Kalman and Particle Filtering for Geoacoustic Parameter Tracking

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Outline

I. Introduction
Geoacoustic Inversion vs. Tracking

II. Tracking Filter Theory
Extended (EKF), Unscented Kalman (UKF), Particle (PF) Filters

III. Geoacoustic Parameter Tracking
Yardim C., P. Gerstoft, and W. S. Hodgkiss, “Tracking of geoacoustic parameters using Kalman and particle filters”, JASA, 125(2), pp.746-760, 2009.

IV. Source Tracking
Yardim C., P. Gerstoft, and W. S. Hodgkiss, “Source tracking in changing geoacoustic environments”, JASA, to be submitted, 2009.

V. Results

VI. Conclusions

Introduction

What is geoacoustic tracking? What is a tracking filter?

- Geoacoustic tracking is the estimation of the evolution of geoacoustic parameters sequentially, temporal and/or spatial. (estimates and underlying posterior densities)
- A tracking filter is a recursive Bayesian estimator.

Why do it?

Efficient way of doing sequential estimation. A framework that handles both the previous values of the parameters and the sequential data at each index *k.*

How to do it?

- Kalman Framework, the optimal recursive Bayesian estimator for linear/Gaussian.
- Sequential Monte Carlo Techniques.

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Inversion vs. Tracking

Geoacoustic Inversion

$$\mathbf{d}^{obs} = h(\mathbf{m}) + \mathbf{e}$$

Forward model

- $\mathbf{m}$: state vector
- $\mathbf{d}^{obs}$: measurement vector
- $\mathbf{e}$: measurement noise vector

PPD: $p(\mathbf{m} \mid \mathbf{d})$

Geoacoustic Tracking

Environmental evolution model

$$\mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}, \mathbf{v}_k) \quad \text{state equation}$$

$$\mathbf{y}_k = h_k(\mathbf{x}_k, \mathbf{w}_k) \quad \text{measurement equation}$$

Forward model

- $\mathbf{x}_k$: state vector
- $\mathbf{y}_k$: measurement vector
- $\mathbf{v}_k$: process/state noise vector
- $\mathbf{w}_k$: measurement noise vector

$$p(\mathbf{x}_k \mid \mathbf{X}_{k-1}, \mathbf{Y}_k)$$

$$\mathbf{X}_{k-1} = \mathbf{x}_{k-1}, \cdots, \mathbf{x}_0 \qquad \mathbf{Y}_k = \mathbf{y}_k, \cdots, \mathbf{y}_0$$

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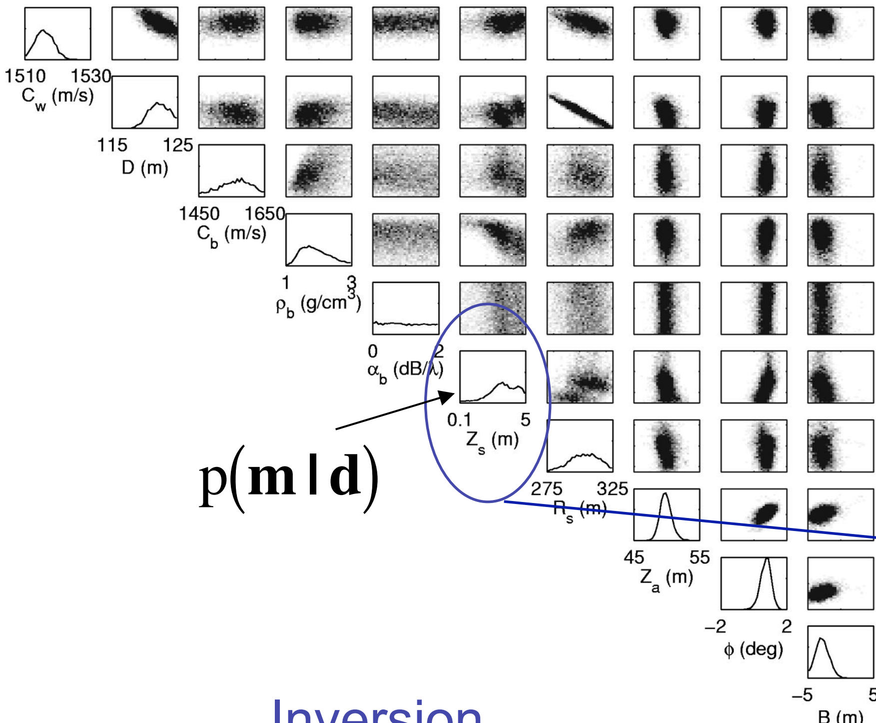
Inversion vs. Tracking

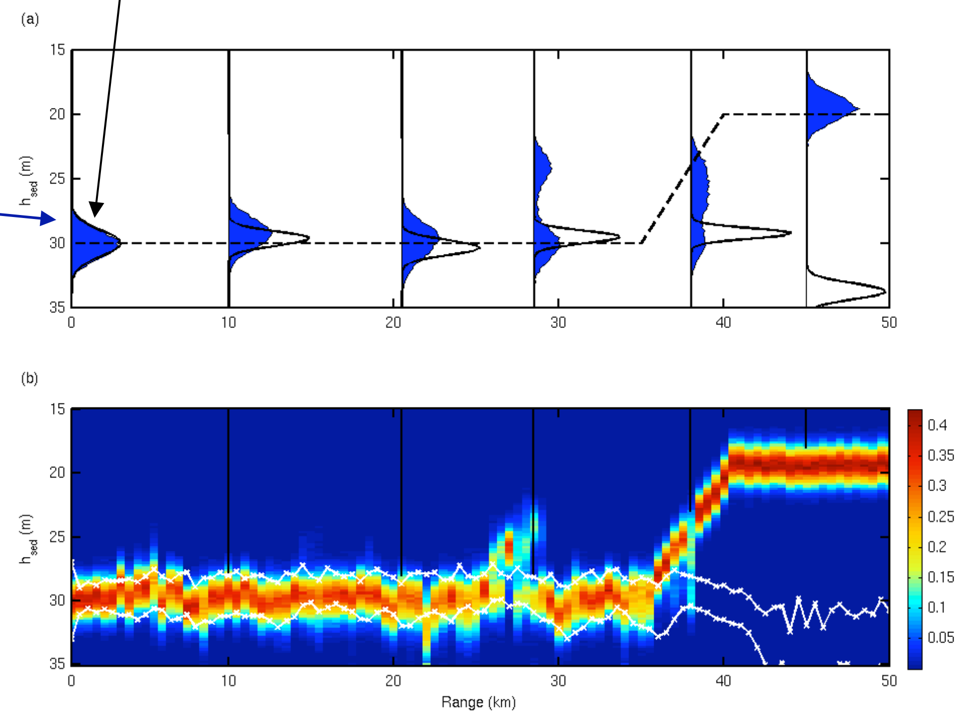
Tracking

$$p(\mathbf{x}_k \mid \mathbf{X}_{k-1}, \mathbf{Y}_k)$$

$$\mathbf{X}_{k-1} = \mathbf{x}_{k-1}, \cdots, \mathbf{x}_0 \qquad \mathbf{Y}_k = \mathbf{y}_k, \cdots, \mathbf{y}_0$$



$$p(\mathbf{m} \mid \mathbf{d})$$



Inversion

A quick guide to filter selection

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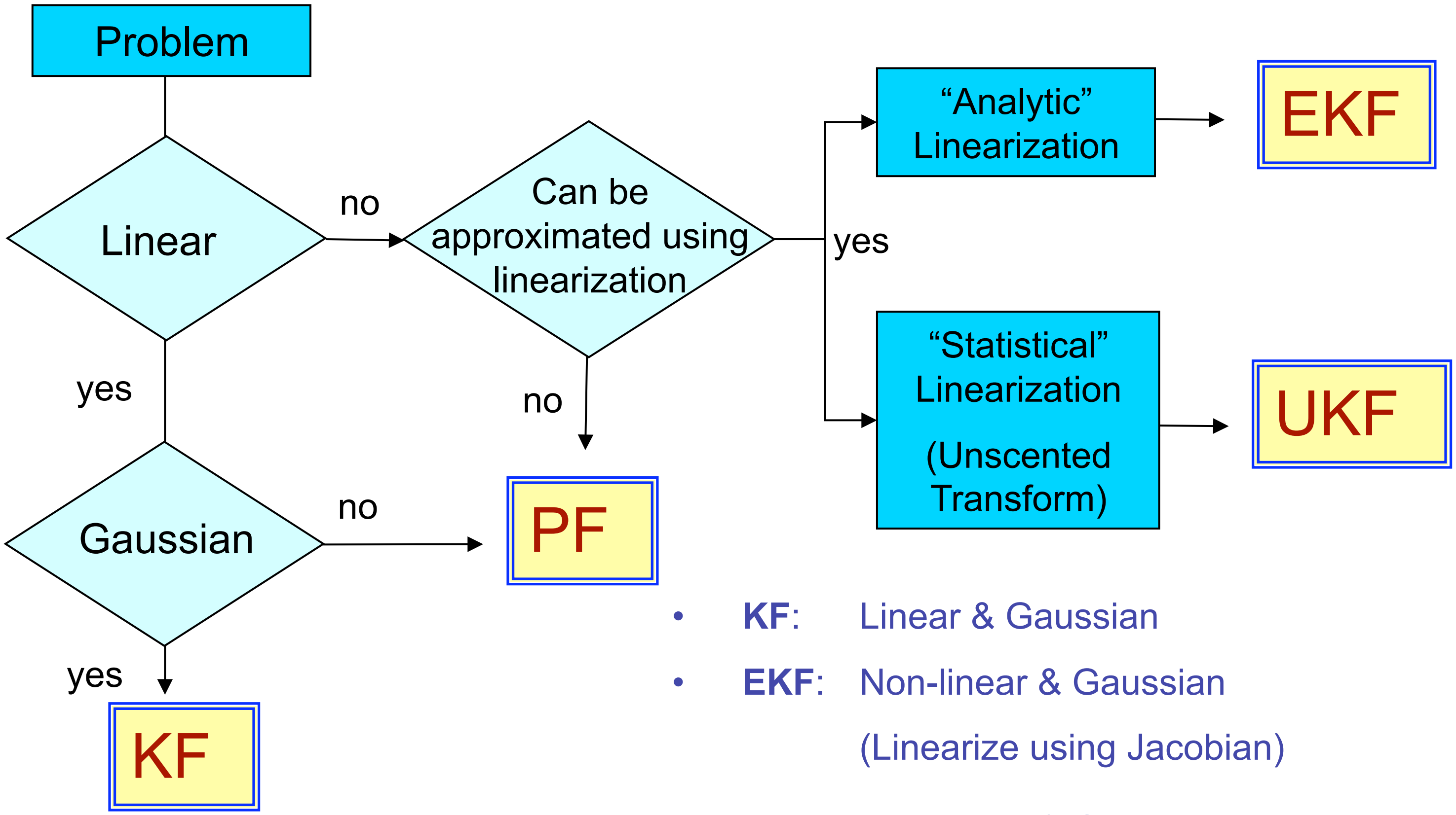

- **KF**: Linear & Gaussian
- **EKF**: Non-linear & Gaussian
  (Linearize using Jacobian)
- **UKF**: Non-linear & Gaussian
  (Assume Gaussian input – Gaussian output)
- **PF**: Non-linear & Non-Gaussian

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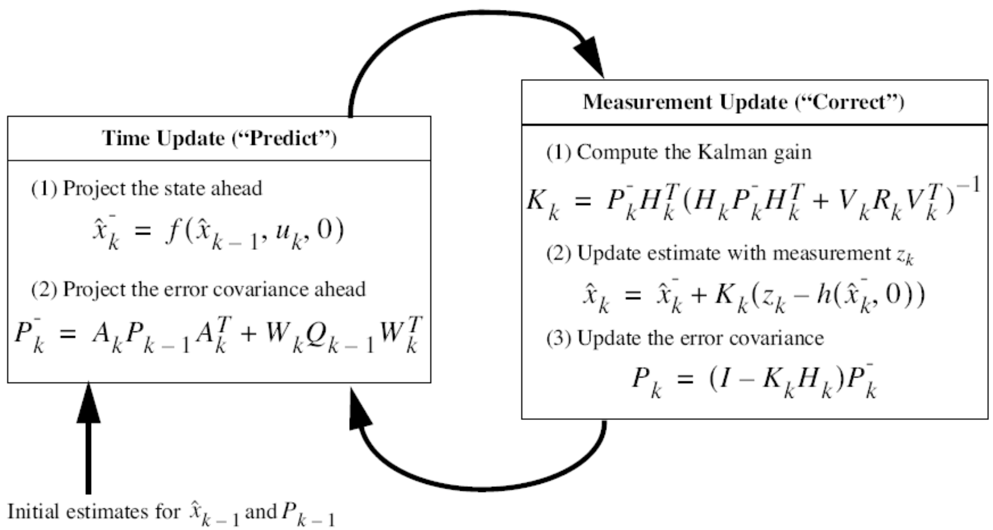
EKF and PF

$$\mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}, \mathbf{v}_k)$$
$$\mathbf{y}_k = h_k(\mathbf{x}_k, \mathbf{w}_k)$$

f, h : nonlinear

$\mathbf{x}_k$, $\mathbf{y}_k$, $\mathbf{v}_k$, $\mathbf{w}_k$: non-Gaussian

EKF



from
Greg Welch
Gary Bishop

PF

$$\mathrm{p}(\mathbf{x}_o) \sim \left\{\chi_o^i\right\}_{i=1}^{N_p}$$
$$\mathrm{p}(\mathbf{x}_k \mid \mathbf{y}_{k-1}) \sim \left\{\chi_{k|k-1}^i\right\}_{i=1}^{N_p}$$
$$\mathrm{p}(\mathbf{x}_k \mid \mathbf{y}_k) \sim \left\{\chi_{k|k}^i\right\}_{i=1}^{N_p}$$

Scenarios and Possible Applications

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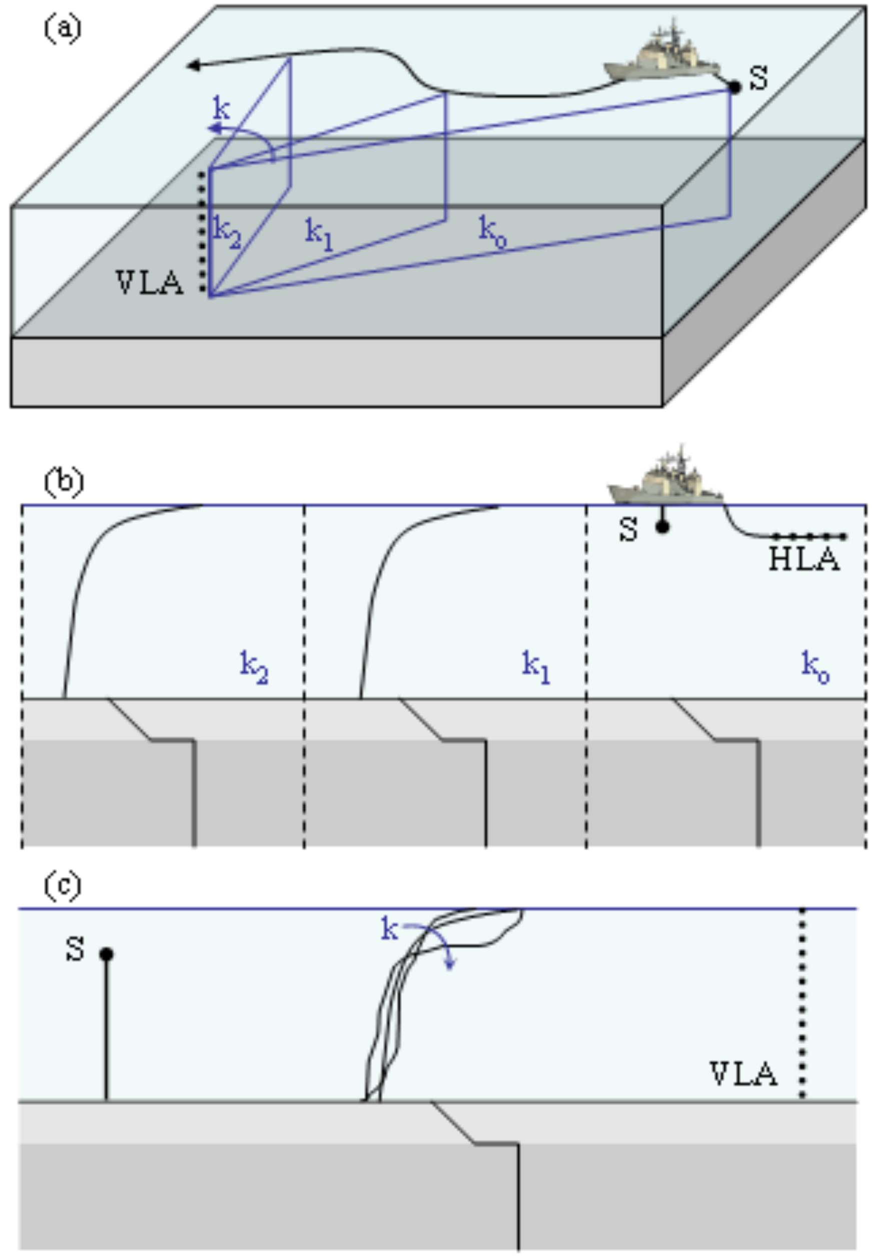
- Towed source/fixed HLA, VLA
- Towed source/HLA platform
- Fixed hydrophone on the seafloor and a towed source
- Tow ship self noise data acquired via a towed HLA
- Passive fathometer from the ocean ambient noise field measured by drifting array
- Fixed source/receiver. Track sound speed evolution

SWARM95, SWAMI98, MAPEX2000, SCARAB98, ASCOT01, Boundary03, Yellow Shark94, MREA/BP07, SW06

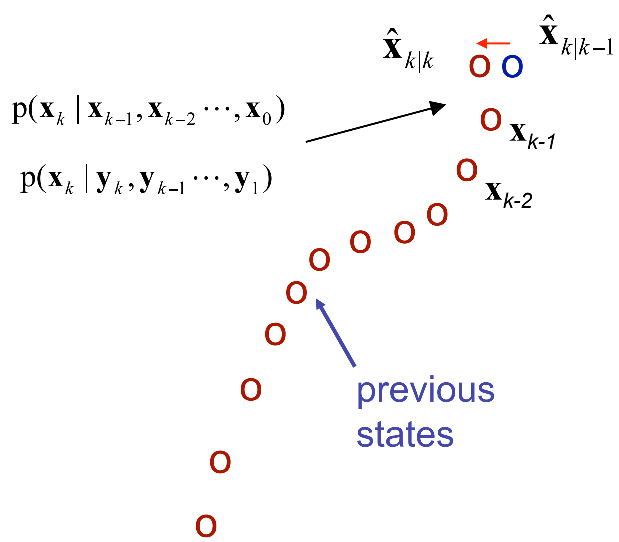


Kalman Framework

A Single Kalman Iteration

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{v}_k$$
$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{w}_k$$

$$\mathbf{x}_{k|k} \sim \mathcal{N}(\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k})$$



1. Predict the mean $\hat{\mathbf{x}}_{k|k-1}$ using previous history.

   $\mathrm{p}(\mathbf{x}_k \mid \mathbf{x}_{k-1})$

   $$\hat{\mathbf{x}}_{k|k-1} = \mathrm{E}\{\mathbf{x}_k \mid \mathbf{x}_{k-1}\} = \int \mathbf{x}_k \, \mathrm{p}(\mathbf{x}_k \mid \mathbf{x}_{k-1}) d\mathbf{x}_k$$

2. Predict the covariance $\mathbf{P}_{k|k-1}$ using previous history.

PREDICT

3. Correct/update the mean using new data $\mathbf{y}_k$

   $\mathrm{p}(\mathbf{x}_k \mid \mathbf{Y}_k)$

   $$\hat{\mathbf{x}}_{k|k} = \mathrm{E}\{\mathbf{x}_k \mid \mathbf{Y}_k\} = \int \mathbf{x}_k \, \mathrm{p}(\mathbf{x}_k \mid \mathbf{Y}_k) d\mathbf{x}_k$$

4. Correct/update the covariance $\mathbf{P}_{k|k}$ using $\mathbf{y}_k$

UPDATE

$$\cdots \Rightarrow \mathrm{p}(\mathbf{x}_{k-1} \mid \mathbf{Y}_{k-1}) \Rightarrow \mathrm{p}(\mathbf{x}_k \mid \mathbf{Y}_{k-1}) \Rightarrow \mathrm{p}(\mathbf{x}_k \mid \mathbf{Y}_k) \Rightarrow \cdots$$

PREDICTOR-CORRECTOR

DENSITY PROPAGATOR

Geoacoustic Parameter Tracking

Full state equation

$$\mathbf{m}_k = \mathbf{F}^{\mathbf{m}}_{k-1}\mathbf{m}_{k-1} + \mathbf{B}^{\mathbf{m}}_{k-1}\mathbf{v}^{\mathbf{m}}{}_{k-1}$$

$$\mathbf{F}^{\mathbf{m}}_{k-1} = \mathbf{I} \quad \mathbf{B}^{\mathbf{m}}_{k-1} = \mathbf{I}$$

Filter Performance and PCRLB

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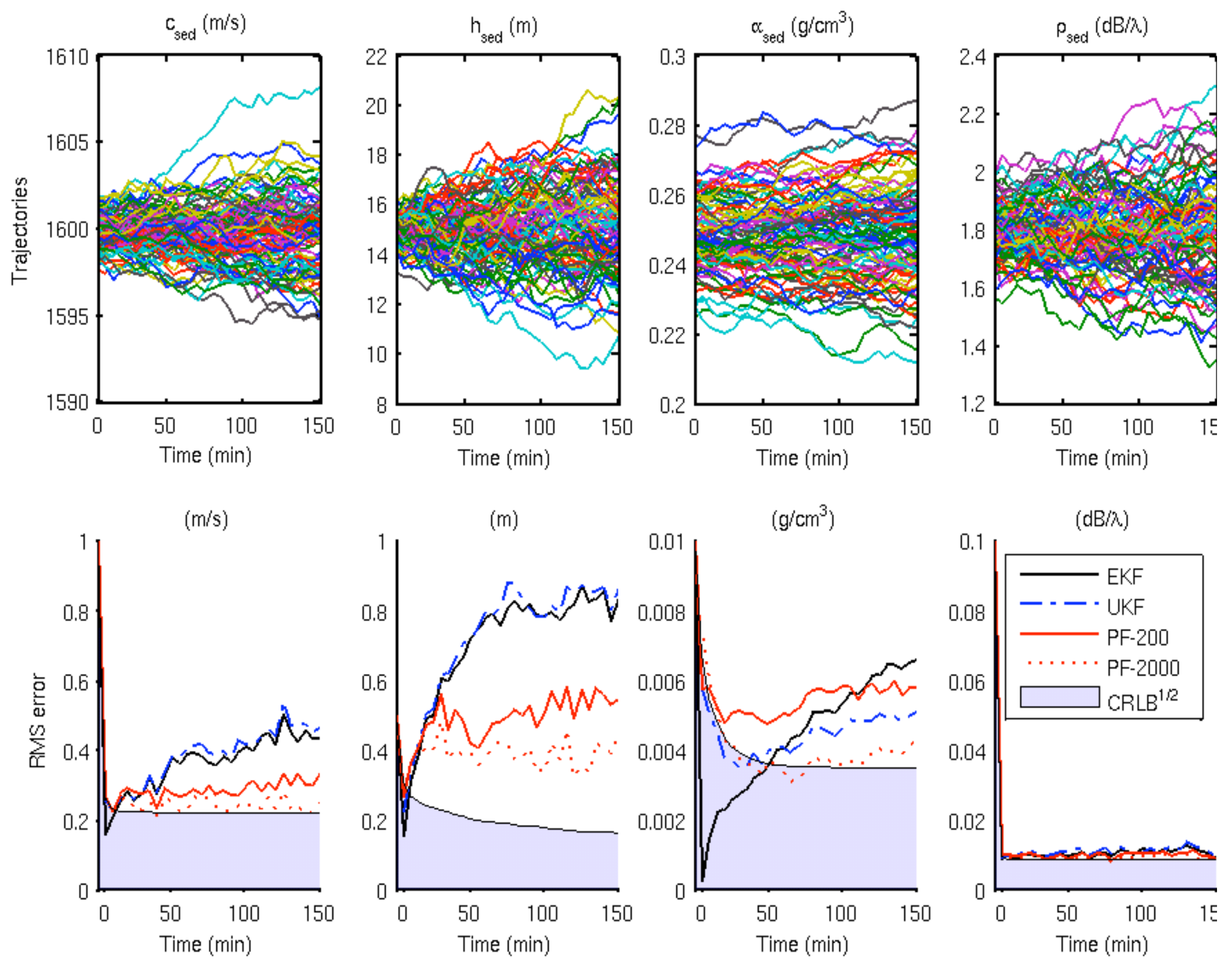
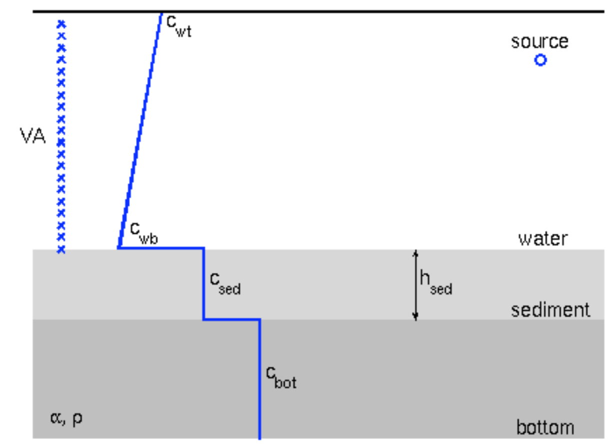
- 5 km range
- Performance of 100 tracks
- EKF, UKF, PF-200, PF-2000
- Posterior or Bayesian CRLB (MC sampled)





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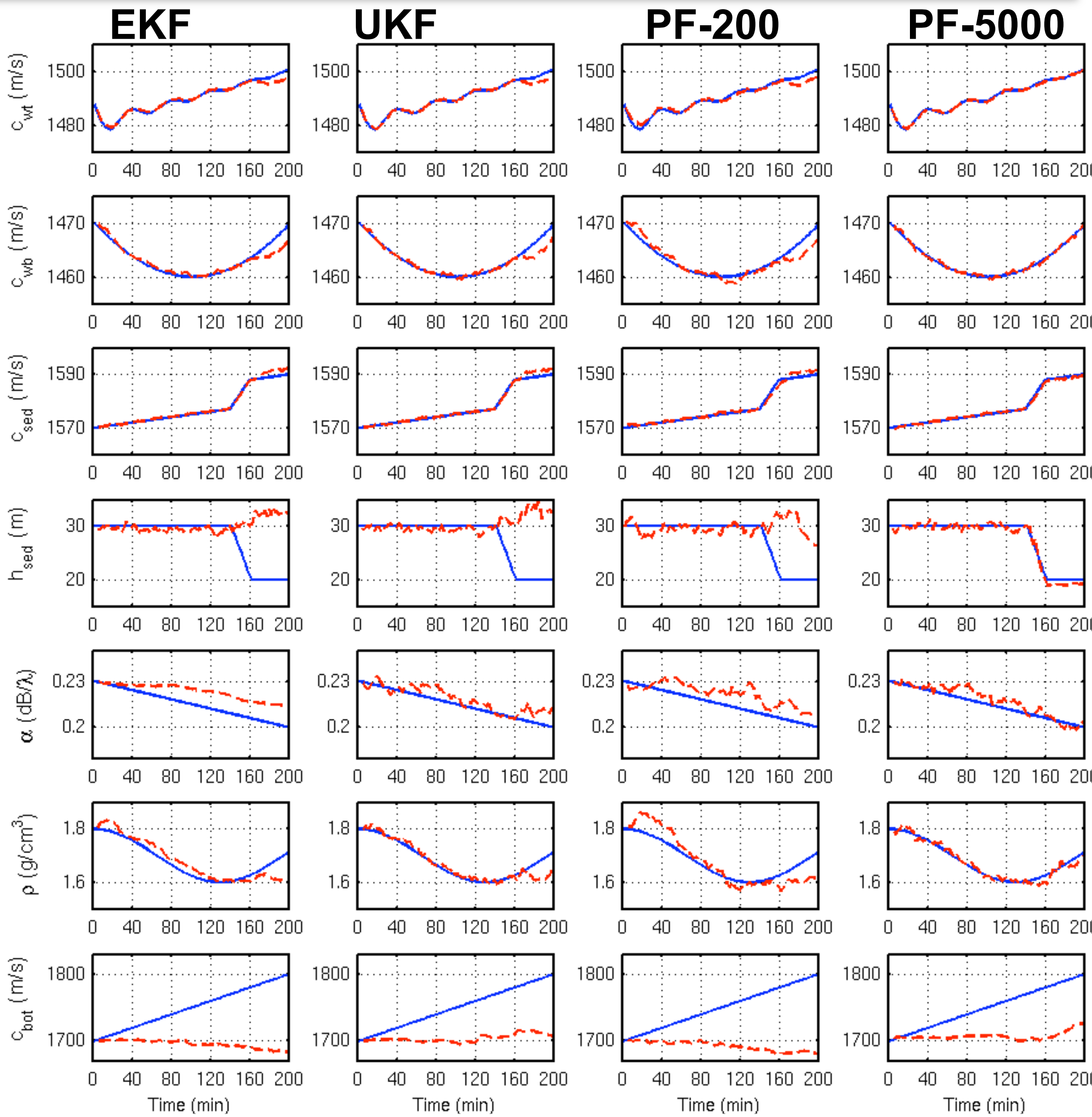
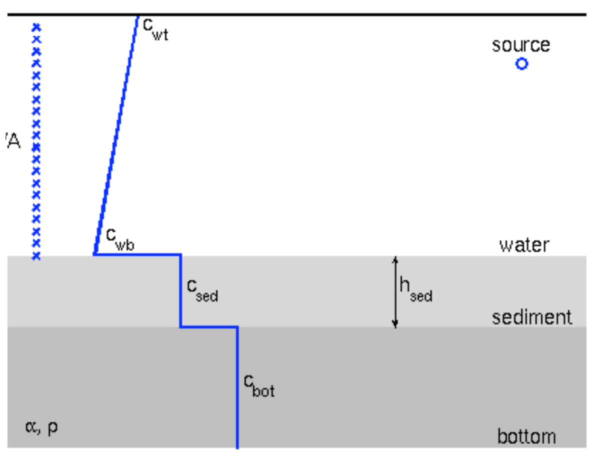
Tracking Example 2

- Evolution of a 200 min track with jump in sediment, VLA 5km range
- True environment
- Tracked environment

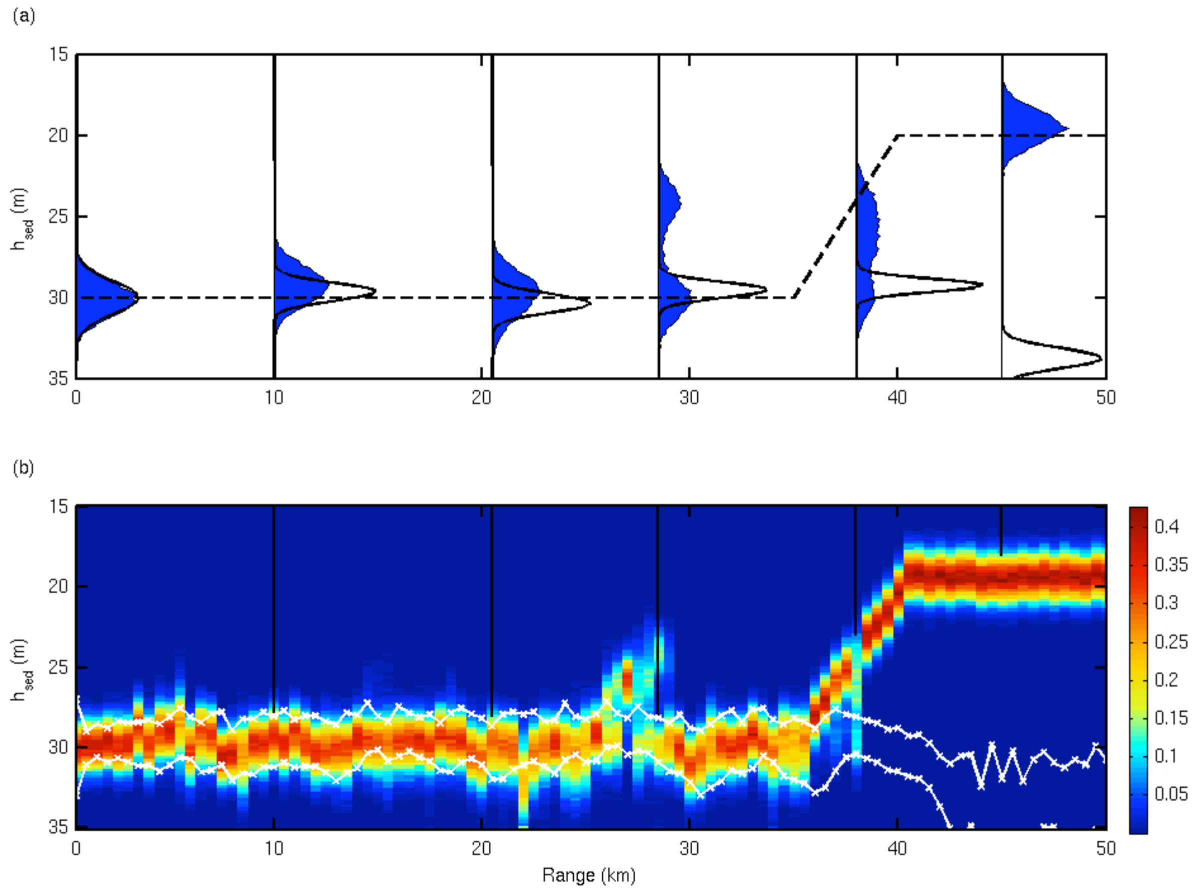
- PF-5000 tracks sediment jump





Evolution of PPD

- PPD of sediment thickness
- Black curves: EKF (Gaussian)
- PF with 10k particles
- MCMC requires typically 100 k to 1 M particles
- PF requires less particles, because it is based on the history



Source Tracking

environment

$$\mathbf{m}_k = \mathbf{F}^{\mathbf{m}}_{k-1}\mathbf{m}_{k-1} + \mathbf{B}^{\mathbf{m}}_{k-1}\mathbf{v}^{\mathbf{m}}{}_{k-1}$$

$$\mathbf{F}^{\mathbf{m}}_{k-1} = \mathbf{I} \quad \mathbf{B}^{\mathbf{m}}_{k-1} = \mathbf{I}$$

Source (CV)

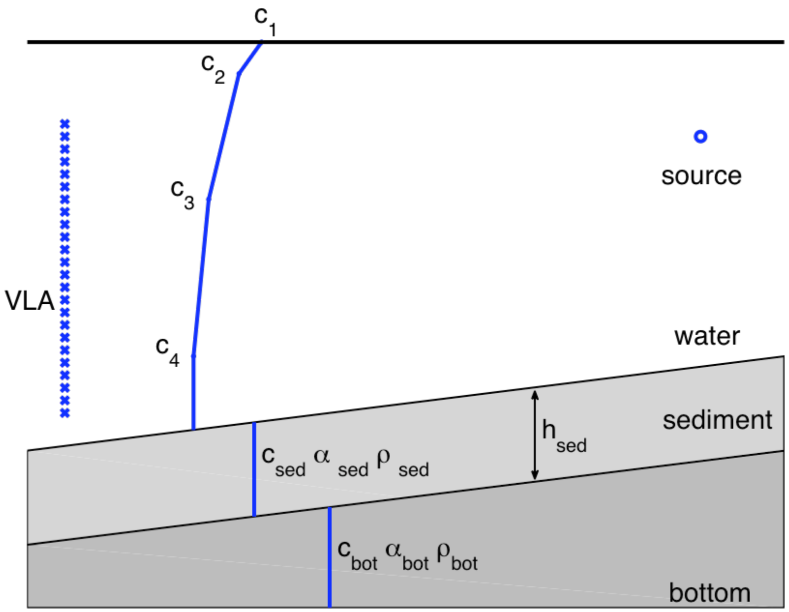
$$\mathbf{s}_k = \mathbf{F}^{\mathbf{s}}_{k-1}\mathbf{s}_{k-1} + \mathbf{B}^{\mathbf{s}}_{k-1}\mathbf{v}^{\mathbf{s}}{}_{k-1}$$

$$\begin{bmatrix} z_s \\ r_s \\ v_s \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_s \\ r_s \\ v_s \end{bmatrix}_{k-1} + \begin{bmatrix} 1 & 0 \\ 0 & \frac{\Delta t^2}{2} \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} \mathrm{v}_{z_s} \\ \mathrm{v}_{a_s} \end{bmatrix}_{k-1}$$

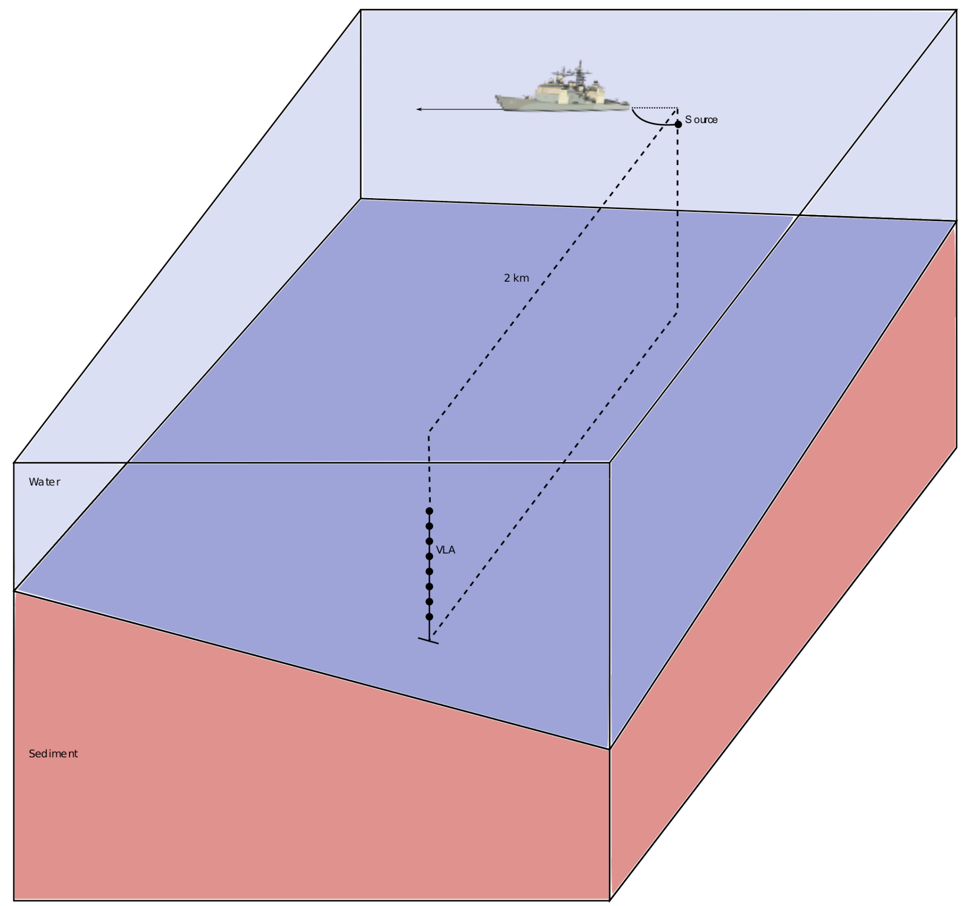
Full state equation

$$\begin{bmatrix} \mathbf{s} \\ \hline \mathbf{m} \end{bmatrix}_k = \begin{bmatrix} \mathbf{F}^{\mathbf{s}}_{k-1} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{s} \\ \hline \mathbf{m} \end{bmatrix}_{k-1} + \begin{bmatrix} \mathbf{B}^{\mathbf{s}}_{k-1} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{v}^{\mathbf{s}} \\ \hline \mathbf{v}^{\mathbf{m}} \end{bmatrix}_{k-1}$$

Source Tracking



Dosso, JASA, 2008



Source Tracking

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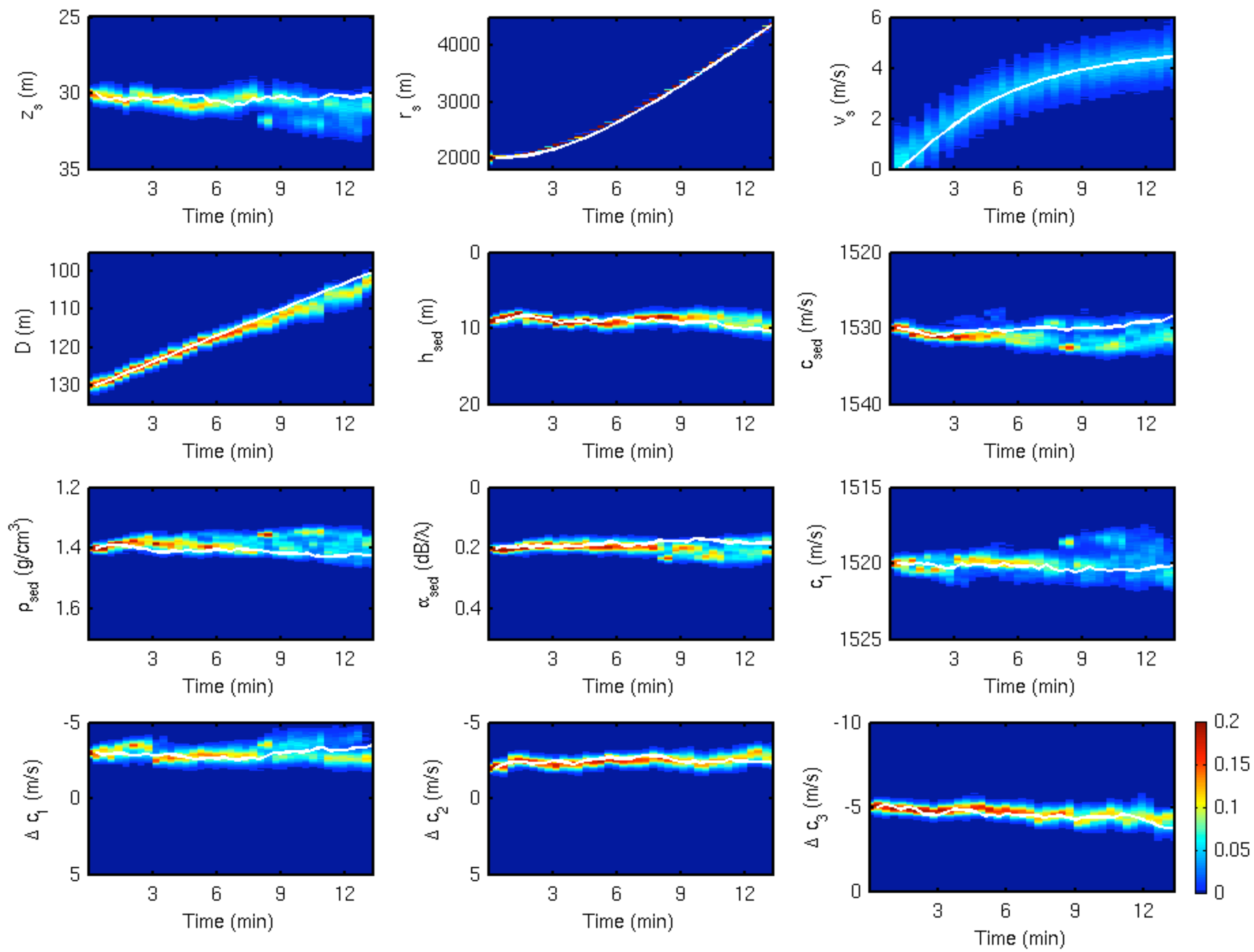
Evolving 1-D Marginal PDFs



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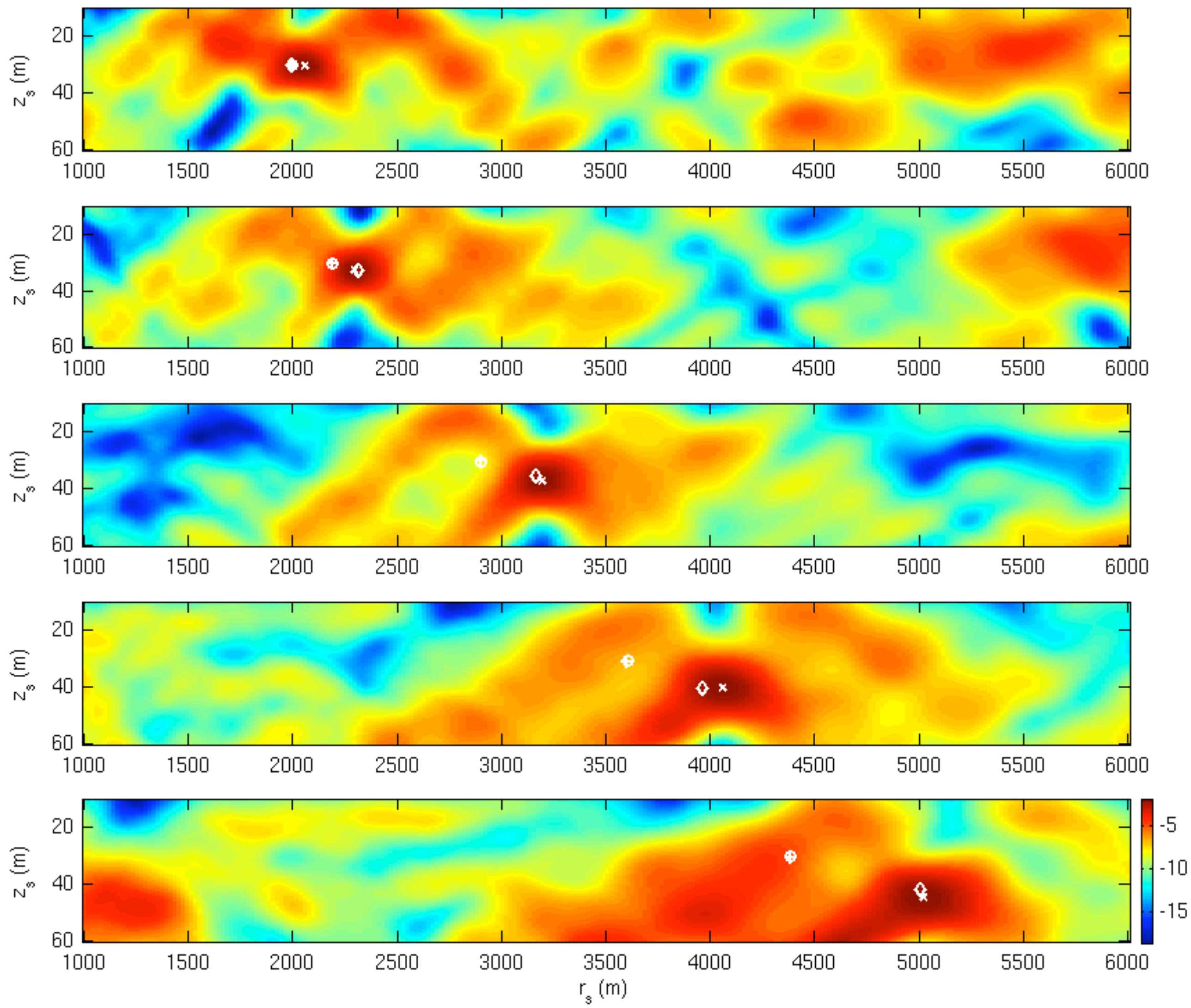
Source Tracking

× Mismatched MFP

○ True Source Location

✚ Full PF

◇ Source-only PF



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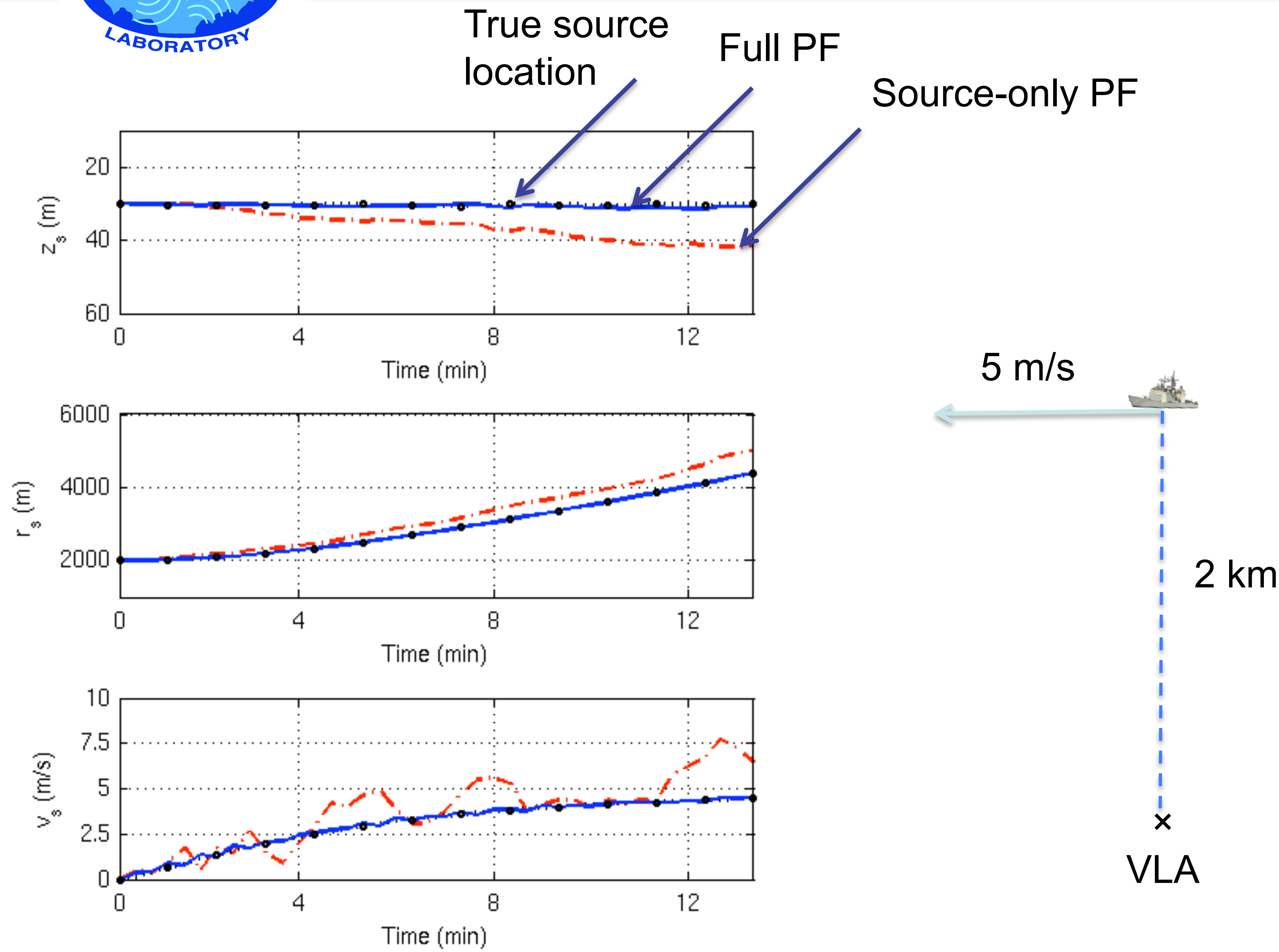
Source Tracking



Conclusions

Geoacoustic tracking can help improve the estimating the evolution of the environmental parameters and their associated uncertainties and can be a useful tool to complement classical geoacoustic inversion algorithms.

- EKF: Easy and fast but not for most geoacoustic tracking problems which can be highly nonlinear and non-Gaussian.
- UKF: Higher order nonlinearities, but still high nonlinearity and non Gaussian pdfs are problematic.
- PF: No assumptions. for nonlinear, non-Gaussian problems.
- Can help to track source successfully in changing geoacoustic environment.