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On the sign of the adaptive passive fathometer impulse response (L)

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Harrison [J. Acoust. Soc. Am. **125**, 3511–3513 (2009)] presented a mathematical explanation for a sign-inversion induced to the passive fathometer response by minimum variance distortionless response (MVDR) beamforming. Here a concise mathematical formulation is offered, which decomposes the cross-spectral density matrix into coherent and incoherent components and allows the matrix inversion to be obtained exactly by eigendecomposition. This shows that, in the region containing the bottom reflection, the MVDR fathometer response is identical to that obtained with conventional processing multiplied by a negative factor.
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I. INTRODUCTION

A drifting vertical array can be used as a passive fathometer by cross-correlating up- and down-ward beams.[1] A more detailed analysis and an introduction to the application of adaptive beamforming to the passive fathometer are given in Ref. 2. Recently it was found that adaptive processing induced a sign change in the fathometer cross-correlation. An explanation was given in Ref. 3 using the Woodbury matrix identity to invert the cross-spectral density matrix (CSDM). Here we offer a simple explanation using eigendecomposition to perform this matrix inversion. This approach is exact given the idealized noise model described below.

A simple physical model is used consisting of a vertically downward propagating signal and its vertically upward propagating reflection in the presence of incoherent background noise. It should be noted that this is an idealization and a realistic ocean environment would likely contain a more complicated spatial structure. The passive fathometer requires the reflected signal to be coherent with the downward propagating signal to extract the depths of the reflecting layer boundaries. Consequently the two coherent signals are not separable and act as a single coherent component in constructing the CSDM. We show that, in the region of interest for the fathometer attributed to this coherent component, the adaptive response obtained using the minimum variance distortionless response (MVDR) beamformer is identical to that obtained using the conventional beamformer multiplied by a *negative* factor.

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II. THEORY

Consider the response of the fathometer to a single frequency plane wave. The conventional response is given by[2]

$$C(\omega) = \mathbf{w}^T\mathbf{R}\mathbf{w}, \tag{1}$$

where $\mathbf{w}$ is the downward directed steering vector, the superscript T denotes the matrix transpose, and $\mathbf{R}$ is the CSDM. The MVDR adaptive response is given by[3]

$$C_{\mathrm{MVDR}}(\omega) = \Lambda(\omega)\mathbf{w}^T\mathbf{R}^{-1}\mathbf{w}, \tag{2}$$

where $\Lambda = |\mathbf{w}^H\mathbf{R}^{-1}\mathbf{w}|^{-2}$ is a positive normalization factor and the superscript H denotes the complex conjugate transpose. Thus, excluding the normalization factor, MVDR processing is of the same functional form as conventional processing with the CSDM inverse in place of the CSDM.

Decomposing the CSDM into coherent and incoherent components,

$$\mathbf{R} = \mathbf{d}\mathbf{d}^H + \sigma^2\mathbf{I}, \tag{3}$$

where $\mathbf{d} = \mathbf{d}_{\mathrm{down}} + \mathbf{d}_{\mathrm{up}}$ is the sum of the downward propagating signal and the upward propagating reflection. $\mathbf{d}_{\mathrm{down}}$ is proportional to the steering vector $\mathbf{w}$. $\sigma^2\mathbf{I}$ is the component due to incoherent background noise, with $\mathbf{I}$ designating the identity matrix. This allows the CSDM to be expanded by eigendecomposition to yield

$$\mathbf{R} = (\sigma^2 + a)\mathbf{u}_1\mathbf{u}_1^H + \sigma^2\sum_{j=2}^{N}\mathbf{u}_j\mathbf{u}_j^H, \tag{4}$$

where N is the number of array elements, a is the eigenvalue component corresponding to the coherent signal, and $\mathbf{u}_j$ are the normalized eigenvectors. The CSDM inverse is

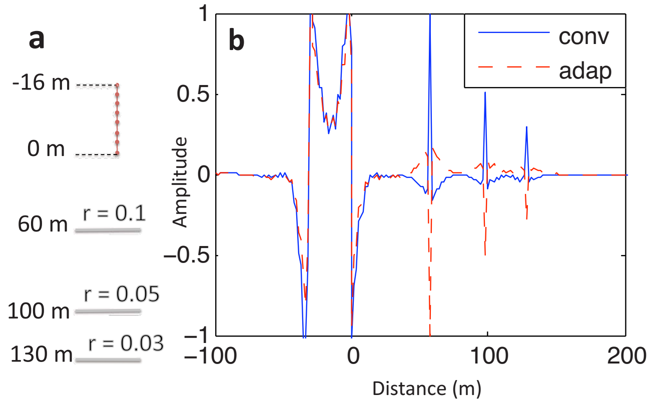

FIG. 1. (Color online) (a) A schematic of the model environment. The array is shown between −16 and 0 m (only every fourth hydrophone is shown) and the reflection layers with the associated reflection coefficients, r. (b) The conventional (solid) and MVDR (dashed) fathometer responses from simulated data.

$$\mathbf{R}^{-1} = \frac{1}{(\sigma^2+a)}\mathbf{u}_1\mathbf{u}_1^H + \frac{1}{\sigma^2}\sum_{j=2}^{N}\mathbf{u}_j\mathbf{u}_j^H$$
$$= \frac{1}{\sigma^2}\left[\frac{\sigma^2}{(\sigma^2+a)}\mathbf{u}_1\mathbf{u}_1^H + \mathbf{I} - \mathbf{u}_1\mathbf{u}_1^H\right]$$
$$= \frac{1}{\sigma^2}\left[\mathbf{I} - \frac{a}{\sigma^2+a}\mathbf{u}_1\mathbf{u}_1^H\right]$$
$$= -\frac{1}{(\sigma^2+a)^2}\mathbf{d}\mathbf{d}^H + \frac{1}{\sigma^2}\mathbf{I}, \tag{5}$$

where we have utilized $\Sigma\mathbf{u}_j\mathbf{u}_j^H=\mathbf{I}$. Thus the CSDM inverse contains the same terms as the CSDM with different coefficients. The component due to the coherent signal has been multiplied by a negative coefficient. The term due to background noise remains positive and the effect of this term is examined separately in both the frequency and time domains.

In the conventional case,

$$C_{\text{noise}} = \sigma^2\mathbf{w}^T\mathbf{I}\mathbf{w}. \tag{6}$$

The effect of this term depends on the choice of reference element. For convenience, the lowermost element of the array is defined as the reference [see Fig. 1(a)]. Thus the kth term of the steering vector is $e^{i\omega\tau_k}$, where τ_k is the vertical propagation time to the kth element. Substituting this yields

$$C_{\text{noise}} = \sigma^2\sum_{k=1}^{N} e^{2i\omega\tau_k}. \tag{7}$$

In the time domain, assuming infinite bandwidth, Eq. (7) corresponds to a series of delta functions at $t=-2[\tau_{N-1}, \tau_{N-2}, \ldots, \tau_2, 0]$. For an equispaced array with inter-element vertical travel time τ, this becomes $t=-2\tau[N-1, N-2, \ldots, 1, 0]$. The adaptive case is identical except the scalar coefficient Λ/σ^2 replaces σ^2. Note that defining the lowermost element as the reference ensures that these delta functions due to the background noise appear at negative times. Equations (1) and (2) are cross-correlations of down- and up-ward propagating beams. In this model, the upward beam is a sum of time-delayed reflections of the downward beam. As the reflections lag the downward beam, the correlation peaks will appear at positive times. Thus the seabed response will not be obscured by the contribution from background noise.

Neglecting the background noise component and substituting Eqs. (3) and (5) into Eqs. (1) and (2), respectively, reduces the fathometer response to

$$C(\omega) = \mathbf{w}^T\mathbf{d}\mathbf{d}^H\mathbf{w} \tag{8}$$

$$C_{\text{MVDR}}(\omega) = -\frac{\Lambda}{(\sigma^2+a)^2}\mathbf{w}^T\mathbf{d}\mathbf{d}^H\mathbf{w}. \tag{9}$$

where $\Lambda/(\sigma^2+a)^2$ is a positive factor.

III. NUMERICAL SIMULATION

A simulation was constructed with a 32-element array with 0.5 m spacing (design frequency of 1500 Hz) over three reflection layers 60, 100, and 130 m below the bottom of the array with reflection coefficients of 0.1, 0.05, and 0.03, respectively [see Fig. 1(a)]. A boxcar function with a signal-to-noise ratio of −10 dB and a width of 1.3 ms was used as the down-going signal. The processing was done in the frequency domain with 1024 frequency bins from 0 to 750 Hz. A 40 Hz high pass filter was applied and the two responses were normalized such that the first reflection peak has an absolute magnitude of one.

The conventional and MVDR responses are shown in Fig. 1(b) against the depth associated with a two-way travel time in a medium with sound speed of 1500 m/s. At depths greater than 0, the conventional and MVDR traces are mirror images. In the region between −32 and 0 m, the conventional and MVDR responses are similar. This is the region dominated by the incoherent noise term. The delta functions predicted in Eq. (7) are twice convolved with the box car signal which obscures the individual peaks.

IV. SUMMARY

This analysis shows that adaptive processing will induce a negative sign to the seabed response given by conventional passive fathometer processing. In addition, it has been shown that the component from incoherent noise which obscures both conventional and adaptive processing can be confined to negative times by referencing the array elements relative to the bottom hydrophone.

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