Beamforming frequency

$$p(f) = \int p(t) e^{-i2\pi ft} dt \qquad \text{FFT}$$

$$p(t) = \int p(f) e^{i2\pi ft} df \qquad \text{IFFT}$$

Pressure field is a sum of plane waves

$$p(f,\mathbf{r}) = \int p(f,\mathbf{k}) e^{i(\mathbf{k}^T\mathbf{r})} d\mathbf{k}$$

$$p(f,\mathbf{k}) = \int p(f,\mathbf{r}) e^{-i(\mathbf{k}^T\mathbf{r})} d\mathbf{r}$$

Based of the observed field $p(f,\mathbf{r}_k)$ at discrete ranges $\mathbf{r}_k$ the $p(f,\mathbf{k}_j)$ is estiamted

$$p(f,\mathbf{k}_j) = \sum_k p(f,\mathbf{r}_k) e^{-i(\mathbf{k}_j^T\mathbf{r}_k)} = \mathbf{w}^H\mathbf{p}$$

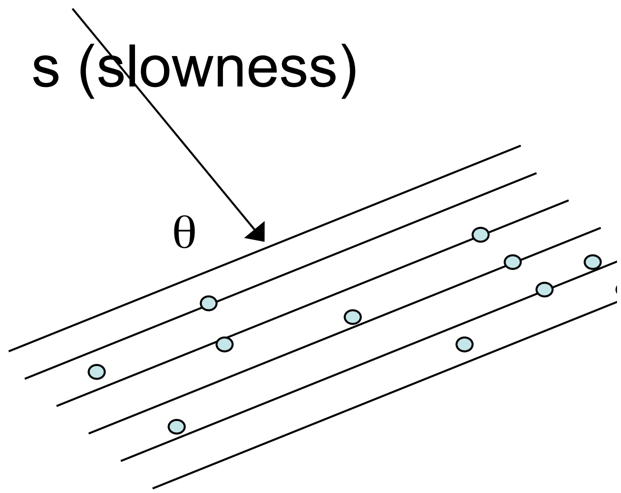
Where

$$\mathbf{w} = \begin{bmatrix} e^{i(\mathbf{k}_j^T\mathbf{r}_1)} \\ \vdots \\ e^{i(\mathbf{k}_j^T\mathbf{r}_N)} \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} p(f,\mathbf{r}_1) \\ \vdots \\ p(f,\mathbf{r}_N) \end{bmatrix}$$

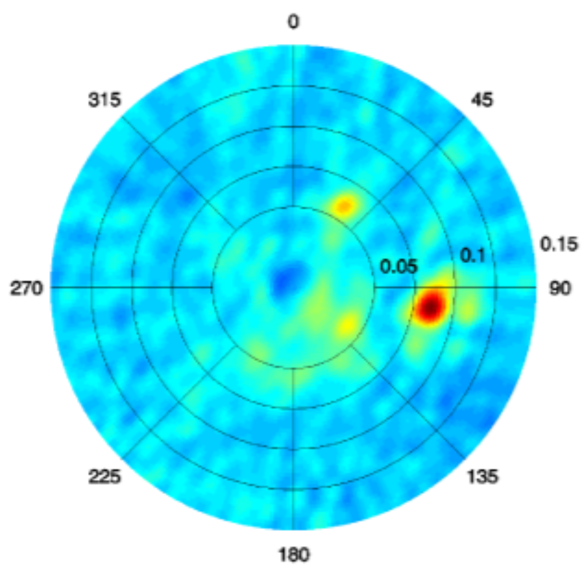
Phase slowness

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \qquad \mathbf{k} = \omega \mathbf{s} \qquad \mathbf{s} = \begin{bmatrix} s_x \\ s_y \end{bmatrix} = s \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$s_x^2 + s_y^2 = s_{Horizontal}^2 = \frac{1}{c_H^2}$$

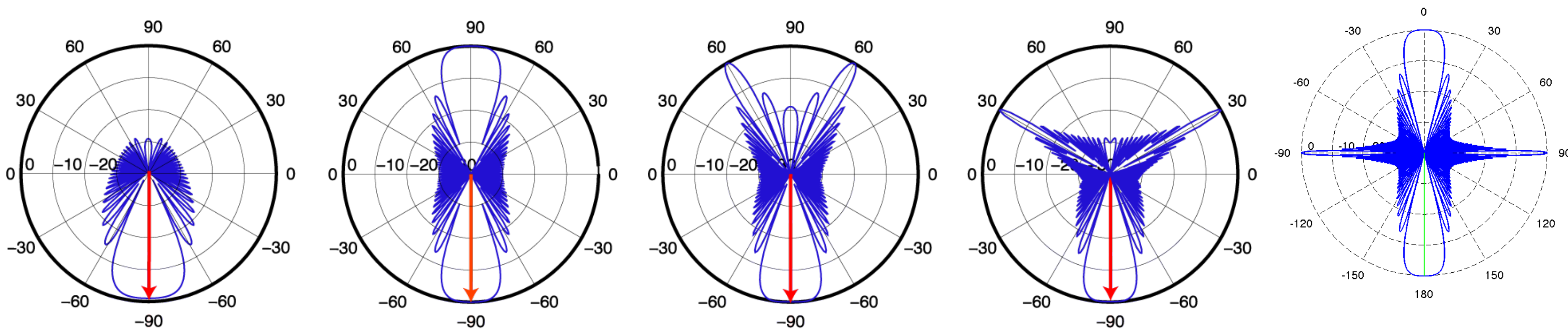


What is the horizontal wave speed
- for R, L?
- Body waves, and what does it tell



Aliazing

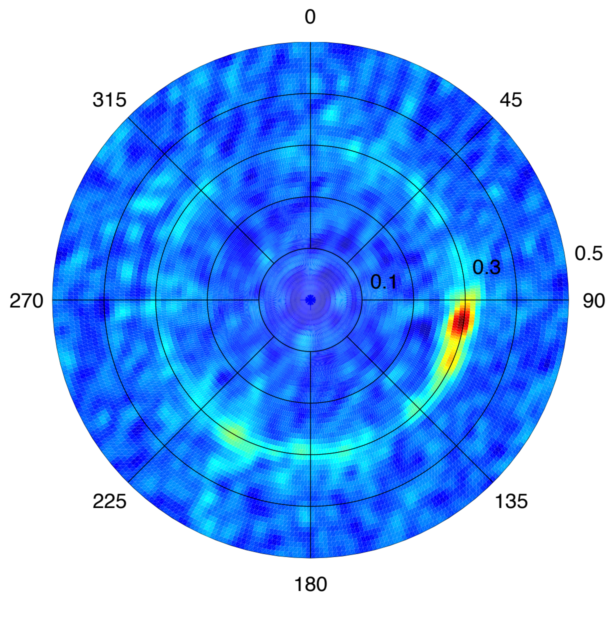
$d < \lambda / 2$ $\quad$ $d = \lambda / 2$ $\quad$ $d > \lambda / 2$ $\quad$ $d > \lambda / 2$ $\quad$ $d = \lambda$



Surface and P-wave on 29 Aug 2005

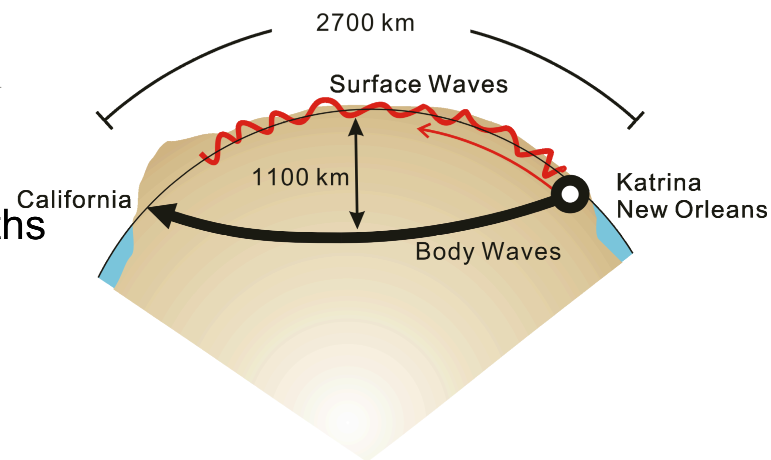
Surface wave

0.1Hz: 3km/s,95 deg



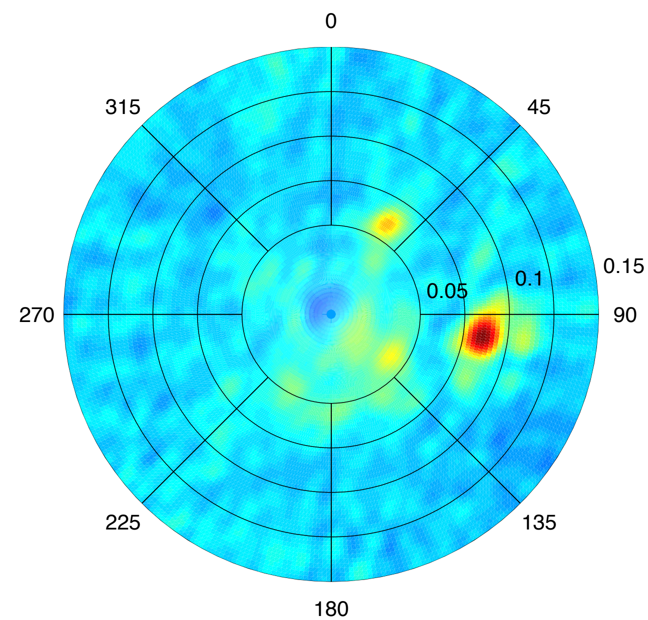
Slowness 0-0.5 s/km
(velocity inf-2km/s)

Microseisms from all azimuths



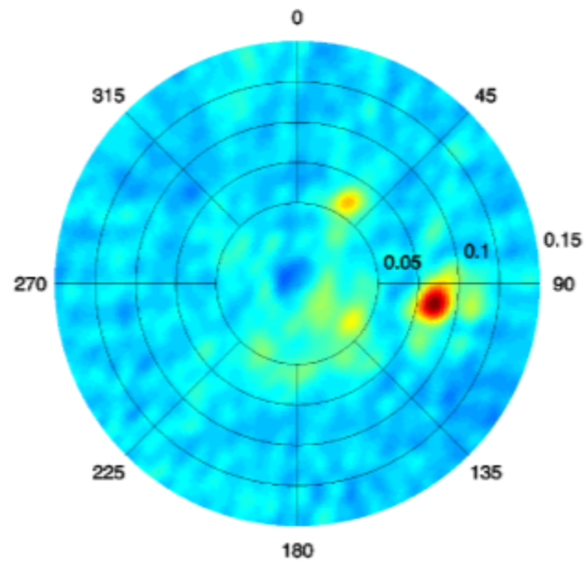
Body P-wave

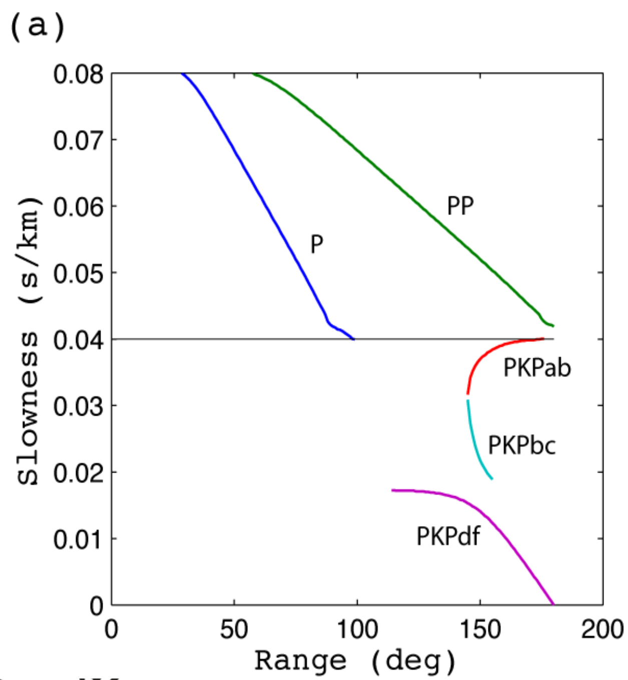
0.2Hz: 12km/s, 95 deg

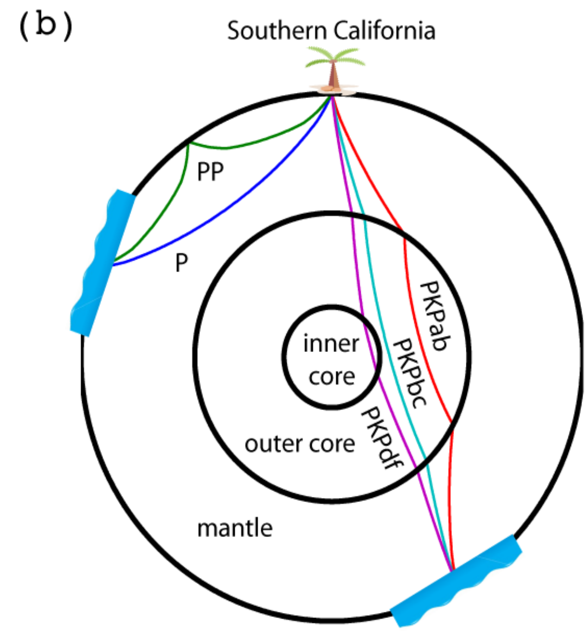


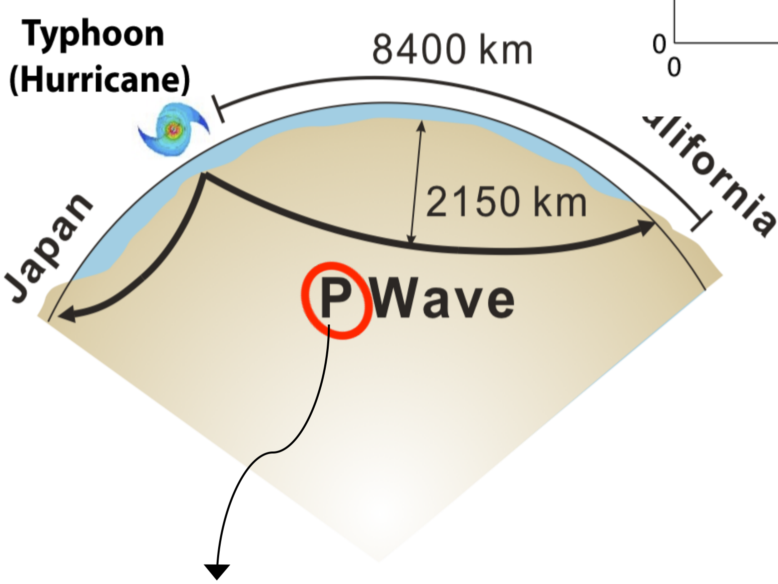
Slowness 0-0.15 s/m
Velocity (inf-6.6Km/s)

Beamformer Output+ travel time table =location



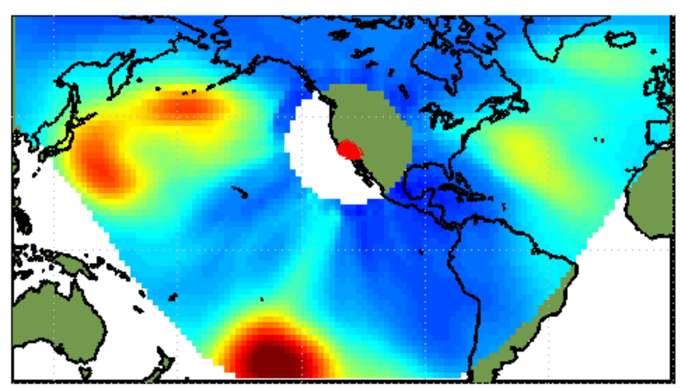






"Primary" or "Pressure"

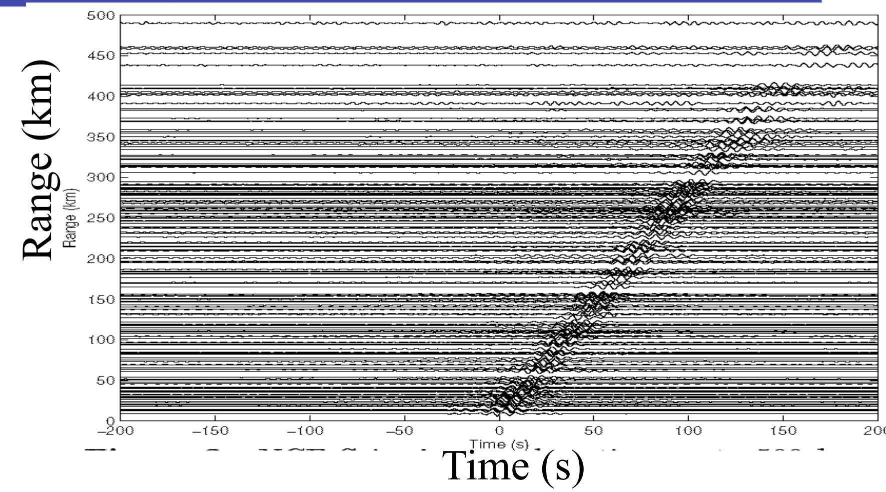
Microseism Source



Beamforming

$$p(f) = \int p(t) e^{-i2\pi ft} dt \qquad \text{FFT}$$

$$p(t) = \int p(f) e^{i2\pi ft} df \qquad \text{IFFT}$$



Pressure field is a sum of plane waves

$$B(t,m) = \sum_k p_k(t - \tau_{km})$$

$$= \sum_k \int [\int p_k(t - \tau_{km}) e^{-i2\pi ft} dt] e^{i2\pi ft} df$$

$$= \sum_k \int e^{-i2\pi f \tau_{km}} [\int p_k(t) e^{-i2\pi ft} dt] e^{i2\pi ft} df$$

$$= \sum_k \int e^{-i2\pi f \tau_{km}} p_k(f) e^{i2\pi ft} df$$

$$B(f,m) = \sum_k e^{-i2\pi f \tau_{km}} p_k(f) = \mathbf{w}^H \mathbf{p}$$

Where

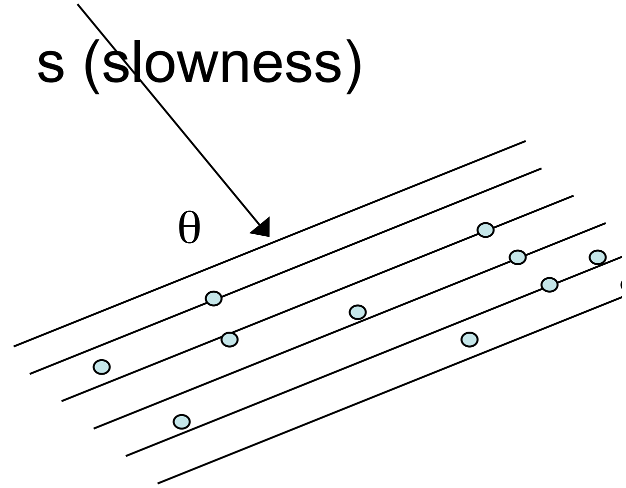
$$\mathbf{w} = \begin{bmatrix} e^{i2\pi f \tau_{km}} \\ \vdots \\ e^{i2\pi f \tau_{km}} \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p(f, \mathbf{r}_1) \\ \vdots \\ p(f, \mathbf{r}_N) \end{bmatrix}$$

Seismic beamforming

Observed signal on N stations is split into N_{time} snapshots (~10 min),

FFTed,

spectral whitining (only phase is retained)



Cross spectral density matrix

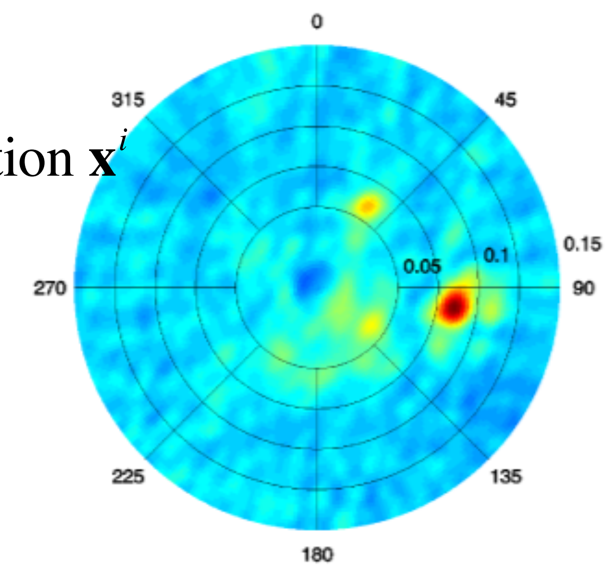
$$\mathbf{R}(\omega) = \frac{1}{N_{time}} \sum_{i}^{N_{time}} \mathbf{p}_i(\omega)\mathbf{p}_i^H(\omega)$$

Plane wave replica

$$\mathbf{w} = \exp\left[i\omega(\mathbf{x}\cdot\mathbf{s})\right] = \exp\left[i\omega s(x_1\cos\theta + x_2\sin\theta)\right]$$

Beamforming, using observed signal $u_i(\omega)$ at sensor location $\mathbf{x}^i$

$$B(\omega) = \mathbf{w}^H(\omega)\mathbf{R}(\omega)\mathbf{w}(\omega) \sim |\mathbf{w}^H(\omega)\mathbf{p}(\omega)|^2$$



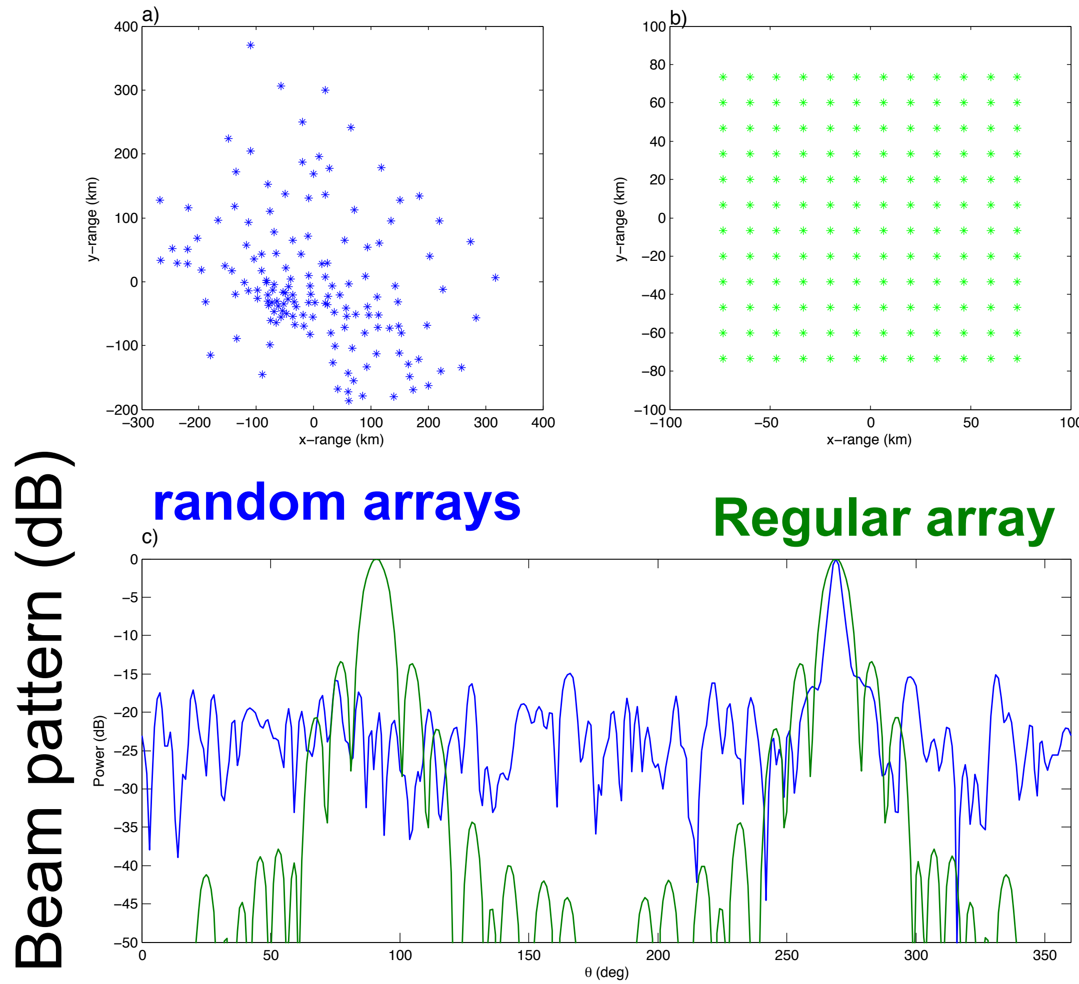
Beamformer depends on

- time, frequency, azimuth and phase slowness
- Using slowness, P-waves can give location.

Beamforming advantages

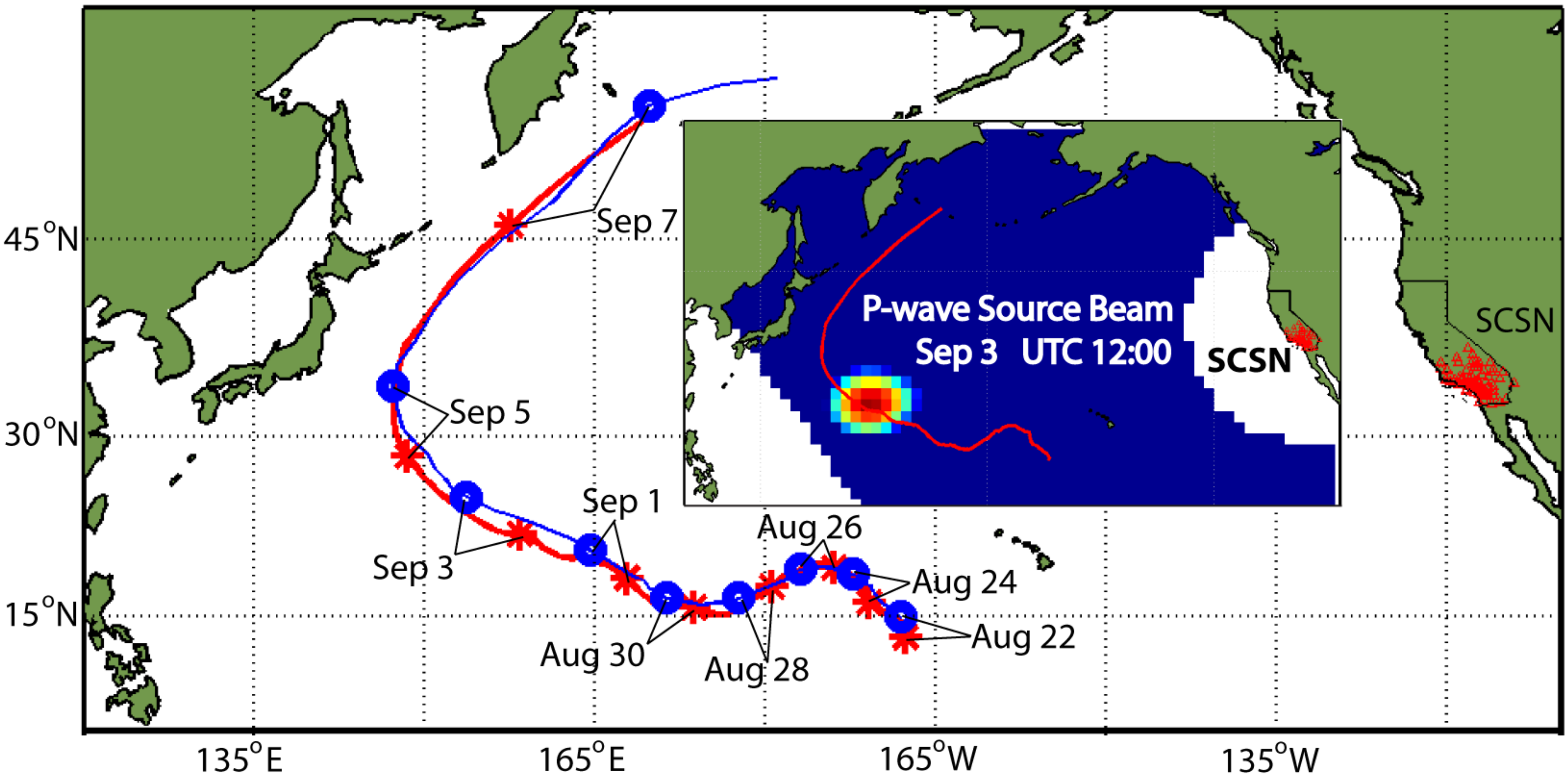
- Solid mathematical and signal processing framework
- Regular array: spacing **d<λ/2** and beampattern decays~ 6dB/sidelope
- Random arrays (like SCSN or Hi-Net) are not sensitive to the array spacing, but velocity inhomogenities is a concern and

$$B(\phi) \rightarrow \frac{1}{N}$$



Tracking Tropical Cyclones

Zhang, Gerstoft, and Bromirski (GRL, 2010)



P-wave microseisms →

- Evidencing nonlinear wave-wave interactions in the deep ocean (Longuet-Higgins, 1950)
- Note: Tracking wave-wave interactions rather than a storm itself