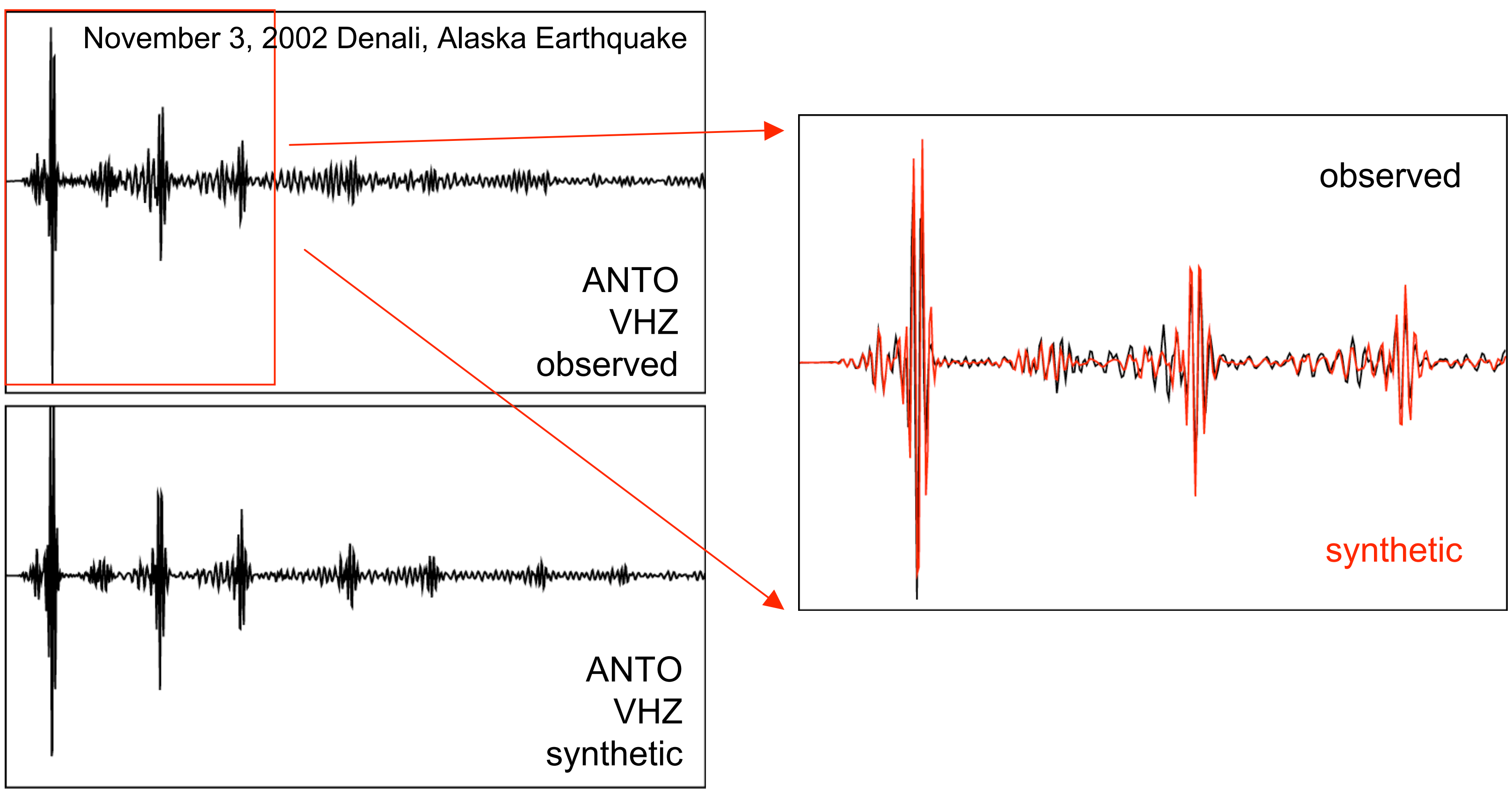


Filtered between 3 and 8 mHz

Synthetic Seismograms

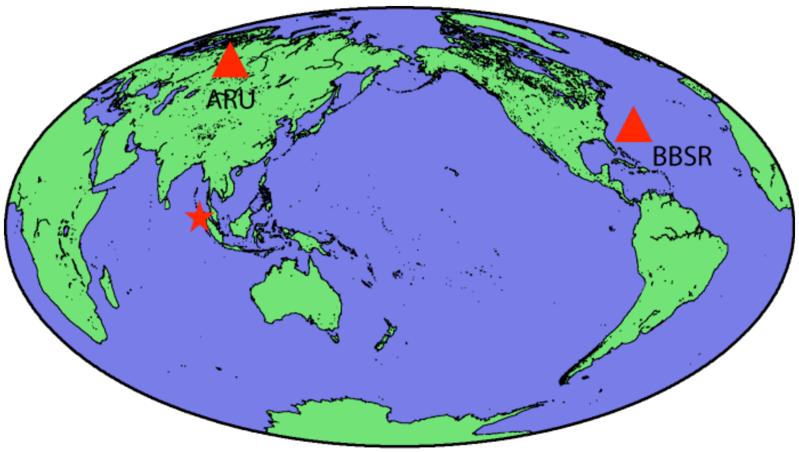
Combination of normal modes can describe ANY motion on spherical Earth

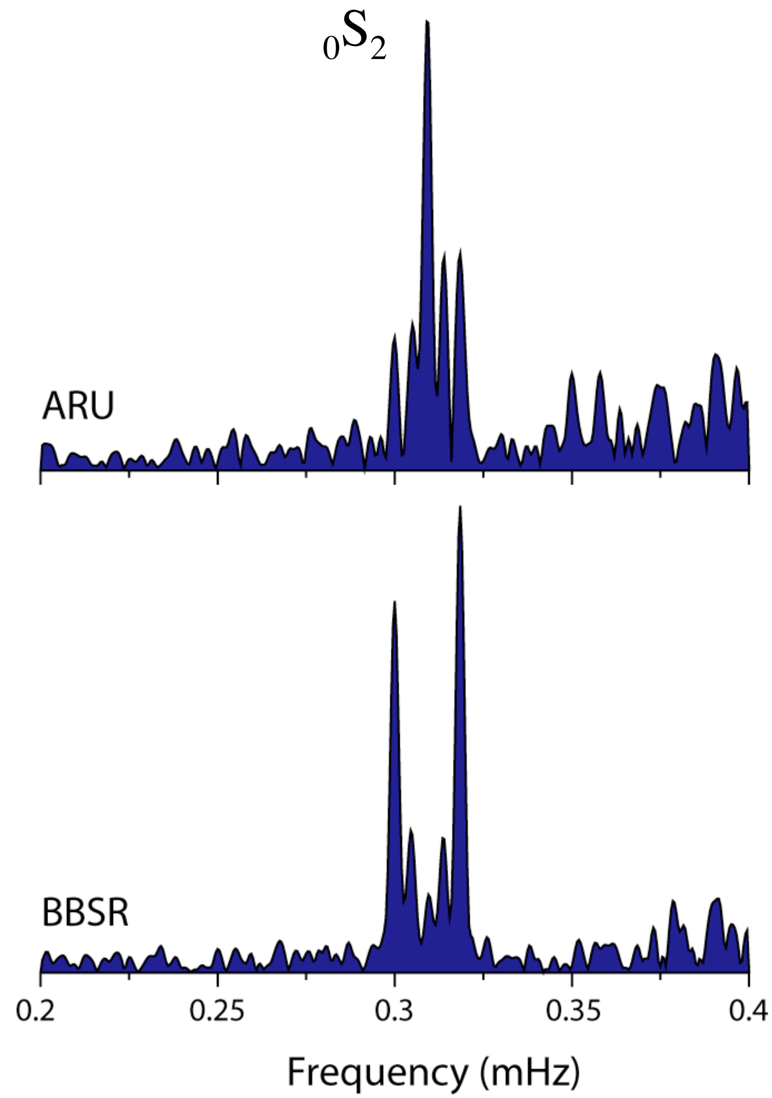
➔ Generate synthetic seismograms



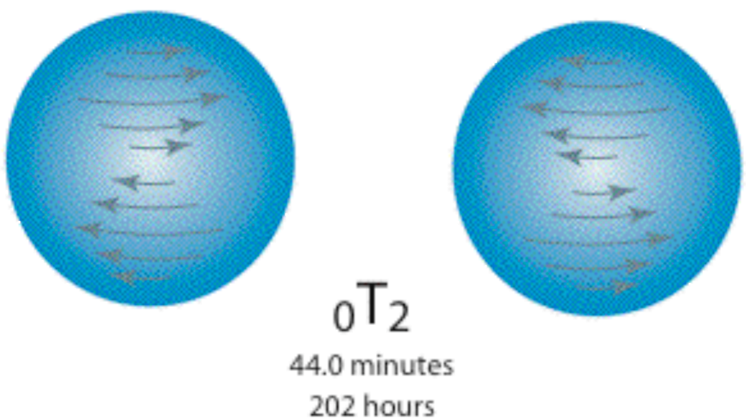
Filtered between 3 and 8 mHz

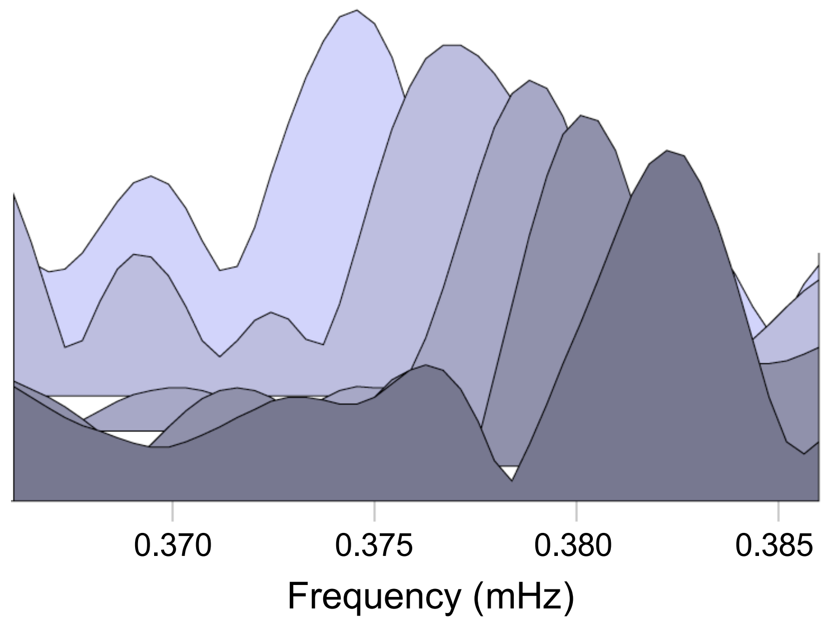
Normal-Mode Splitting





Earth's Free Oscillations ($_0T_2$ Receiver Strips)





Earth's Free Oscillations (Spheroidal-Toroidal Coupling)