Compressive MIMO Beamforming of Data Collected in a Refractive Environment

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Abstract.

The phenomenon of ducting is caused by abnormal atmospheric refractivity patterns, and is known to allow electromagnetic (EM) waves to propagate over the horizon with unusually low propagation loss. It is unknown what effect ducting has on multiple input multiple output (MIMO) channels, particularly its effect on multipath propagation in MIMO channels. A high accuracy angle-of-arrival (AoA) and angle-of-departure (AoD) estimation technique for MIMO communications, which we will refer to as compressive MIMO beamforming, was tested on simulated data, then applied to experimental data taken from an over the horizon (OTH) MIMO testbed located in a known ducting hotspot in southern California. The multipath channel was estimated from the receiver data recorded over a period of 18 days, and an analysis was performed on the recorded data. The goal is to observe the evolution of the MIMO multipath channel as atmospheric ducts form and dissipate to gain some understanding of the behavior of channels in a refractive environment. This work is motivated by the idea that some multipath characteristics of MIMO channels within atmospheric ducts could yield important information about the duct.

Keypoints:

• MIMO channel estimation and beamforming
• Influence of refractivity on MIMO channel
• Evolution of MIMO channel in refractive environments
1. Introduction

For long range wireless electromagnetic (EM) communication signals, the line of sight path between transmitter and receiver can be obscured by the curvature of the earth. Generally signal strength at the receiver fades quickly with range for over the horizon signals, but an exception can be made for marine environments experiencing atmospheric ducting [2, @]. Ducting causes trapping of EM waves propagating at low angles between the ocean’s surface and the top of the duct [4, @]. Ducts are formed by certain atmospheric refractivity profiles, where refractivity is a measure of the deviation of an electromagnetic wave from a straight line path due to variation in air density as a function of height. In a ducting environment an EM wave can be guided over the horizon through a path that it would not otherwise travel [6, @]. This effect is known to decrease signal attenuation over long ranges, but may also cause multipath interference in the form of multipath fading [8, @].

In a ducting environment it is hypothesized that communication channels may have higher throughput [10, @], and may also take on unique properties based on the duct. To test this hypothesis, an experimental testbed was set up where a 4x4 multiple input multiple output (MIMO) communications array was placed on vertical masts 41 km apart in a ducting hotspot in southern California [2, @], see figure 1. Pilot signals from the transmitters were recorded by the receivers at regular intervals for 18 days. The goal of the experiment was to observe the change in the multipath environment for an over the horizon channel during ducting events. Ducting events were identified by significant increases in received power levels at the receiver array.

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We are unaware of any other experiment to date having been performed for the purpose of relating the multipath characteristics of a MIMO channel to atmospheric refractivity. While the over the horizon MIMO channel data that was collected is unique, it is also limited in that it is representative of one static channel which may not be representative of all ducting channels. Our hope is that a better understanding of the impact of ducting on MIMO communications channels will lead to more effective refractivity estimation techniques. For instance if the multipath channel takes on specific properties exclusively during ducting events, this information could be used to more accurately identify the presence and properties of a duct.

In this paper we use compressive MIMO beamforming to determine the angle-of-arrival (AoA) and angle-of-departure (AoD) of signals traveling through a channel. Compressive MIMO beamforming (detailed in section 3.2) is a precision beamforming technique utilizing compressed sensing, and is based on the assumption of signal sparsity [12, @], [14, @] in the transmit and receive angle domain. To obtain a best estimate of the MIMO channel matrix (see section 2), [16, @], [18, @], a set of optimal training sequences [20, @] (see section 5) were sent from the transmitter array. From the channel matrix a virtual array [22, @] is formed, to which compressive MIMO beamforming can be applied to obtain the transmit and receive angles of the signal.

The structure of the paper is as follows. In section 2 the signal model is explained and theoretical framework for channel estimation is laid out. Optimal channel estimation is essential to this work because knowledge of the MIMO channel is required for beamforming. Section 3 gives the theory behind compressive sensing and lays the framework for MIMO beamforming. Section 4 gives compressive MIMO beamforming error rates in...
two metrics for simulated signals of varying signal to noise ratio (SNR) and number of active signal paths. Section 5 describes the experimental testbed and details regarding the transmitted waveforms and their detections. Experimental data is processed and the resulting multipath channels and channel properties are analyzed in section 6.

2. Multipath Signal Model

2.1. MIMO Signal Model

MIMO systems are a well studied topic in antenna communications and offer several benefits over single input and output systems. A MIMO system is defined as an antenna setup utilizing multiple transmitters and receivers, often positioned in an array. By increasing the number of transmitters and receivers the dimension of the channel matrix, whose elements represent the transfer function between each transmitter and receiver, is increased. Higher dimensional channel matrices can be exploited by spatially multiplexing multiple streams of information [24, @]. Having multiple receivers allows for spatial processing (more generally known as beamforming) to estimate the receive angle of arriving signals, while multiple transmitters allows for spatial processing for estimation of the transmit angle of arriving signals.

The position of transmitters and receivers in a MIMO system is flexible, though this paper will focus on transmit and receive elements positioned in uniform linear arrays (ULA). Consider $N_T$ transmitters and $N_R$ receivers with element spacing $r$ and $s$, communicating over a single frequency. The set of transmitters send signals $\mathbf{x}_t \in \mathbb{C}^{N_T}$ at time sample time $t = 1 \ldots T$. Each of $P$ paths have a unique AoD $\phi_p$, AoA $\theta_p$, and corresponding complex path gain.
Assuming a time invariant channel, the received signal at time sample $t$, $y_t \in \mathbb{C}^{N_R}$ is

$$y_t = Hx_t + w_t,$$  \hspace{1cm} (1)

where $w_t \in \mathbb{C}^{N_R}$ is a zero mean symmetric complex Gaussian noise vector at time $t$ and $H \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix which can be further decomposed 12 \cite

$$H = \sum_{p=1}^{P} a_p a_R(\theta_p) a_T(\phi_p)^H,$$  \hspace{1cm} (2)

where $a_p$ is the complex gain of path $p$, $a_R(\theta) \in \mathbb{C}^{N_R}$, and $a_T(\phi) \in \mathbb{C}^{N_T}$ are the receive and transmit steering vectors for angles $\theta$ and $\phi$, whose $i$th element is defined as $a_{T,i}(\theta) = \exp[-j2\pi r \sin(\theta)i/\lambda]$ for $i = [0, \ldots, N_T - 1]$ and $a_{R,i}(\theta) = \exp[-j2\pi s \sin(\theta)i/\lambda]$ for $i = [0, \ldots, N_R - 1]$ with $j = \sqrt{-1}$, $s$ and $r$ are transmitter and receiver array spacings in meters, and $\lambda$ is the carrier wavelength in meters 26 \cite. Eq (2) implicitly assumes that the received signals arrive as plane waves at the receiver.

The channel described by eq (1) and (2) can be equivalently viewed as the sum of channels from every possible signal path. Rather than express the channel in (2) as the sum of array responses from each of $P$ paths, it can also be expressed as the sum of array responses from every possible path over the space of possible transmit and receive angles $(\theta, \phi)$, only a few of which will be active (take non-zero values). Formally,

$$H = \int_{\phi} \int_{\theta} H^a(\theta, \phi) a_R(\theta) a_T(\phi)^H \, d\theta \, d\phi,$$  \hspace{1cm} (3)

where $H^a(\theta, \phi)$ is a function representing the path gain from transmit and receive angles $\theta$ and $\phi$. Since we have made the assumption that $H^a(\theta, \phi)$ is non-zero at only a few points, an approximate discretization can be made

$$H \approx \sum_{i}^{Q_R} \sum_{j}^{Q_T} H_{i,j}^a a_R(\theta_i) a_T^H(\phi_j),$$  \hspace{1cm} (4)

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where \( Q_R \) and \( Q_T \) are the number of points to which the transmit and receive angle space are quantized. The approximation of eq (4) replaces the continuous function \( H^a(\theta, \phi) \) with sparse matrix \( H^a \in \mathbb{C}^{Q_R \times Q_T} \), (having exactly \( P \) non-zero entries) which we will refer to as the angular domain channel matrix [28, @]. Plugging (4) into (1) we have

\[
y_t \approx \sum_{i}^{Q_R} \sum_{j}^{Q_T} H_{i,j}^a a_R(\theta_i) a_T^H(\phi_j) x_t + w_t, \tag{5}
\]

which is an alternative signal model utilizing sparsity. The focus of this paper is the estimation of \( H^a \). From known \( x_t \) and measurements of \( y_t \) we estimate \( H \), then using sparse processing detailed in section 3 an estimate of \( H^a \) is constructed from \( H \). Once recovered, we analyze the transmit and receive angles traveled by multipath signals propagating through ducting environments to determine any special properties they may have which could be used to characterize ducts.

The elements of the steering vectors \( a_R(\theta) \) and \( a_T(\phi) \) are unit magnitude complex values representing the relative phase shift of the signals arriving or departing from one antenna with respect to a reference antenna. From their definitions, the phase shift between vector elements is dependent on carrier signal wavelength, arrival/departure angle, and inter element array spacing. Ideally inter element spacing should be one half the carrier signal wavelength, which results in maximum angular resolution of \( \pm 90^\circ \) and is known as critical spacing. Sparsely spaced array elements result in angle ambiguities caused by aliasing, while densely spaced arrays have smaller array aperture leading to lower angular resolution. For sparsely spaced arrays, \( \theta_{\text{max}} = \sin^{-1}\left(\frac{\lambda}{2r}\right) \) where \( \theta_{\text{max}} \) is the maximum resolvable receive angle, \( r \) is the inter element spacing, and \( \lambda \) is the wavelength of the carrier signal in meters.

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2.2. Channel Estimation

Compressive MIMO beamforming takes the estimated channel matrix $\mathbf{H}$ as input. The estimate, $\hat{\mathbf{H}}$ (where $\hat{}$ indicates an estimate) must be made from the known transmitted and measured received signals. In this section we derive criteria for the transmitted waveform such that the resulting channel estimate is optimal. Later in section 5 the waveform transmitted from the experimental testbed is described and can be shown to satisfy the criteria for optimal channel estimation.

Consider a matrix $\mathbf{X} \in \mathbb{C}^{N_T \times T}$ whose columns $\mathbf{x}_t$ represent the symbols sent from all transmitters at time $t = [1, \ldots, T]$ and matrix $\mathbf{Y} \in \mathbb{C}^{N_R \times T}$ whose columns $\mathbf{y}_t$ contain the received symbols at time $t = [1, \ldots, T]$. $\mathbf{Y}$ can be expressed as

$$\mathbf{Y} = \mathbf{H} \mathbf{X} + \mathbf{W},$$

(6)

where $\mathbf{W} \in \mathbb{C}^{N_R \times T}$ is complex Gaussian sensor noise distributed such that each column $\mathbf{w}_t \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_T)$ for $t = [1, \ldots, T]$, where $\mathbf{I}_T$ is the identity matrix of dimension $T$. In this case the least squares (LS) channel estimator is $[18, @]

$$\hat{\mathbf{H}} = \mathbf{Y} \mathbf{X}^\dagger,$$

(7)

where $\mathbf{X}^\dagger = \mathbf{X}^H (\mathbf{XX}^H)^{-1}$ is the pseudoinverse. Training matrix $\mathbf{X}$ is constrained by the transmitted power as

$$\|\mathbf{X}\|_F^2 = \text{tr}(\mathbf{XX}^H) = \mathcal{P},$$

(8)

where $\text{tr}[.]$ is the trace operation and $\mathcal{P}$ is a known constant representing the total power transmitted and $\|\|_F$ is the Frobenius norm. We wish to find a training matrix $\mathbf{X}$ which minimizes the channel estimation error subject to the power constraint (8). This is equiv-
alent to solving the optimization problem

$$\min_X \mathbb{E} \left[ \| H - \hat{H} \|^2_F \right] \quad \text{s.t.} \quad \text{tr}(XX^H) = P,$$

(9)

where $\mathbb{E}[.]$ is the expected value. Combining (6) and (7) we see $H - \hat{H} = WX^\dagger$. Continuing from (9) we get the objective function

$$J = \mathbb{E} \left[ \| H - \hat{H} \|^2_F \right]$$

$$= \mathbb{E} \left[ \| WX^\dagger \|^2_F \right]$$

$$= \sigma_n^2 N_R \text{tr} \left[ X^HH \right]$$

$$= \sigma_n^2 N_R \text{tr} \left[ (XX^H)^{-1} \right],$$

(10)

where $\mathbb{E}[W^HW] = \sigma_n^2 N_R I$. Plugging (10) into (9), an equivalent equation is

$$\min_X \text{tr} \left[ (XX^H)^{-1} \right] \quad \text{s.t.} \quad \text{tr}(XX^H) = P,$$

(11)

From here any training matrix satisfying

$$XX^H = \frac{P}{N_T} I,$$

(12)

is optimal for (11), [18, @]. The optimality criteria also simplifies the channel estimation equation. Plugging (12) into (7) we have

$$\hat{H} = \left( \frac{N_T}{P} \right) YX^H.$$

(13)

All matrices with orthogonal rows with the same norm satisfy the optimality criteria of eq (12). The waveforms transmitted by the MIMO testbed described in section 5 satisfy (12).

3. Compressive MIMO Beamforming

3.1. Review of Compressive Sensing
Compressive Sensing (CS) is a relatively new field of signal processing wherein a measurement vector is reconstructed as a sparse linear combination of predetermined dictionary vectors. Consider the classical linear measurement model

\[ r = \Psi t + w, \]

where \( w \) is Gaussian noise and \( r \) is a known measurement composed of some linear combination of the columns of known dictionary matrix \( \Psi \). The goal of CS is to reliably determine \( t \) from knowledge of \( r \) and \( \Psi \) given that \( t \) is sparse, (has few non-zero values) or approximately sparse (has entries that decay rapidly when reordered by magnitude).

The central tenet of CS is that if \( t \) is sparse then most of the salient information in \( r \) can be captured by a few dictionary vectors (for appropriately designed dictionaries). Additionally, recent theoretical results have established that \( t \) can be solved using tractable mixed norm optimization programs \([30, @]-[34, @]\), efficient greedy algorithms, \([36, @]\), fast iterative thresholding algorithms \([38, @]\), or Bayesian probabilistic methods \([40, @]\).

Proofs establishing the reliability of the mentioned reconstruction procedures depend on a certain property of the dictionary matrix \( \Psi \), and the sparsity of \( t \). Specifically, the key property of \( \Psi \) for proving the optimality of reconstruction is the restricted isometry property (RIP) \([42, @]\).

There are currently no known algorithms that check the RIP for a given matrix in polynomial time, though one of the reasons that has lead to the widespread use of CS in many fields is the discovery that certain probabilistic constructions of matrices satisfy the RIP with high probability \([44, @]\). In this paper we assume the RIP holds for the dictionary matrix defined in section 3.2, and test this theory with simulations performed in section 4.

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Of the many CS reconstruction algorithms noted earlier, we use the LASSO \cite{30, @} (which is sometimes also referred to as basis pursuit denoising \cite{34, @}). The LASSO is a well studied method for solving compressive sensing problems which has good reconstruction error bounds \cite{46, @} and is computationally attractive due to the many publicly available software packages for computing it. The LASSO in Lagrangian form is

\[
\min_t \| r - \Psi t \|_2^2 + \mu \| t \|_1, \tag{15}
\]

where \( \mu \) is a positive regularization parameter satisfying \( 0 \leq \mu \leq 2\| \Psi^H r \|_\infty \) \cite{14}. In the next section we will show that the path angles traveled by a multipath MIMO signal can be solved using the LASSO.

### 3.2. MIMO Beamforming

From section 2.2 it is clear that an estimate of a channel \( H \) can be made from MIMO systems transmitting certain sequences. Additionally, from eq (2) we see that a MIMO channel can be written as the sum of a small number of paths, thus the channel is sparse in the dictionary formed from the array responses from each possible path angle. In general we will only have estimates of the channel, \( \hat{H} \) (from eq (13)), which are used to produce angular domain channel estimates, \( \hat{H}^a \).

Define \( h = \text{vec}(H) \) where \( \text{vec}(.) \) stacks the columns of a matrix, and \( \tilde{a}(\theta_p, \phi_p) = a_R(\theta_p) \otimes a_T(\phi_p) \) where \( \otimes \) is the Khatri-Rao product \cite{48, @}. The dictionary \( A_D \in \mathbb{C}^{Q_TQ_R \times N_RN_T} \) of path angles is defined as

\[
A_D = \begin{bmatrix}
\tilde{a}(\theta_1, \phi_1), & \ldots, & \tilde{a}(\theta_{Q_R}, \phi_{Q_T})
\end{bmatrix}, \tag{16}
\]

where \( Q_T \) and \( Q_R \) represent an arbitrary number of transmit and receive angles evenly spaced between \( (\phi_{\min}, \phi_{\max}) \) and \( (\theta_{\min}, \theta_{\max}) \). In practical terms, \( Q_T \) and \( Q_R \) are the
number of grid points which the transmit and receive angle space will be divided into.

For unknown sparse vector $\mathbf{h}^a = \text{vec}(\mathbf{H}^a) \in \mathbb{C}^{Q_{R}Q_{T}}$, 

$$
\hat{\mathbf{h}} = \mathbf{A}_D \mathbf{h}^a + \eta, \quad (17)
$$

where $\eta \in \mathbb{C}^{N_{R}N_{T}}$ is the noise due to approximation which is assumed to be from a Gaussian distribution.

Solving eq (15) with variables from eq (17), the LASSO objective function [50, @] produces a sparse estimate of the angular domain channel matrix in vector form.

$$
\hat{\mathbf{h}}^a = \min_{\mathbf{h}^a} \| \hat{\mathbf{h}} - \mathbf{A}_D \mathbf{h}^a \|_2^2 + \mu \| \mathbf{h}^a \|_1. \quad (18)
$$

In this paper we set $\mu$ to half its maximum value (see 14 [@] for further discussion).

Once $\hat{\mathbf{h}}^a$ has been solved it is cast back into matrix form, $\hat{\mathbf{H}}^a = \text{vec}^{-1}(\hat{\mathbf{h}}^a)$ which was introduced in eq (4). Each element of the angular domain channel matrix is associated with an AoD and AoA through which the signal could have traveled. The magnitude of each element of $\mathbf{H}^a$ represents the path gain of the signal traveling through the path angle pair associated with that element.

4. Simulation

Compressive MIMO beamforming was simulated and tested against two other beamforming techniques; conventional beamforming (CBF) [52, @] and 2D MUSIC [54, @], both described in the appendix. CBF and MUSIC provide estimates of the angular domain power spectrum, which are compared to the estimate produced from compressive MIMO beamforming.

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A simulated signal \( y_t \in \mathbb{C}^{N_R} \) was constructed according to eq (5), repeated here for convenience

\[
y_t = \sum_{j=1}^{Q_R} \sum_{i=1}^{Q_T} H^a_{i,j} a_R(\theta_j) a_T^H(\phi_i) x_t + w_t = \tilde{y}_t + w_t, \tag{19}
\]

where \( t = [1, \ldots, T] \), \( Q_T = Q_R = 30 \) and \( H^a \) is a randomly generated sparse matrix whose non-zero elements are unit magnitude with phase drawn from a uniform distribution over the range \([0, 2\pi]\). The sparsity of \( H^a \) was controlled by parameter \( P \) (\( P = 3 \) in figure 2), and each \( x_t \) was a known realization from a complex normal distribution \( x_t \in \mathbb{C}^{N_T} \sim \mathcal{CN}(0, I) \) (Gaussian sequences obey eq (12) with high probability) which was generated independently for each \( t \). \( y_t \) was first generated from random but known realizations of \( H^a \) and \( x_t \), then complex Gaussian noise \( w_t \in \mathbb{C}^{N_R} \sim \mathcal{CN}(0, \sigma^2 I) \) was added such that any desired SNR could be met, where

\[
\text{SNR(dB)} = 10 \log \left( \frac{\|Hx_t\|_2^2}{\|w_t\|_2^2} \right) = 10 \log \left( \frac{\|\tilde{y}_t\|_2^2}{\|w_t\|_2^2} \right). \tag{20}
\]

and \( \|.\|_2^2 \) is the squared \( \ell_2 \) norm.

From \( y_t \) and \( x_t \), \( \hat{H} \) was estimated according to eq (13) with \( \mathcal{P} = N_T T \). Given \( \hat{H} \), eq (18) was used to solve for \( \hat{H}^a \). Figure 2 shows plots of \( |H^a|^2 \) alongside the angular power spectrum recovered from CBF, (\( |H^a_{\text{CBF}}|^2 \) specified in eq (27) of the appendix) and MUSIC (\( F(\theta, \phi) \) from eq (30) in the appendix) applied to the simulated noiseless data with \( N_T = N_R = 16 \) and \( N_T = N_R = 4 \).

Note that both CBF and MUSIC, are limited by the maximum rank of \( H \), which results in the blurred spectrum seen in figure 2. The maximum rank of \( H \) is determined by the number of transmitters and receivers in the MIMO system \( \text{rank}(H^a) \leq \min(N_T, N_R) \).

Compressive MIMO beamforming is not limited in precision by the rank of \( H \), but rather by how many columns of dictionary \( A_D \) can be formed before the dictionary ceases to
satisfy the RIP property. When $A_D$ is populated by too many column vectors $\tilde{a}(\theta, \phi)$ (which is equivalent to quantizing $(\theta, \phi)$ space too finely) there will be high coherence between the columns of $A_D$ to the point where the dictionary will cease to satisfy the RIP. It is infeasible to check that any given $A_D$ satisfies the RIP because there is no known algorithm that works in polynomial time, however we assume that a dictionary formed from quantizing the angle space into $Q_R = Q_T = 30$ is acceptable because the simulation recovers simulated paths with satisfactory accuracy.

The performance of compressive MIMO beamforming is tested using two error metrics for channels of varying sparsity and SNR given $Q_R = Q_T = 30$. The first error metric, $\epsilon_A$, represents angle error and is defined as the euclidean distance (in $(\theta, \phi)$ space) between the $P$ true path angle pairs of $H^a$ and the best matching set of $P$ path angle pairs from $\hat{H}^a$.

$$\epsilon_A = \frac{1}{P} \sum_{p=1}^{P} \sqrt{(\hat{\theta}_p - \theta_p)^2 + (\hat{\phi}_p - \phi_p)^2},$$

(21)

where $\theta_p$ and $\phi_p$ are the receive and transmit angles from path $p$ of $H^a$, $\hat{\theta}_p$ and $\hat{\phi}_p$ are the receive and transmit angles from path $p$ of $\hat{H}^a$, and the units of $\epsilon_A$ are degrees. The best set of matching paths from $\hat{H}^a$ was found as the set of paths producing the smallest $\epsilon_A$ using exhaustive search. It was found that the number of non-zero elements in $\hat{H}^a$ was always equal to or greater than that of $H^a$. Note that simulations were performed over an angle space ranging from $[-90^\circ, 90^\circ]$, so an average error of $18^\circ$ represents a $10\%$ error over the full space. In section 6 we will present findings over a much smaller angle space, for which we expect the same percentage error rather than absolute error.
The second error metric is $\epsilon_N$, is normed error defined as
$$\epsilon_N = \frac{1}{P} \| \mathbf{H}^a - \hat{\mathbf{H}}^a \|^2_F.$$ Normed error is simply a measure of the mean squared difference between the true and estimated angular domain channel matrices.

Each error metric was calculated for $N = 200$ Monte Carlo trials of simulated signals of varying SNR and sparsity, then plotted in figure 3. The random number generator seed was reset for each unique parameterization. We observe that both metrics show positive correlation between error and SNR. Angle error $\epsilon_A$ appears nearly constant for signals composed of $P > 1$, indicating that more paths does not lower the angular accuracy of the estimated paths. Normed error curves indicate that the mismatch between true and reconstructed angular channel matrices is much greater for signals from multiple paths. Each curve appears to flatten for SNR above $-15$ dB. We conclude compressive MIMO beamforming will have sufficient accuracy for received signals whose SNR is above $-15$ dB.

5. Experimental Setup and Data

The coast of southern California is known as a hotspot for atmospheric ducts [2, @], which can act as leaky waveguides for EM waves. The goal of the experiment was to observe the impact of a ducting environment on a MIMO communications channel, particularly the change in multipath properties of the channel with time. No measurements of the atmospheric refractivity profile were taken, so the observations at the receiver array are understood to reflect the typical characteristics of a refractive channel rather than being the result of any specific type of duct.
5.1. Transmitters and Receivers

Data was collected from vertically positioned ULAs of 4 transmitters and 4 receivers. The transmitter array was positioned at the end of Scripps Pier, 332 m from shore, with array spacing of 3.8 m while the receiver array was located 1000 m inland at Camp Pendleton with array spacing of 4 m. The transmitters will be referred to as T1-T4, where T1 is the topmost transmitter. Likewise, the receivers will be referred to as R1-R4 where R1 is the topmost receiver. From the array spacing the maximum resolvable angles were $\phi_{\text{max}} = 1.63^\circ$ and $\theta_{\text{max}} = 1.55^\circ$, which is within the normal range of expected AoA and AoD’s for such long range over the horizon setups [10, @]. The topmost elements of the transmitter and receiver arrays were approximately 34 and 32 m above sea level respectively. The arrays were located 40.72 km apart, far enough that the line of sight path between all antennas was obstructed by the horizon. The elements of both arrays were pointed facing each other in azimuth, and with an elevation angle of 0°.

A known, narrowband, length $2^{13}$ Zadoff-Chu (ZC) signal, $z$, [20, @] was sent from each transmitter on carrier frequency 1.385 GHz. ZC signals are complex, constant magnitude, and satisfy the following property

$$z^H(Pz) = 0,$$  \hspace{1cm} (22)

where $P$ is any cyclic permutation of the identity matrix with $P \neq I$. Eq (22) should be interpreted to mean any ZC sequence is orthogonal to any circularly shifted version of itself. By sending identical ZC signals with unique circular shifts from each transmitter, a training matrix which satisfies (12) is formed. The transmitted waveforms from \{T1,...,T4\} were identical, circularly shifted ZC sequences, $X \in \mathbb{C}^{N_T \times 2^{13}} = [z_0, z_{500}, z_{1200}, z_{2000}]^T$

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where $z_i$ is a ZC sequence circularly shifted by $i$ samples. Each snapshot is $8 \times 2^{13} = 2^{16}$ samples.

All receivers coherently recorded four snapshots of $2^{16}$ samples taken at sample rate 1.25 MHz every fifteen minutes. Figure 4 shows matched filter outputs between the transmitted ZC sequence and all receivers for a single snapshot. The matched filter output $m_i$ of receiver $i$ is defined as

$$m_i[t] = \sum_{m=-\infty}^{\infty} y_i[m] z[t - m],$$

(23)

where $y_i[t]$ represents index $t$ of the measurement vector from receiver $i$ and $z$ is the unshifted ZC sequence transmitted by T1. Each peak in figure 4 confirms the arrival of a ZC sequence, the first peak being from T1, the second from T2, and so on. Because the four transmitters each sent repeating $2^{13}$ sample waveforms, the four arrival spikes in the matched filter are expected to repeat every $2^{13}$ samples (8 repetitions per snapshot). The magnitude of each spike roughly represents the strength of the path between each transmitter and receiver.

We note the arrival of the ZC sequence from T3 and T4 in figure 4 are significantly weaker than the others. We explore two possible explanations, first that both T3 and T4 are positioned at significantly lower elevations than T1 and T2, thus the horizon presents a greater obstacle. The decreased receive power may be the result of signal attenuation from propagating over the horizon, however this does not explain why the signal from T4 arrives stronger than that of T3. Second, multipath environments can cause fading, resulting in null zones where the signal and its reflections effectively cancel each other out. It is possible that the receiver array lies in a null zone for the signals from T3.
6. Results

Receiver noise variance was not recorded during the measurements. We assume each receiver was subject to the same noise level for the entire testing period, thus received power represents some scaling of SNR. Average received power was calculated as $P_r = \frac{1}{N_R} \|Y\|_F^2$ for each snapshot $Y \in \mathbb{C}^{N_R \times 216}$. The average received power over the full 18 day test period between January 23 and February 09, 2016 is plotted in figure 5. Note that the array went down for 18 hours on January 31st, leading to a short gap in the data.

It is well documented that atmospheric ducts, particularly evaporation ducts, can increase the SNR of over the horizon EM signals [2, @], [10, @]. We assume that periods for which the received power is at a maximum are indicative of some form of ducting [60, @]. Using the periods between January 25-29 and Feb 5-8 as examples of ducting events, we analyze the impact of ducting on the reconstructed path angle pairs of $\hat{H}^a$ from the experimental data.

All receiver returns were processed using compressive MIMO beamforming. The objective was to visualize the evolution of the multipath channel over time. The data was divided into two hour intervals, 32 snapshots per interval (4 snapshots per 15 minutes = 32 snapshots per two hours), and the estimate $\hat{H}^a$ from each snapshot was normalized to limit local SNR fluctuations. All normalized estimates of $\hat{H}^a$ within an interval were averaged to produce $\bar{\hat{H}}^a_{\text{int}}$, representing the fraction of path gain through each angle during the interval, normalized to account for power fluctuations between snapshots. Formally,

$$
\bar{\hat{H}}^a_{\text{int}} = \frac{1}{N} \sum_{n=1}^{N} \frac{\hat{H}^a_n}{\sum_{i,j} |\hat{H}^a_{n,i,j}|},
$$

where $\hat{H}^a_{n,i,j}$ is the $(i,j)$th element of the estimated angular domain channel matrix from snapshot $n$, and $N$ is the number of snapshots in the interval. Note that the elements

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of each $\tilde{H}_{\text{int}}$ sum to one. $\tilde{H}_{\text{int}}$ is plotted in figures 6-8 for all two hour intervals between January 24th and January 26th, 2016.

We observe from the received power measurements of figure 5 that an atmospheric duct existed on January 26th. Figures 6-8 show the evolution of the multipath channel from January 24th through January 26. On January 24th the channel appears stable. There is little fluctuation in received power (mostly occurring between midnight and 6 AM) and two dominant path angle pairs are present at each interval. On January 25th, the received power measurements from figure 5 indicate the possible formation of a weak duct between midnight and 10 AM, and a strong duct after 6 PM. Active path angle pairs from January 25th presented in figure 7 do not show significant variation from those of January 24th. Again two dominant path angle pairs are present in the majority of the data.

Received power on January 26th (see figure 5) indicate the presence of a duct. Figure 8 shows that the multipath channel varied rapidly during daylight hours, but was stable and indistinguishable from a non-ducting channel during nighttime. The impact of ducting appears to be a lack of stability in the active path angle pairs, particularly in daylight hours. Figure 9 shows $\tilde{H}_{\text{int}}$ over much longer intervals of low and high measurements of received power. During low received power measurements of active path angle pairs are distributed tightly about specific receive angles, while high received power measurements lead to more unpredictable path angle pairs. It is possible that during periods of high receive power non-linearities in the receiver hardware caused inaccuracies in the data and is responsible for the increased variability of active path angle pairs during ducting, but this does not explain why these path angle pairs appear to fluctuate only during daylight hours in figure 8.
7. Conclusion

Compressive MIMO beamforming was simulated and applied to data recorded from a MIMO system. Simulations showed that compressive MIMO beamforming is capable of accurately identifying multipath signals with higher resolution than conventional beamforming and the 2D MUSIC algorithm. A MIMO array was set up in a ducting hotspot along the coast of southern California for the purpose of monitoring the evolution of its multipath channel over time. Matched filter processing of the received data confirmed the arrival of signals from the transmitters. An optimal estimate of the channel was made four times every fifteen minutes. From each channel measurement an angular domain channel matrix was calculated using compressive MIMO beamforming. The evolution of active path angle pairs from the angular domain channel matrix were examined over the measurement period.

The received power from the channel was used to determine the presence of ducting with periods of high received power assumed to indicate ducting and low received power to indicate a standard atmosphere. Two ducting events lasting multiple days were observed over the measurement period. The MIMO paths did not appear strongly related to received power or predictive of ducting. The active path angle pairs observed during ducting events were found to come from a wider range of path angles. Path angle pairs traveled by communications signals during ducting events were found to sometimes change quickly and unpredictably compared to those observed when no ducts were present.

No conclusive evidence was found that active path angle abnormalities from received MIMO communications signals can be used to predict ducting. The data, however, is only representative of one nearshore channel between points in southern California, and
results may differ for channels in alternate locations. More data collection is required before conclusions can be drawn about the relation between ducting and the shape of a MIMO multipath channel. We suggest that refractivity profiles be taken alongside MIMO data in future related work so that the precise shape of the duct can be compared with the active path angle pairs of the MIMO signal.

8. Appendix

8.1. 2D Conventional Beamforming

Like compressive MIMO beamforming, conventional beamforming (CBF) takes in a channel estimate $\hat{H}$ and outputs an estimate of the angular domain channel matrix $H^{a}_{CBF}$. From (2), a channel matrix $H$ is the sum of $P$ rank 1 matrices, each representing a path from transmitter to receiver. With this understanding we define dictionaries $[12, @]$

$$S_{R} \in \mathbb{C}^{N_{R} \times Q_{R}} = \left[ a_{R}(-\theta_{\text{min}}), \ldots, a_{R}(\theta_{\text{max}}) \right],$$

$$S_{T} \in \mathbb{C}^{N_{T} \times Q_{T}} = \left[ a_{T}(-\phi_{\text{min}}), \ldots, a_{T}(\phi_{\text{max}}) \right],$$

where $Q_{R}$ and $Q_{T}$ are the number of bins into which the transmit and receive angle space is divided. The dictionaries in (25) and (26) have columns representing the array responses from signals arriving and departing from different angles. Larger $Q_{R}$ and $Q_{T}$ result in dictionaries that more finely divide the angular spectrum.

The angular domain representation of a channel $H$ for conventional beamforming $[12, @]$ is defined as

$$H^{a}_{CBF} = S_{R}^{H}HS_{T}.$$
Unlike compressive MIMO beamforming, the formulation of $H_{CBF}^a$ in eq (27) does not assume sparsity in $H_{CBF}^a$, so the CBF estimate of $H^a$ is inherently different from that of compressive MIMO beamforming.

8.2. 2D MUSIC

The 2D MUSIC algorithm [62, @], is a variation of the popular MUSIC subspace method [54, @]. Increased resolution is achieved by separating the signal and noise subspaces of the channel matrix through eigen-decomposition. The idea is that contributions to the channel $H$ from noise are contained in the smallest eigenvalues of the channel covariance matrix, and by removing such eigenvalues, a de-noised version of the channel is created.

The channel covariance is

$$R = E[hh^H],$$

where $h = \text{vec}(H)$, which has eigen-decomposition

$$R = U_S\Lambda_S U_S^H + U_N\Lambda_N U_N^H. \quad (29)$$

Here $U_S$ and $U_N$ are the matrices whose columns contain the eigenvectors of the signal and noise subspaces, and $\Lambda_S, \Lambda_N$ are diagonal matrices whose elements are the corresponding eigenvalues. If the signal primarily travels through $n$ paths, the dimension of the signal subspace is $n$. Under this assumption $U_S$ will contain the $n$ eigenvectors corresponding to the $n$ largest eigenvalues. The spectrum function for 2D MUSIC is given as [22, @]

$$F(\theta, \phi) = \frac{1}{\tilde{a}^H(\theta, \phi) U_N U_N^H \tilde{a}(\theta, \phi)}. \quad (30)$$

$F(\theta, \phi)$ is a continuous function which represents the relative strength of the signal traveling through angles $(\theta, \phi)$, similar to estimates of $H^a$ from CBF and compressive MIMO beamforming. An example of $F(\theta, \phi)$ is shown in figure 2.
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Figure 1. Shortest path between transmitter (bottom) and receiver (top) superimposed on satellite image of Southern California.
Figure 2. Angular power spectrum from various beamforming techniques performed on simulated noiseless signals with $T = 1000, P = 3, \mu = 0.5\mu_{\text{max}}$. (Top) $N_T = N_R = 16$. (Bot) $N_T = N_R = 4$. (Left) Conventional beamforming, (Middle) 2D MUSIC, (Right) Compressive MIMO beamforming with $Q_R = Q_T = 30$. Transmitter and receiver spacing each set to $\frac{\lambda}{2}$. 

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Figure 3. Average error of $N = 200$ Monte Carlo reconstructions of $\hat{H}^a$ using compressive MIMO beamforming with $\mu = .5\mu_{\text{max}}$ on simulated signals of varying SNR and sparsity $P$. a) Angle error ($\epsilon_A$), b) Normed error ($\epsilon_N$).
Figure 4. Single snapshot matched filter between each receiver and ZC sequence taken on January 24, 2016 at 00:00:18. Each spike represents the arrival of a length $2^{13}$ sample ZC signal from a transmitter.
Figure 5. Average received power (dB) over the test period from Jan 23 - Feb 09, 2016.
Figure 6. Averaged normalized path gain $\hat{H}_{\text{int}}^a$ (from eq (24)) over 2 hour (32 snapshot) intervals vs. AoA and AoD taken on January 24, 2016.
Figure 7. Averaged normalized path gain $\hat{H}_{\text{int}}$ (from eq (24)) over 2 hour (32 snapshot) intervals vs. AoA and AoD taken on January 25, 2016.
Figure 8. Averaged normalized path gain $\hat{H}_{int}$ (from eq (24)) over 2 hour (32 snapshot) intervals vs. AoA and AoD taken on January 26, 2016.
Figure 9. Averaged normalized path gain $\hat{H}_a^{int}$ (from eq (24)) over long intervals of high and low received power.