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A SIMPLIFIED METHOD FOR DYNAMIC ANALYSIS OF A GUYED MAST

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Summary

The dynamic response of a guyed mast is relatively larger than for other structures. Thus a proper evaluation of the dynamic response is of major importance. The proposed simplified method separates the dynamic response into a low-frequency, background region and a high-frequency, resonant region. The background response has been estimated using patch loading. The resonant responses can be taken into account by multiplying the background response by a dynamic magnification factor. This factor depends, in a systematic way, on the average structural properties of the mast. The approach is illustrated by examples, and compared to a statistical method.

1. Introduction

A guyed mast is relatively less reliable than other structures. For a guyed mast, wind and ice are the major loads. This is contrary to most other structures where the dead load is major. Therefore, a proper evaluation of the response due to wind and/or ice may decrease the occurrence of failures. This paper concentrates on the along-wind buffeting of the mast.

In design practice the dynamic response is often neglected. Instead, a higher static load, for example the static load multiplied by a gust factor, is used. In the case of a guyed mast this is inadequate, because the shapes of the static and dynamic responses differ significantly. Further, if a dynamic analysis is carried out, the static loads considered are kept within the linear range. This justifies the neglect of some of the non-linearities when a dynamic analysis is used.

The guyed mast itself is characterized as a beam on elastic supports. These guy supports are non-linear for large deflections but can be treated as linear for small amplitude dynamic motions. A complicating factor in the treat-

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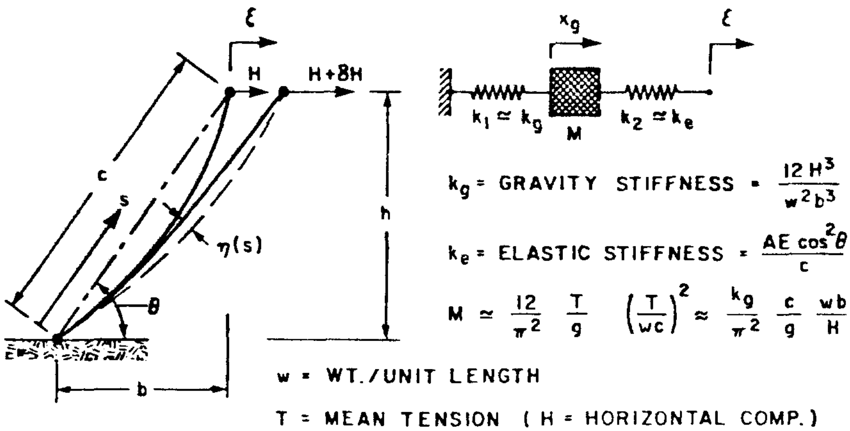

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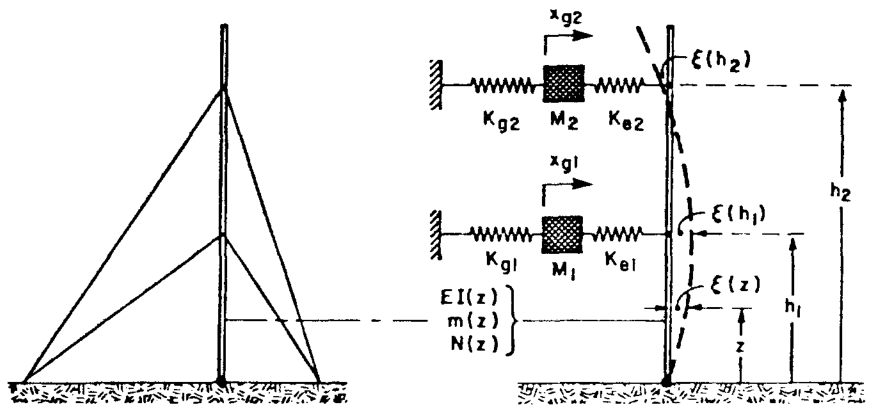

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Fig. 1. (A) Guy, notation and equivalent spring—mass—spring system. (B) Guyed mast, notation and equivalent system.

ment of guys is the effect of the cable mass. This can be simplified, as depicted in Fig. 1, using the equivalent spring—mass—spring [1]. For higher frequencies the guy system approaches the elastic stiffness.

The work presented here is a continuation of the research by Davenport and his co-workers [1—8].

2. Methods of dynamic analysis

The dynamic response is stochastic in nature. Therefore, it seems reasonable to calculate the dynamic response by a stochastic method as in the modal approach. This has been described by Hartmann and Davenport [1] and Vellozzi [10]. This method is complicated and hence rarely adopted. This dynamic analysis had originally been developed for cantilever structures, where only a few modes contribute to the response. For guyed masts, where the modes are closely spaced, it may be unconservative and inaccurate to neglect the cross coupling between modes.

For design this method is not practical. First, it requires large computational effort. Second, it is difficult to predict the change in response due to change in design. The static deflection has to be established before this analysis can be done. Therefore, the dynamic analysis is only used as a final check of design, even though the dynamic and static loads are of the same order of magnitude.

One approach to this problem is to use "patch loading" as suggested by Cohen [11]. He suggested removing 25% of the static wind load between any adjacent spans. This is similar to the procedure in varying live loads on floor systems for calculating maximum moments. This approach has been adopted in some codes, for example, ANSI [12] and the IASS recommendations [9]. This patch load method seems to give a reasonable approximation to the shape of the dynamic moments. However, the magnitude of the moments differ.

2.1. *A simplified statistical method*

The approach described in this section has been developed and described extensively by Davenport and Allsop [4] and Allsop [3].

In Fig. 2 a typical response of a mast is shown; it indicates a bulk of energy at low frequencies, and for the higher frequencies there is a contribution from many modes. The energy at low frequency is due to excitation by the turbulence of the wind. The gusts of wind occur at frequencies considerably lower than the natural frequencies, therefore the low-frequency, background response can be analysed using quasi-static methods. This requires only knowledge of the correlation of the wind along the structure and the magnitude of the dynamic load. This eliminates the problem of knowing the exact shape of the force spectrum at the lower frequencies, i.e. the wind-spectrum.

In refs. 3 and 4, eqns. (1)—(9) for the moments were derived.

2.1.1. *Background moment (BM)*

$$\mathrm{BM}(z) = q(z)[2i(z)]\,\mathrm{CD}(z)\,gL^2\mathrm{BMF}(z) \tag{1}$$

Where z is the local height as a fraction of the total height H, $q(z) = \frac{1}{2}\rho U^2$ is the velocity pressure, $i = \sigma_u/U = [\ln(zH/z_0)]^{-1}$ is the intensity of turbulence, z_0 is the roughness length, CD is the effective aerodynamic width of the structure, g is the peak factor (~ 4), and L is the span length. BMF is the

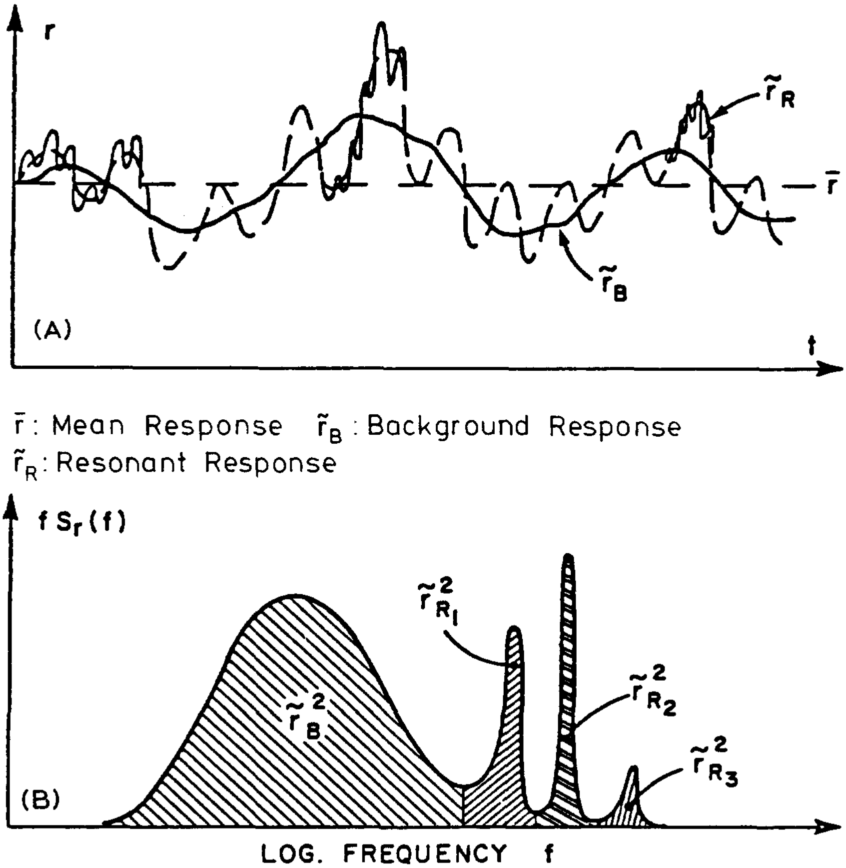

Fig. 2. (A) Typical response of guyed mast to wind. (B) Spectrum of response.

background moment factor, which can use influence lines, and the correlation of the wind along the structure:

$$\mathrm{BMF}^2(z) = \left(\frac{H}{L}\right)^2 \int_0^1 \int_0^1 \exp\left(-|z_1 - z_2|\frac{H}{\lambda}\right) [\phi_u \phi_{\mathrm{CD}} \phi_I(z_1)]\, [\phi_u \phi_{\mathrm{CD}} \phi_I(z_2)]\, \mathrm{d}z_1 \mathrm{d}z_2 \quad (2)$$

where H is the height of the structure. The exponential function in the integral expresses the correlation of the wind, λ is the vertical scale of turbulence or correlation length, which is roughly 50—60 m. The variations in velocity, drag and influence lines are expressed dimensionlessly by

$$\phi_u(z) = \frac{U(z)}{U_0}, \phi_{\mathrm{CD}}(z) = \frac{\mathrm{CD}(z)}{\mathrm{CD}_0}, \phi_I(z,z_1) = \frac{I(z,z_1)}{I_{\mathrm{ref}}} \quad (3)$$

where $I(z,z_1)$ is the influence line for moment at point z due to a unit load at point z_1, I_{ref} is a characteristic scale factor for the influence line; for moment, it is chosen as the span length L.

For real masts the magnitude of the influence line $I(z,z_1)$ is large when z is close to z_1, whereas when $|z - z_1|$ is large $I(z,z_1)$ is small. The windspeed varies slowly with height compared to $I(z,z_1)$. Thus ϕ_u can be treated as a constant. The variation in drag has also been neglected. Hence the background moment factor is only dependent on the scale of turbulence and the influence line.

2.1.2. The resonant response (RM)

The total resonant moment is given by

$$\mathrm{RM}(z) = [q(2i)\mathrm{CD}]_{\mathrm{ref}} g L^2 Q \mathrm{RMF}(z) \tag{4}$$

where ref means a reference value, RMF is the dimensionless resonant moment factor, Q is the inertial resistance factor. Q is mainly dependent on the structural properties and is given by

$$Q = 0.37\left[\left(\frac{m_0}{\rho_D^2}\right)^4 \frac{D^4}{I}\,\frac{D}{H}\,\frac{q_0}{E}\,\frac{1}{C^3}\right]^{1/6} \tag{5}$$

Where D, H, I, E, C and m_0 are the width, height, average moment of inertia, modulus of elasticity, average drag coefficient and average mass per unit length of the mast, respectively, q_0 is the reference wind pressure and ρ is the density of air.

The total resonant moment is found as a sum of the moment in each mode. Thus the total resonant moment factor (RMF) is given by

$$\mathrm{RMF} = \sqrt{(\textstyle\sum_j \mathrm{RMF}_j^2)} \tag{6}$$

Equations for the RMF_j similar to that for BMF involve the spectrum of turbulence, mode shape and damping. They are discussed in more detail in refs. 3 and 4.

2.1.3. Total dynamic moment

The total dynamic peak moment is given by

$$\mathrm{DM}(z) = q(z)2i(z)\mathrm{CD}(z)L^2 g\mathrm{BMF}(z)\mathrm{DMF}(z) \tag{7}$$

here DMF is the dynamic magnification factor. If CD is assumed constant and the wind is varying as a power law with exponent α, then the DMF can be expressed as

$$\mathrm{DMF}(z) = \sqrt{[1 + Q^2\mathrm{DT}(z)]} \tag{8}$$

$$\mathrm{DT}(z) = z^{-4\alpha}\,[\mathrm{RMF}(z)/\mathrm{BMF}(z)]^2 \tag{9}$$

where DT is a dynamic term.

3. Simplified estimation of the dynamic response

The method in Section 2.1 avoids the problem of coupling between modes. For this reason it is more accurate than the classical modal approach. Both of these methods require large computational effort, mainly because the natural frequencies and the mode shapes have to be calculated. This is acceptable for some consulting engineers, but in a practical design situation it is preferable to have a simpler approach. Further, the many uncertainties in the analysis of a guyed mast do not justify an accurate analysis. It should be accepted that a guyed mast will always be analysed under higher uncertainties than other structures.

The cornerstone in this simplified method is eqn. (7). Here the dynamic response is found by estimating the background response and then multiplying this by the dynamic magnification factor (DMF) which takes the resonant response into account.

3.1. Background moment (simplified)

The background moment has been estimated using the patch loading or checkerboard method. This is not new, as it was first introduced by Cohen [11] and is used in codes [9, 12]. However, the magnitude of the load and the way of applying the patch loading are different from the proposed method.

The magnitude of the background moment is given by eqn. (1). The background moment factor (BMF) is determined by eqn. (2) and the simplifying assumptions given in Section 2.1.1. The influence lines are found by unit loads, therefore the BMF is also determined by unit loads.

Usually, when patch loading is used, the background moment is found as the maximum of the moments from all the loading cases. Figure 3A shows the loading system put forward in the IASS recommendations. However, if the statistical equation is examined, it seems reasonable to sum the moments from patch loading as root sum square (RSS):

$$\mathrm{BMF}(z) = \sqrt{[\Sigma\, \mathrm{BMF}_i^2(z)]} \tag{10}$$

where BMF_i is the moment from the i-th loading system, as shown in Fig. 3B, and the background moment factor (BMF) is a dimensionless expression for the background moment.

It was found that the root sum squared (RSS) approach gave the best results when the BMF for supports was calculated by applying the patch loading between each support, as illustrated in Fig. 3B. For midspan response the patch loading should be applied between each midspan (see Fig. 3B). This assumes the span length to be about equal to the length scale of turbulence.

Normally the span length, L, is not larger than the turbulence length scale, λ, but it could be smaller. When the length is smaller, BMF will decrease. This is most pronounced for midspans. Some attempts have been made to

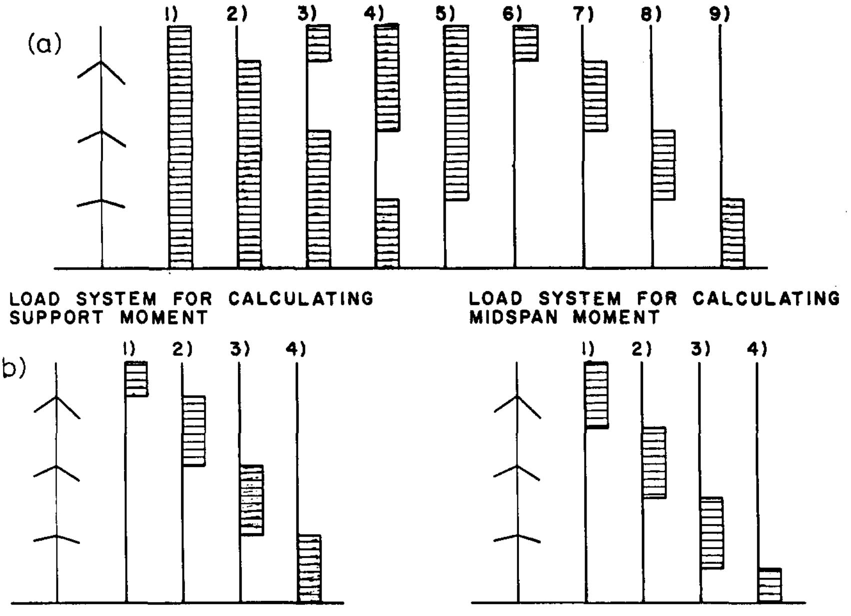

Fig. 3. Methods of applying patch loading for calculating background response. (All loads are unit magnitude and the height is normalized by the total height H.) (a) IASS method of applying patch loading. The total dimensionless moment is $\mathrm{BMF}(z) = \max[\mathrm{BMF}_i(z)]$, where BMF_i is the bending moment for the i-th loading system. (b) Root sum square method for applying patch loading. The total dimensionless moment is $\mathrm{BMF}(z) = \sqrt{[\Sigma \mathrm{BMF}_i^2(z)]}$, where $\mathrm{BMF}_i(z)$ is the contribution from the i-th unit load.

include this in the size and length of the patch loads, as yet without success. Based on figures in refs. 3 and 5, the following approximate corrections are suggested for $L < \lambda$

$$\mathrm{BMF} = \begin{cases} \mathrm{BMF}_0 (L/\lambda)^{0.2} & \text{for midspans, except first} \\ \mathrm{BMF}_0 & \text{for supports and first midspan} \end{cases} \tag{11}$$

where BMF_0 is the background moment factor for $L = \lambda$.

Figure 4 compares the response of the statistical method and the two patch loading approaches for three different masts. The IASS method seems to overestimate the response, while the RSS method gives a closer agreement to the statistical approach.

3.2. Dynamic magnification factor (simplified)

It was found that the shapes of the background and the resonant responses looked rather similar. Thus, the dynamic response can be estimated by multiplying the background response by a dynamic magnification factor

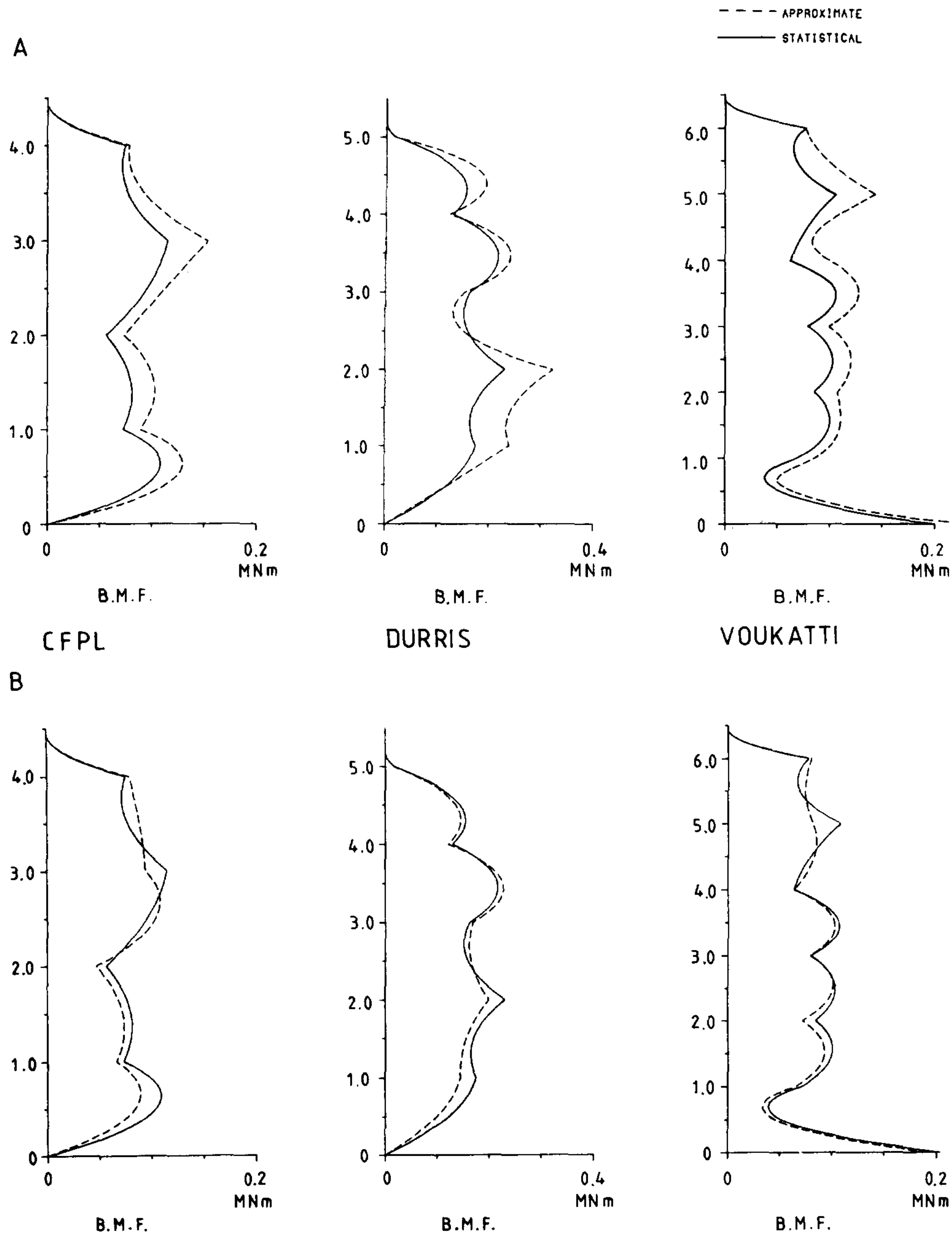

Fig. 4. Comparison between the statistical method and the patch loading approaches for calculating the background moment factor (BMF). The loading system is either: (A) the IASS or (B) the root sum square.

as described by eqns. (8) and (9). Based on studies in ref. 5, it was found that the dynamic term DT (eqn. 9) can be approximated by

$$\mathrm{DT} = \begin{cases} 4 & \text{open; forest, } z > 1/3 \\ 7 & \text{forest, } z < 1/3 \end{cases} \tag{12}$$

If the base of the mast is fixed then

$$\mathrm{DT} = \begin{cases} 2.5\ \mathrm{DT} & \text{lowest midspan} \\ 0.5\ \mathrm{DT} & \text{base} \end{cases} \tag{13}$$

The DMF will always be larger than 1.0. If the background response is larger than the resonant response then the DMF $< \sqrt{2}$. The dynamic magnification factor (DMF) can now be found by eqn. (8).

When the magnitude of the Q factor is large, DMF is also large. Thus, the resonant response dominates the dynamic response. The above method, which is based on the background response, is most accurate when the resonant response is small. Therefore, it is required that

$$Q < 0.7 \tag{14}$$

This will limit the approach to communications masts, i.e. masts with a low mass distribution. Most guyed stacks have a Q value larger than 0.7; the simplified method will be less reliable for a guyed stack.

3.3. Examples of moment calculation

The total dynamic moment of a mast can be determined by eqns. (7)–(14). Figure 5 compares the moments from this codified approach to the statistical method in Section 2.1. The masts have either 4, 5 or 6 guy levels, and the terrain class is either open country or forest. Other examples of the moment calculation are shown in ref. 5. They also confirm the encouraging agreement, as long as the Q factor is less than 0.7.

4. Uncertainties in the model

The structural analysis of a guyed mast is complex and the load systems from wind, ice and/or temperature cannot be exactly defined. The analysis may incur higher uncertainties than other structures. A difficult problem in the analysis concerns the guy cables.

4.1. The guy cables

For a dynamic analysis of a taut cable the primary mode of interest is the first. This is represented quite accurately by the spring–mass–spring model (Fig. 1). As the cable becomes more slack, contributions to the response from the higher modes will increase. The "slackness" can be de-

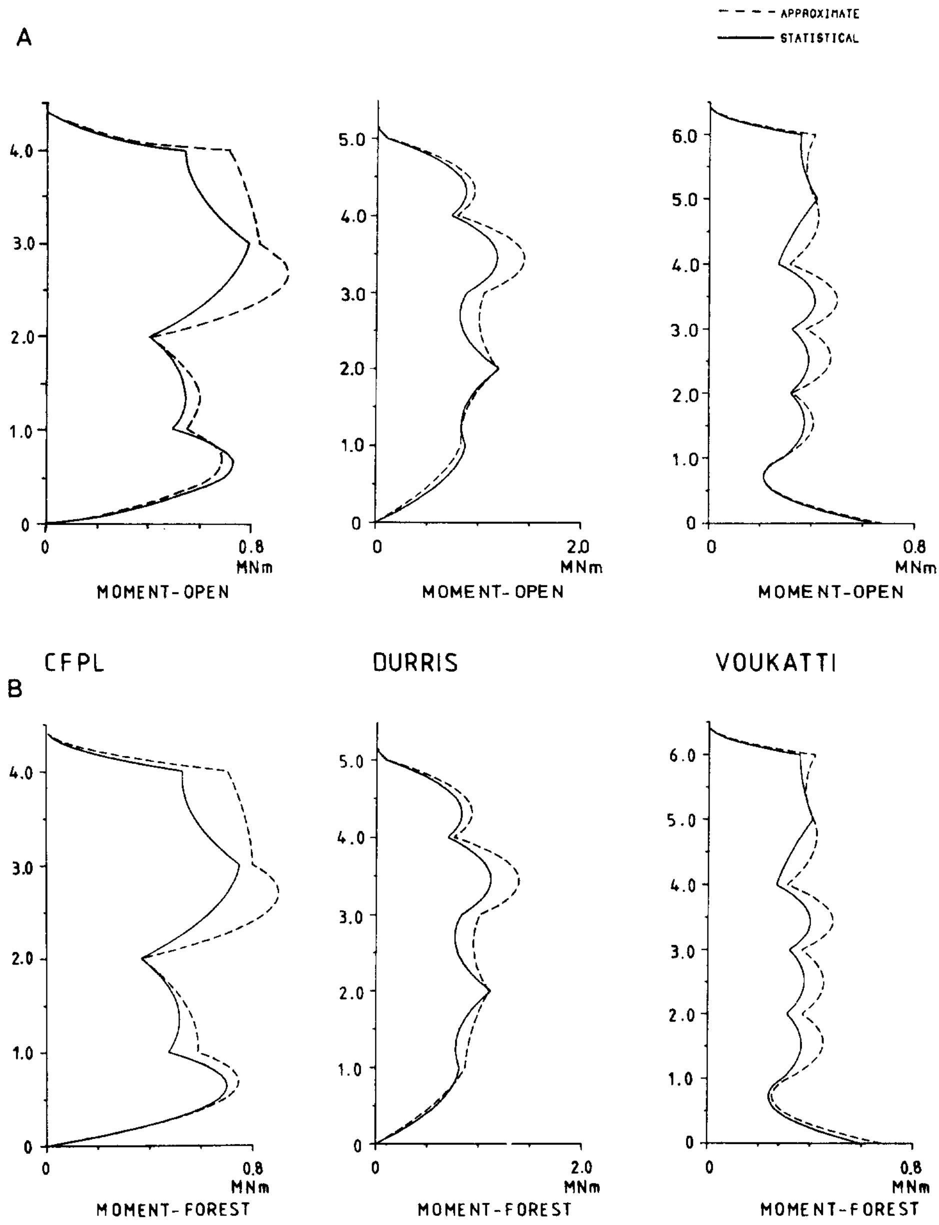

Fig. 5. Total dynamic moment calculated by the statistical method and the simplified approach. The moments are calculated for (A) open country or (B) forest.

scribed by the ratio between the gravity stiffness, k_g, and the elastic stiffness, k_e

$$k_g/k_e = 12\,\frac{\sigma_c^3}{(\rho_c\,gb)^2E} \tag{15}$$

Where σ_c is the stress in the cable, ρ_c is the effective density of the cable, E is the modulus of elasticity, b is the base length of the cable and g the acceleration of gravity.

A taut cable will have $k_g/k_e \to \infty$, a slack cable $k_g/k_e \to 0$.

It can be shown that the frequency of the symmetric modes of the cable varies with this stiffness ratio, while the antisymmetric mode is nearly independent of this ratio. The frequencies of the lowest symmetric and antisymmetric modes couples strongly when [5, 13] k_g/k_e is in the range 0.5—0.3. This question has been discussed in some detail by Karna [13].

The reason for considering the region of crossover can be explained as follows. The antisymmetric mode is easy to activate because it is inextensible. At crossover, the coupling between the modes will be large, hence it will be possible to transfer energy from the antisymmetric mode to the symmetric mode. The symmetric mode is strongly coupled with the mast and it is thus possible to increase the transfer of energy to the mast.

For a design practice, this situation should be avoided. In order to avoid this situation it is advised that

$$\frac{k_g}{k_e} > 1.0 \tag{16}$$

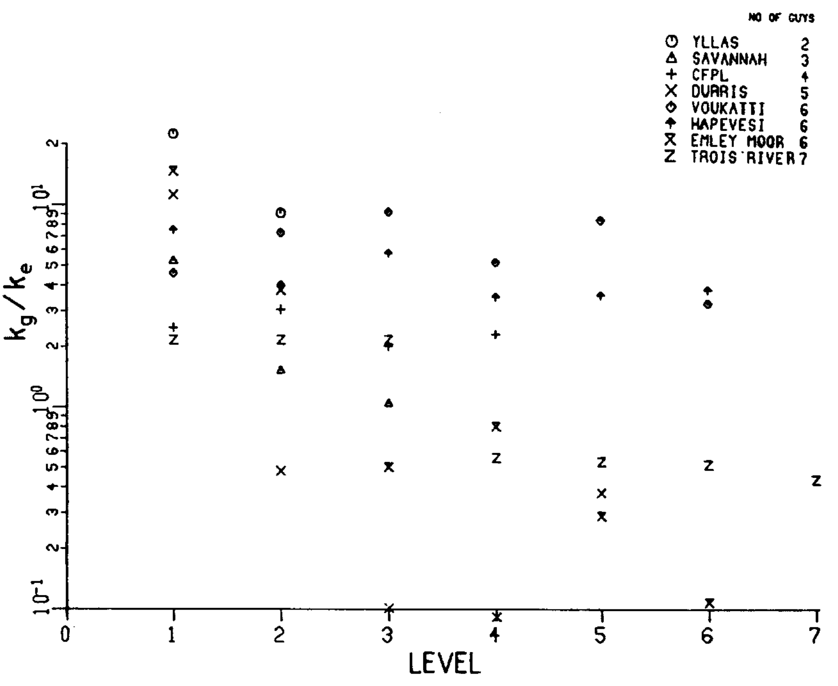

Fig. 6. Ratios of gravity and elastic stiffness for guy cables.

It should be noted that under wind and ice loads, the value of k_g/k_e can alter significantly. The above equation is not a guarantee for avoiding crossover. From parabolic cable-theory and assuming the cable fixed at the ends, it is found that if $k_g/k_e > 0.5$ the apparent stiffness will decrease for a lateral wind or iceload.

Figure 6 depicts the ratio of k_g/k_e as a function of span for some real masts. This includes several masts which suffered from large vibrations of the guy cables and were in a range where the crossover condition is likely to have occurred in the deflected shape.

The equality (16) advocates a high stress in the cables. In order to avoid high bending moments in the mast through the P-delta effect it is, from a static point of view, preferable to have a low stress in the cables. Practical considerations will also limit the value of the stress in the cables. No optimal value of the stress in a cable has yet been found.

The crossover phenomenon is not included in the dynamic approach described here, nor is the guy mass (Fig. 1), nor any excitations due to wind and/or ice. When the cable is taut, $k_g/k_e > 1$, these phenomena are less likely to be of importance. However, many existing masts have such low values of k_g/k_e that the phenomena are important.

4.2. The mast

The axial force in a guyed mast is largely due to the vertical load from the guy cables. For the static load this can give a large contribution to the moment through the so-called P-delta effect. When the axial force approaches the buckling load the natural frequencies will decrease. For practical values of the axial load the change in the resonant response due to the inclusion of the axial load is insignificant (less than 8%).

The simplified approach was developed for a "standard mast", with constant bending stiffness, constant mass per unit length and the guy stiffness inversely proportional to height. Real masts will deviate from this, and thus the response factors found for these masts will deviate from the standard masts. The expected error for this was investigated. It was found [5] that the resonant and background responses were sensitive to variation in the structural parameters. The dynamic magnification factor was approximately a factor 10 less sensitive. This supports the proposed approach where the background moment is based on the actual mast, and the dynamic magnification factor is approximately calculated from the average structural values.

5. Conclusion

A simplified method for estimating the dynamic wind response of a guyed mast has been presented. This method separates the dynamic response into a low-frequency, background region and a high-frequency, resonant region. The background response has been estimated using patch loading. The resonant responses can be taken into account by multiplying the background response by a dynamic magnification factor. This factor depends, in a systematic way, on the average structural properties of the mast. The approach is illustrated by examples, and compared to a statistical method.

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