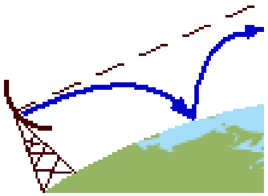
AOS SEMINAR II



Application of a Hybrid Genetic Algorithm - Markov Chain Monte Carlo Sampler in Bayesian Parameter Estimation

February 03, 2005

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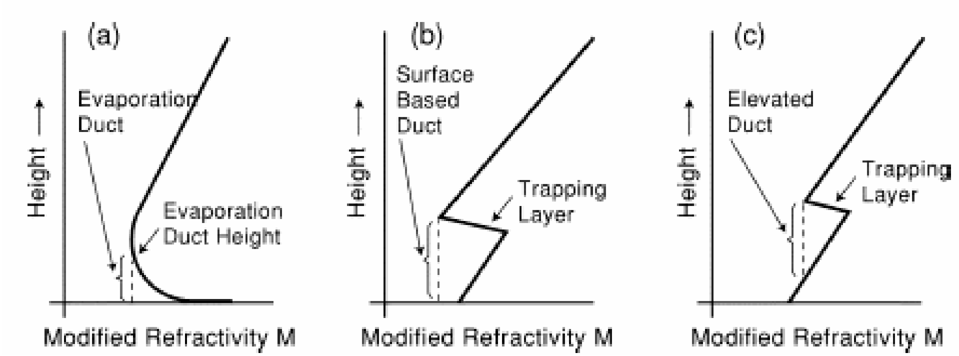
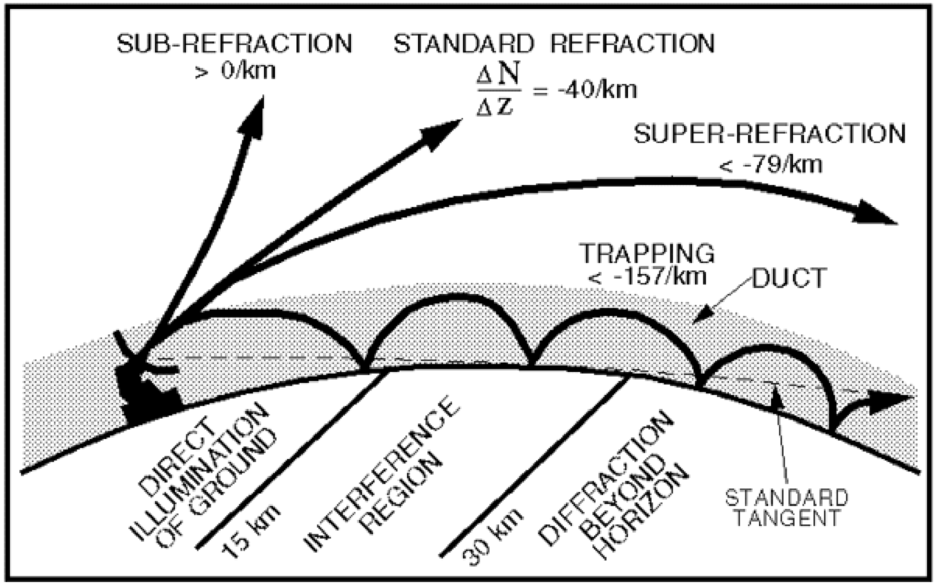
OUTLINE

- Introduction
- Refractivity From Clutter (RFC) Problem
- Inversion Problem and Bayesian Framework
- Implementation of RFC Inversion
  - Genetic Algorithm (GA)
  - Markov Chain Monte Carlo Methods (MCMC)
    - Metropolis Sampler
    - Gibbs Sampler
  - GA-MCMC Hybrid
- Results
- Conclusions

INTRODUCTION

EM Duct in Sea-borne Radar Applications





Three most common ducting profiles

n : the index of refraction
c : the speed of light in vacuum
v : the speed of light in the medium

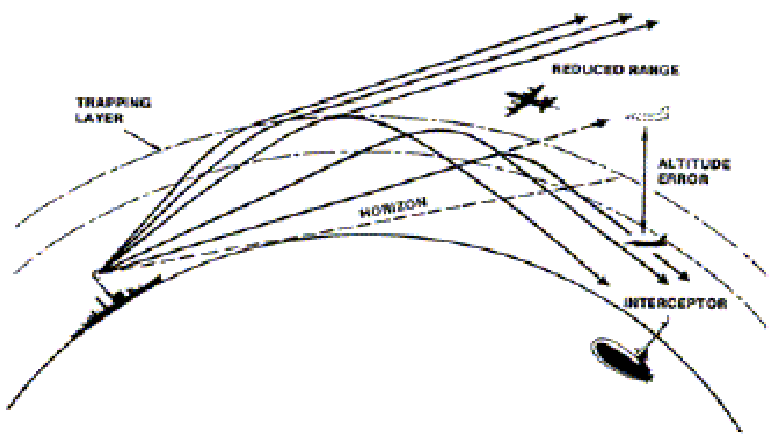
$n = c/v$

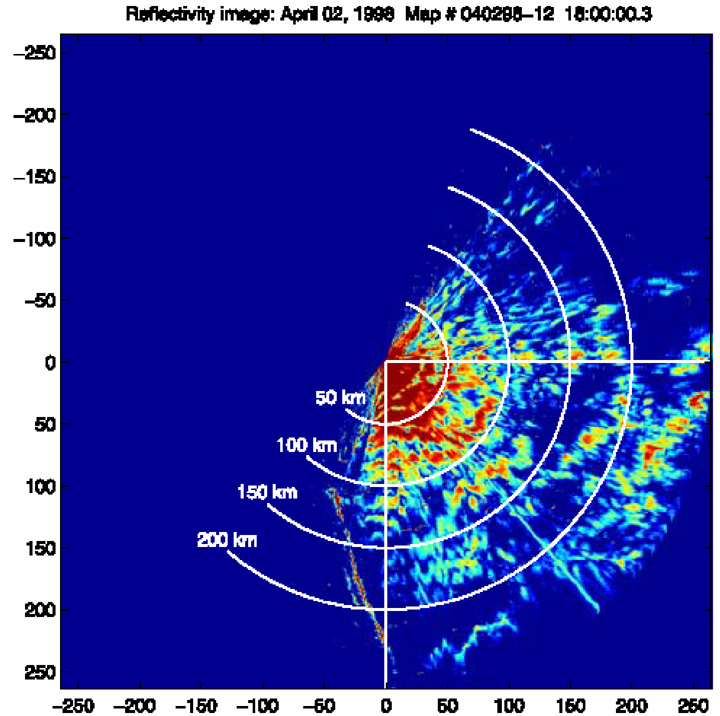
$M \sim n$

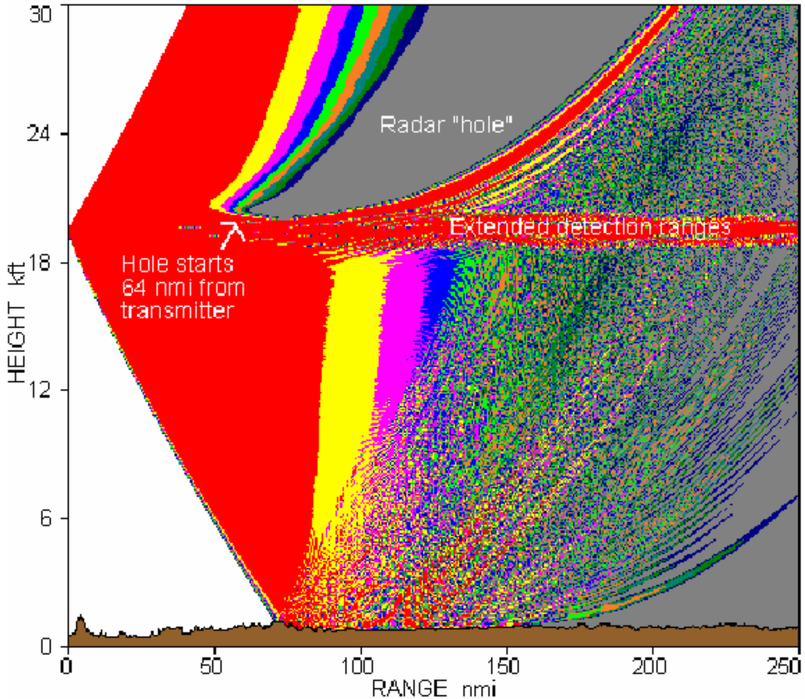
Effects of Ducting

➢ Why do we care about it?
What are the effects on EM Propagation?

1. Blind Zones (Radar Holes)
2. Height Error for 3-D Radars
3. Clutter Rings
4. Extended Range

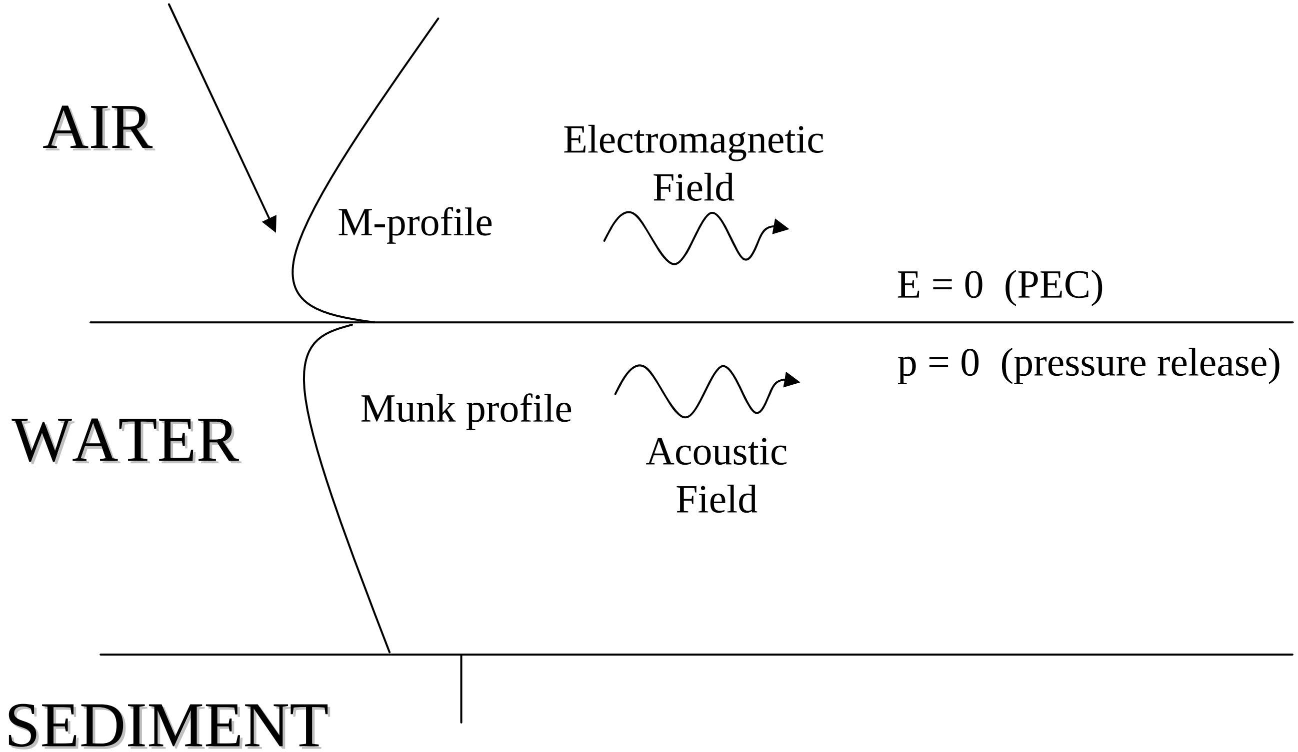






EM vs Acoustic Inversion

Marine Boundary Layer
Humidity & Temperature Profiles



Estimation of the M-Profile

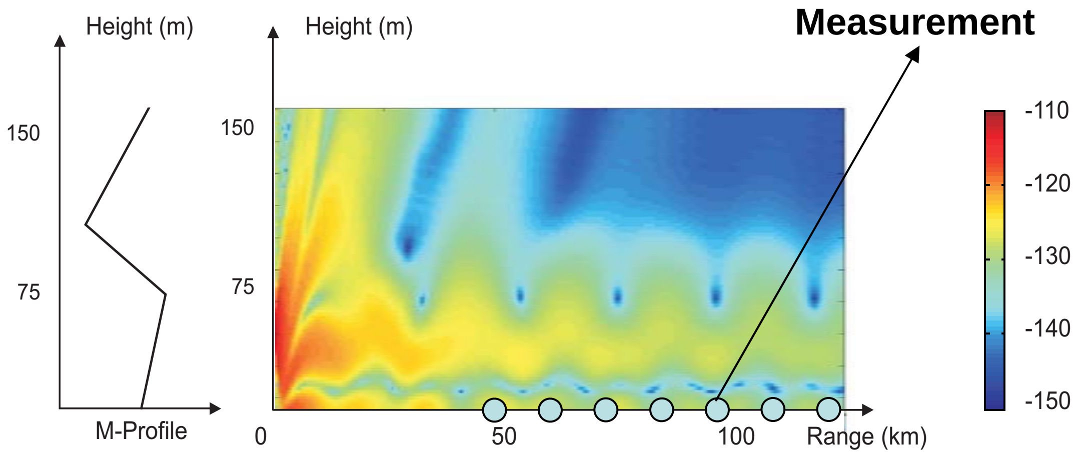
- Conventional Duct Measurement Techniques

  - Bulk Measurements (radiosonde, helicopter soundings, etc)
  - Numerical Weather Prediction Models

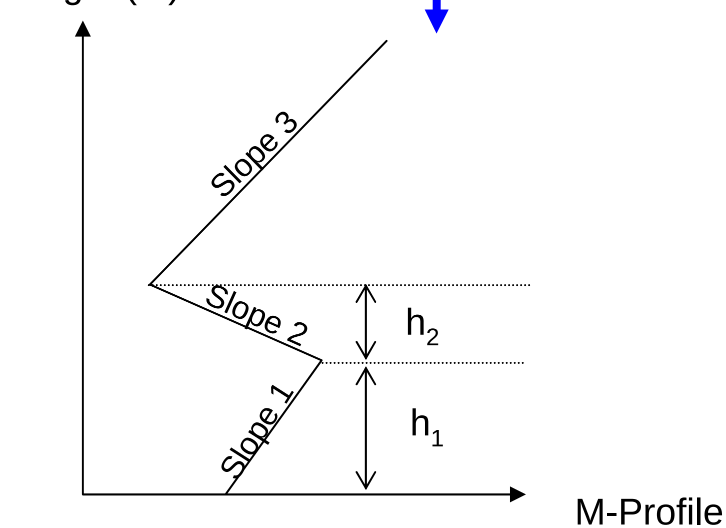
- Alternative Method

  - Refractivity From Clutter (RFC)
    1. No ship based equipment or measurement
    2. No additional signal, Inversion is performed the data acquired during the normal radar operation
    3. Near real-time range dependent refractivity profile

RFC Inversion



Unknown Environment



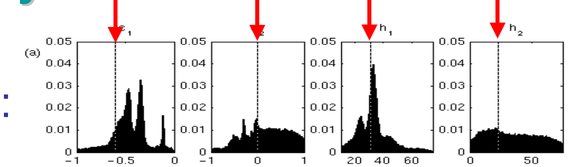
Point Estimation

$\hat{\mathbf{m}} = [\ c_1 \quad c_2 \quad h_1 \quad h_2\ ]$

$= [\ 0.13 \quad -2.5 \quad 20 \quad 40\ ]$

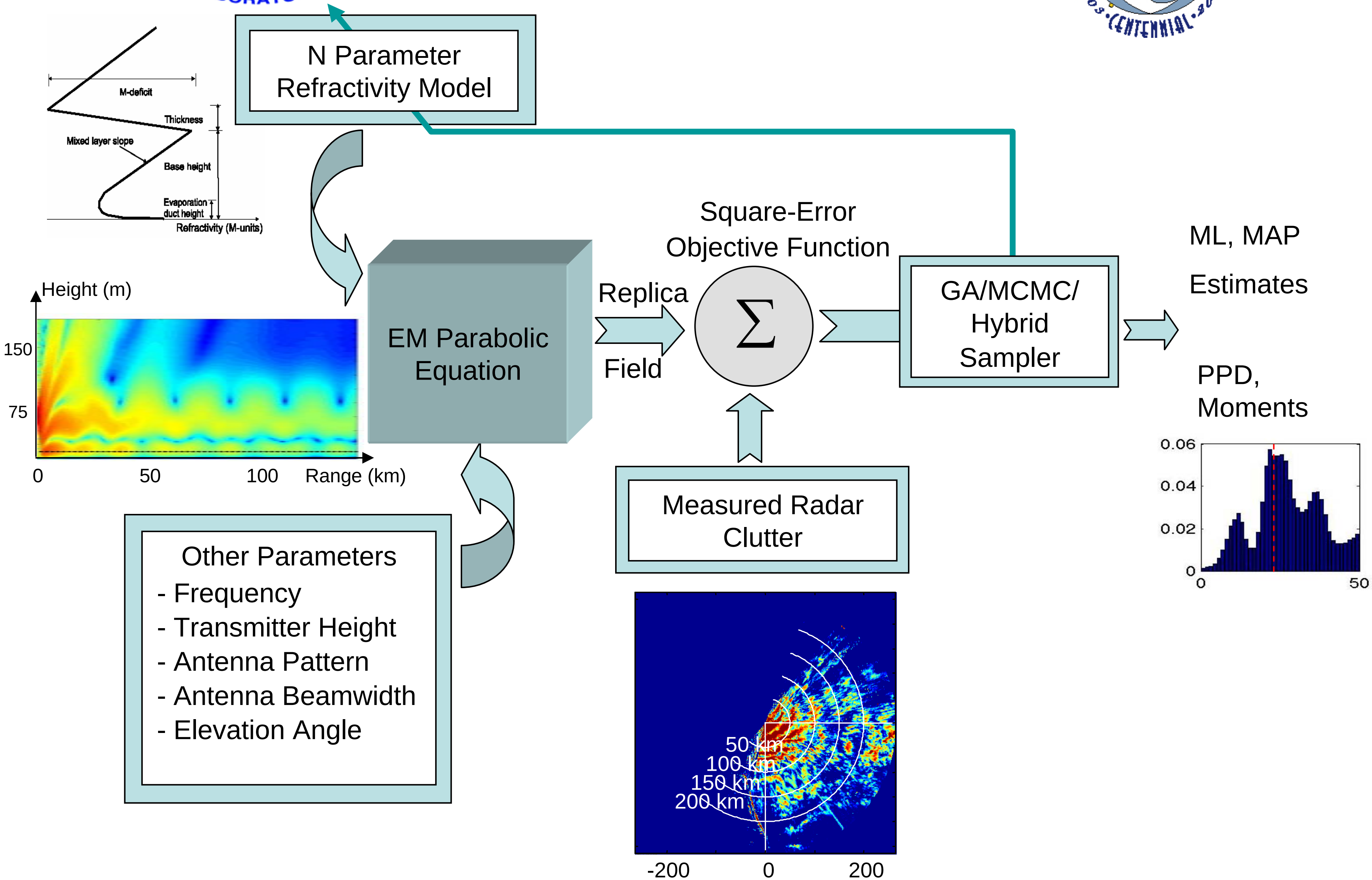
Bayesian Framework

PPD:

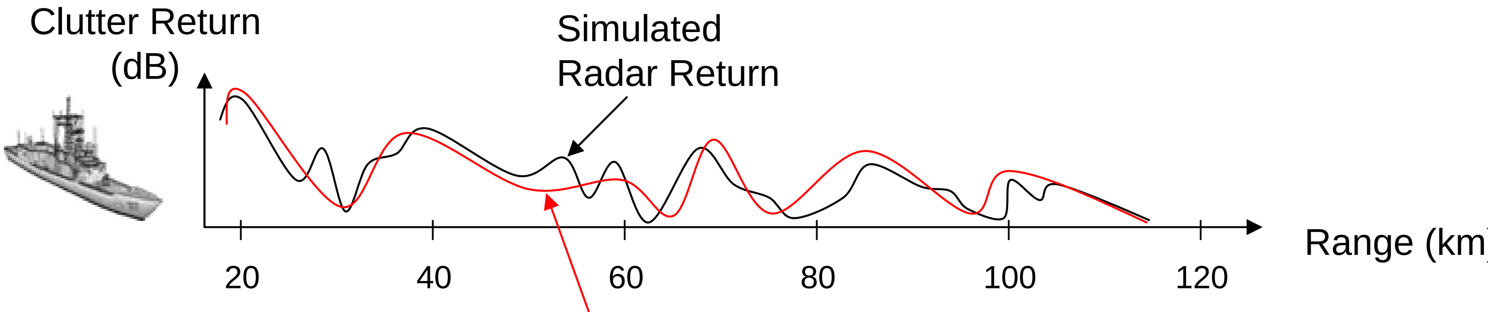


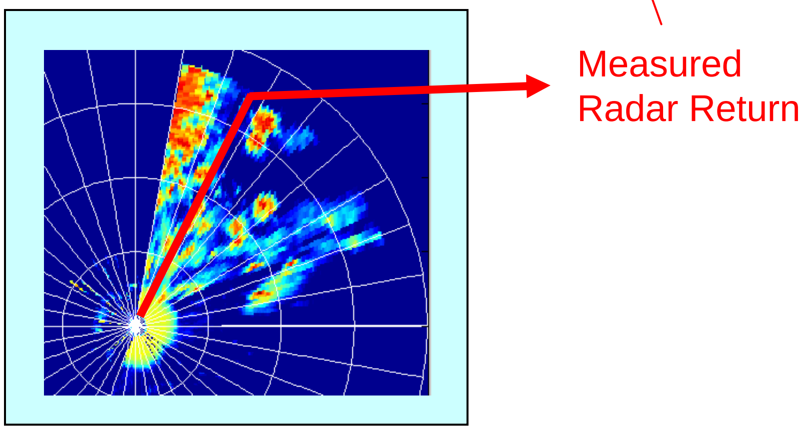
N-dimensional Posterior Probability Density (PPD)

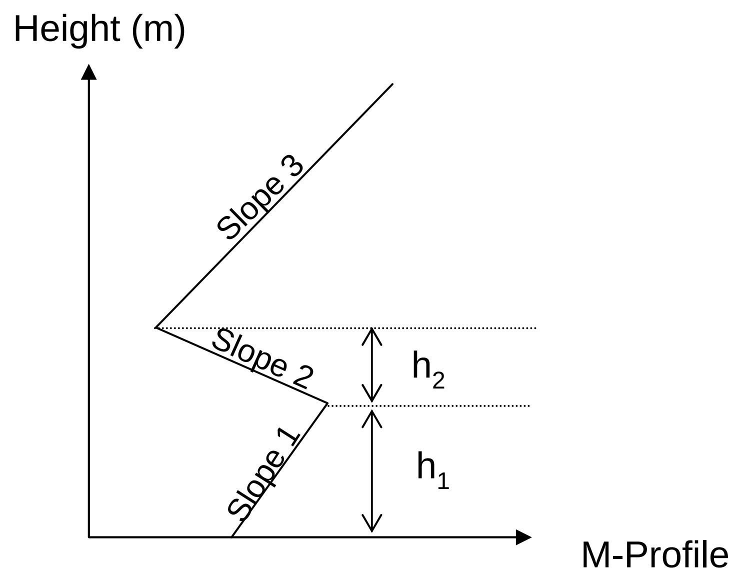
RFC as an Inversion Problem



Inversion ….continued



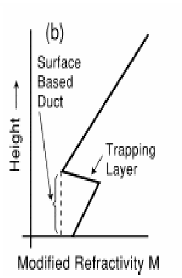




Minimize Objective Function

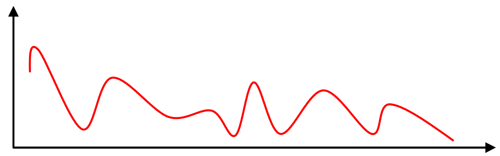
$$\textit{Objective Function} = \sum_{range=R_{\min}}^{R_{\max}} \left[d^{obs} - d(\mathbf{m}) \right]^2$$

Bayes' Theorem



***m**, model*

***m** : [m_1, m_2, m_3,…, m_N]*



***d**, data*

Desired Quantities:

1. p(*model*|*data*) = p(**m**|**d**)
2. Probability distribution of each parameter, pdf, p(m_i|**d**)
3. Means, variances, medians of each parameter

$$\underbrace{PPD = p(\mathrm{m}|\mathrm{d})}^{\text{POSTERIOR}} = \frac{p(\mathrm{d}|\mathrm{m})\,p(\mathrm{m})}{p(\mathrm{d})} = \frac{\overbrace{p(\mathrm{d}|\mathrm{m})}^{\text{LIKELIHOOD}}\,\overbrace{p(\mathrm{m})}^{\text{PRIOR}}}{\underbrace{\int p(\mathrm{d}|\mathrm{m})\,p(\mathrm{m})\,d\mathrm{m}}_{\text{EVIDENCE}}}$$

$$PPD \propto L(\mathrm{m}) = \frac{1}{\sqrt{(2\pi\sigma^2)^R}} \exp\left(-\frac{\sum_R \left(d^{obs} - d(\mathrm{m})\right)^2}{2\sigma^2} \right)$$

Desired Quantities

N-Dimensional Posterior Probability Density

$$PPD \equiv p(\mathbf{m}|\mathbf{d})$$

$$p(m_i|\mathbf{d}) = \iiint \ldots \int_{m_j\ j\neq i} p(\mathbf{m}|\mathbf{d})\, dm_1 dm_2 \ldots . dm_{i-1} dm_{i+1} \ldots . dm_N$$

Marginal Posterior Probability Density

$$\mu_i = < m_i > = \int_{\mathbf{m}} m_i\, p(\mathbf{m}|\mathbf{d})\, d\mathbf{m}$$

$$\sigma_i^2 = \int_{\mathbf{m}} (m_i - \mu_i)^2\, p(\mathbf{m}|\mathbf{d})\, d\mathbf{m}$$

How are we going to get $PPD,\ p(\mathbf{m}|\mathbf{d})$ ⇒

Exhaustive ?

GA ?

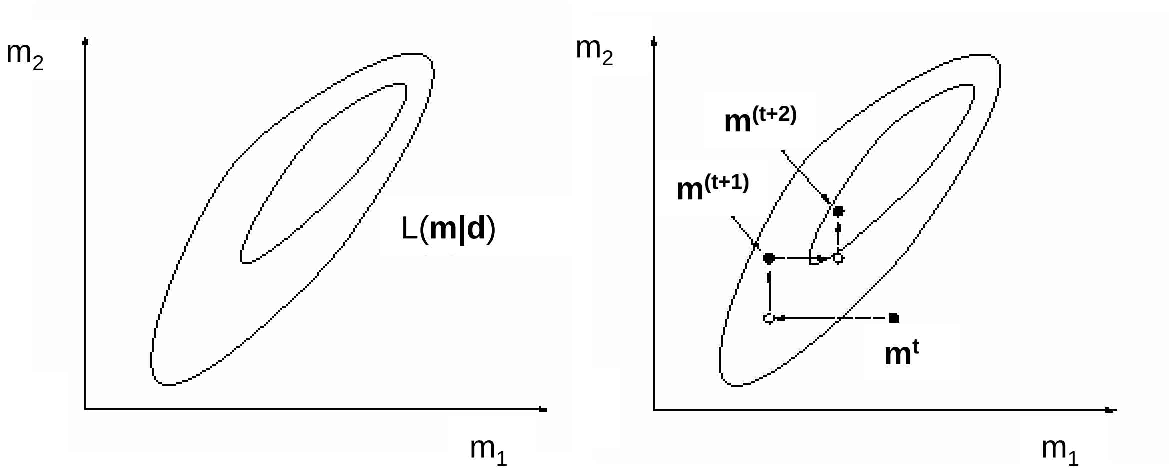
MCMC ?

Other ?

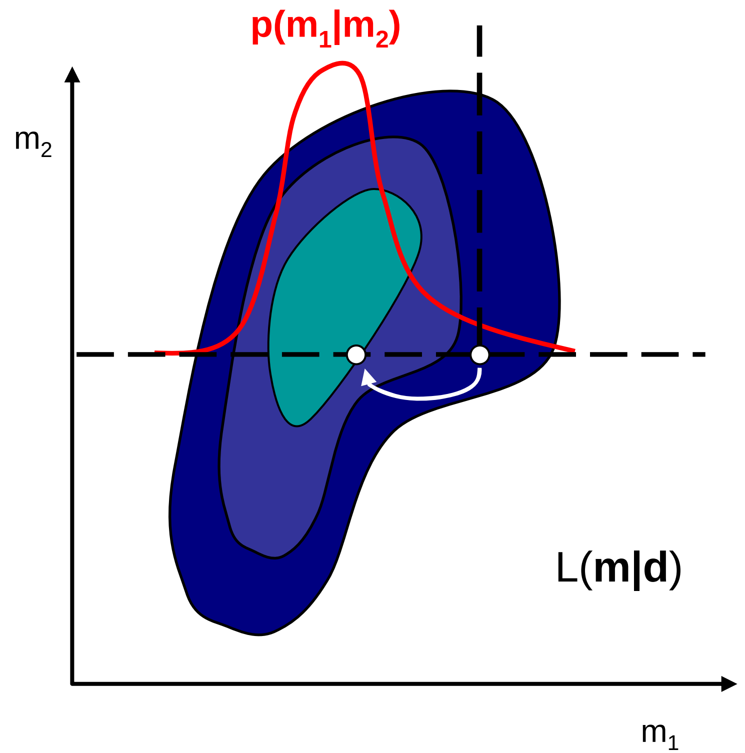
Efficient Sampling Techniques – Markov Chain Monte Carlo

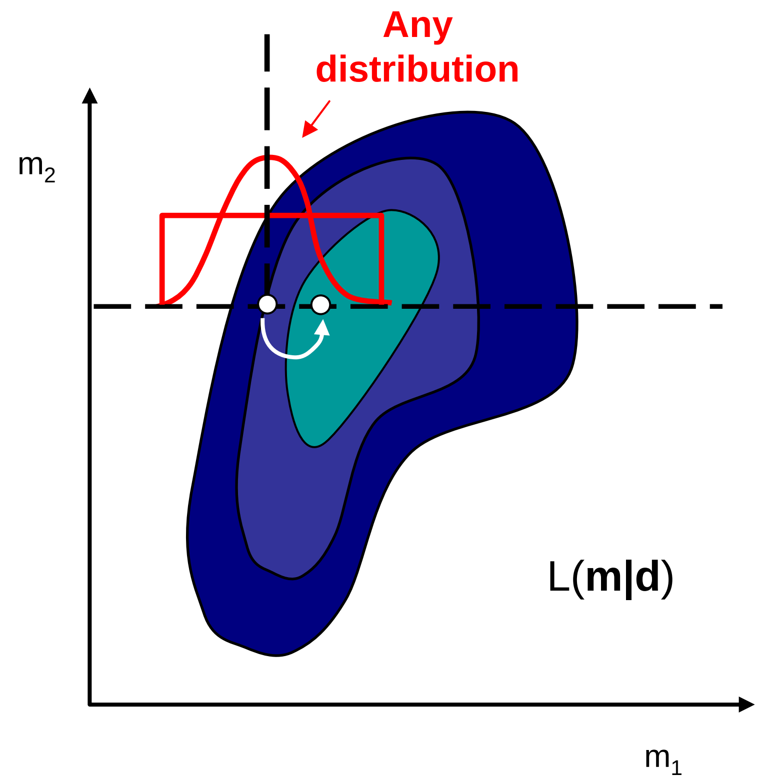
MCMC are algorithms that are mathematically proven to sample the state space in such a way that PPD can be found using these few samples. (Metropolis – Hastings Algorithm, Gibbs Sampling, Slice Sampling,…)

Metropolis Algorithm :



Gibbs and Metropolis Samplers





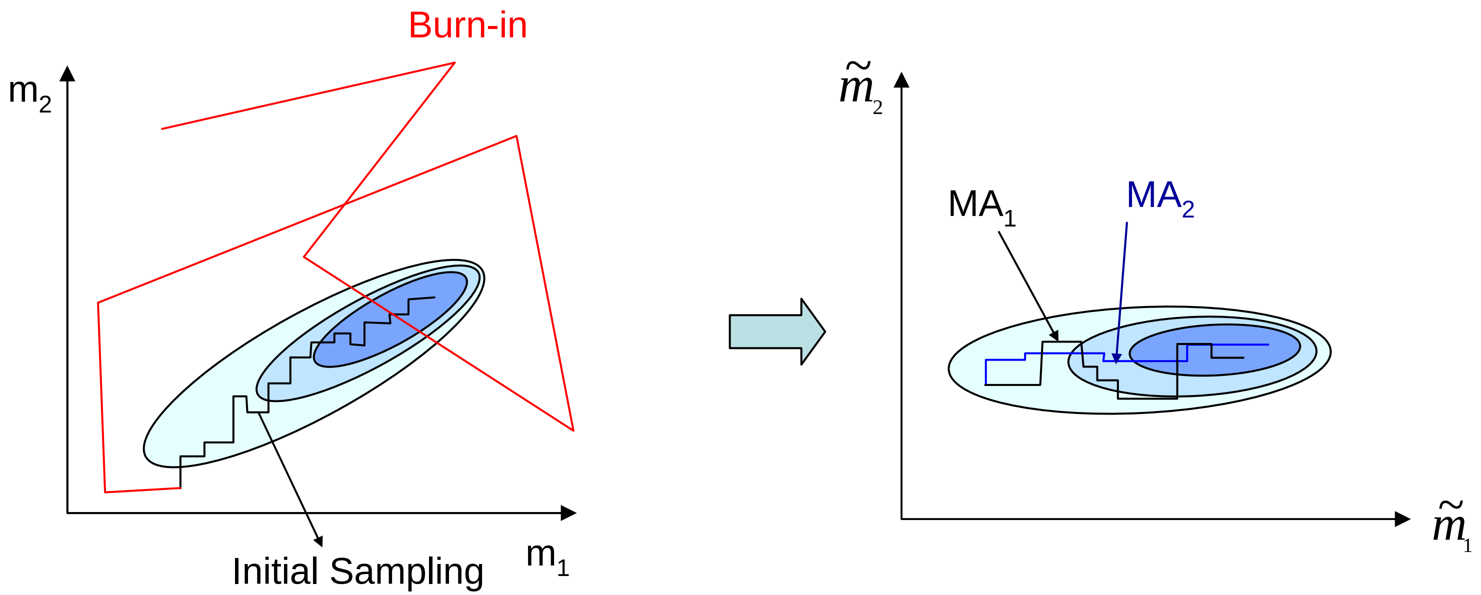
Gibbs Sampler

New point always accepted as the new sample

Metropolis Sampler

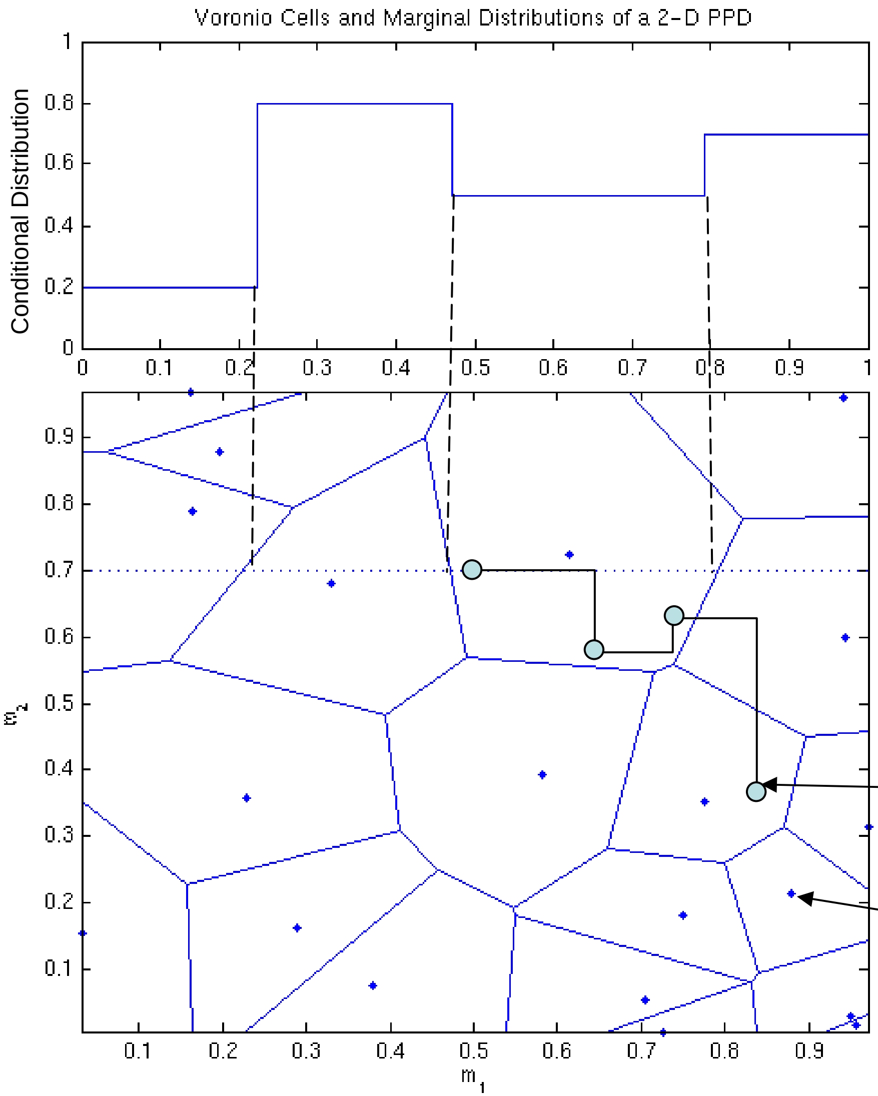
New point accepted only if it passes the Metropolis acceptance test

Illustration of How the Algorithm Works



GA-MCMC Hybrid Method

We know the conditional density
Perfect for Gibbs sampler
p(m1|m2)



1. GA Phase

   (forward model calculations)

2. Voronoi Cells &

   Creation of Approximate PPD

3. MCMC Phase (Gibbs)

   (no forward model calculations)

MCMC samples

GA points

Based on Sambridge's neighborhood Algorithm

RESULTS

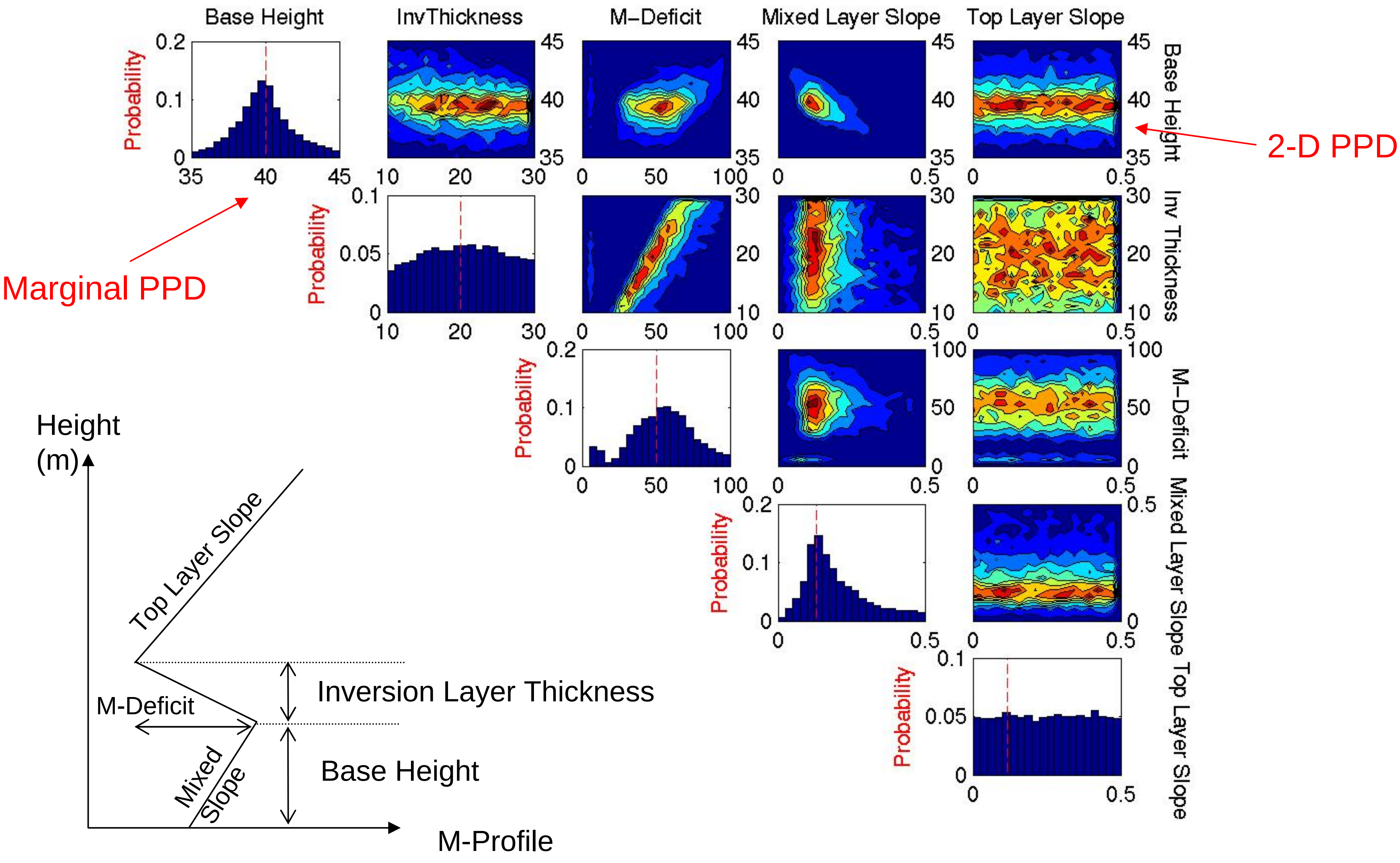
CASE I Bayesian – classical parameter estimation comparison

CASE II Comparison of 5 different methods:
Exhaustive / GA / Metropolis / Gibbs / Hybrid

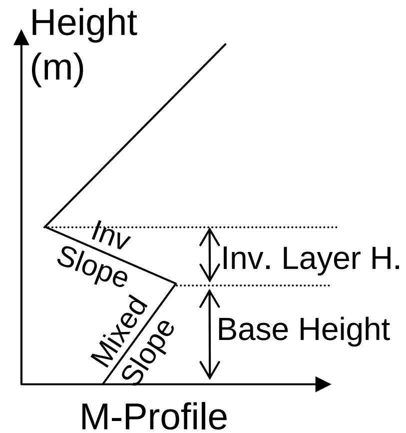
CASE III Application to experimental measurements

CASE I

Bayesian vs. Classical Inversion



CASE II - 4 Parameters



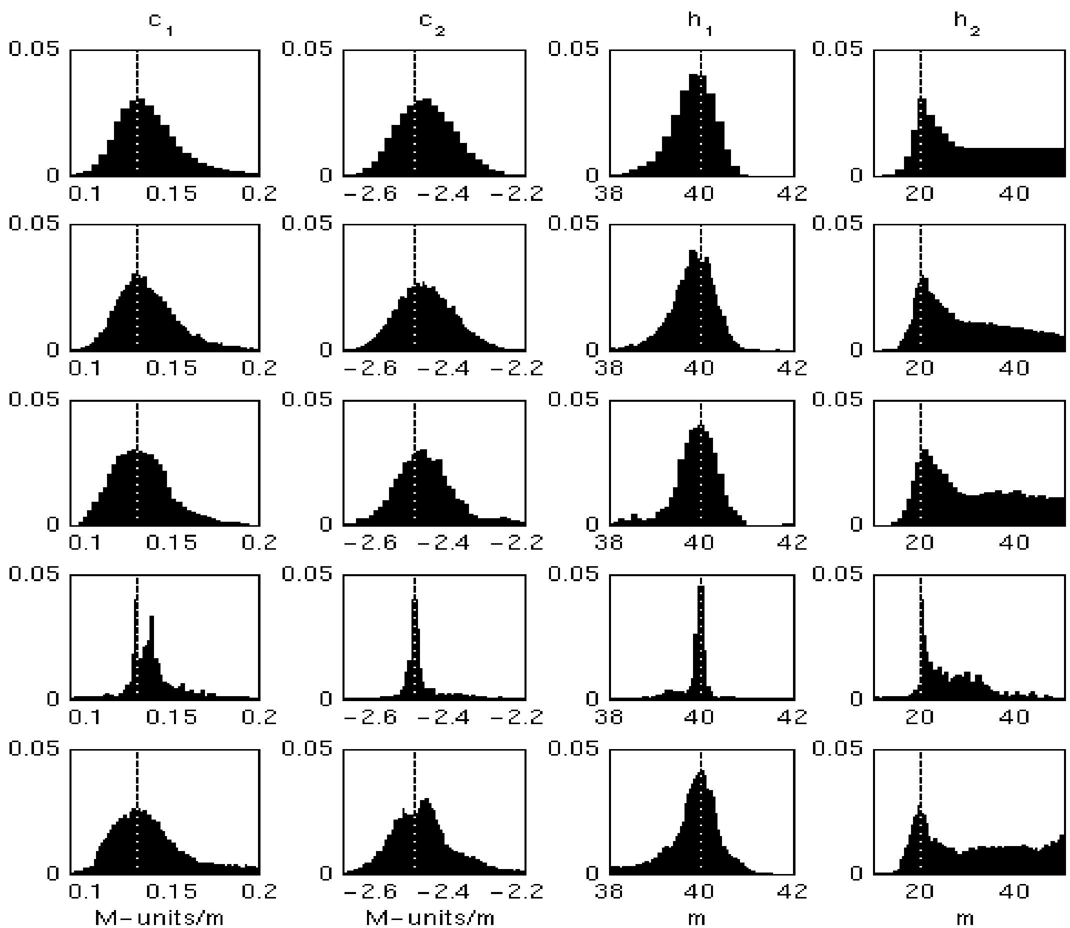
Frequency: 2.84 GHz

HPBW: 0.4°

Mixed Layer Slope : 0.13 M-units/meter   Thickness : 40 meters

Inversion Layer Slope : -2.5 M-units/meter   Thickness : 20 meters

Top Layer Slope : 0.118 M-units/meter (constant)



Exhaustive

390k samples

True Distribution

Metropolis

80k samples

Gibbs

80k Samples

MCMC Distributions

(Gibbs & Metropolis)

Converges to the True Distribution

GA

5k samples

GA Distributions

GA-MCMC Hybrid

5k GA + Fast MCMC section

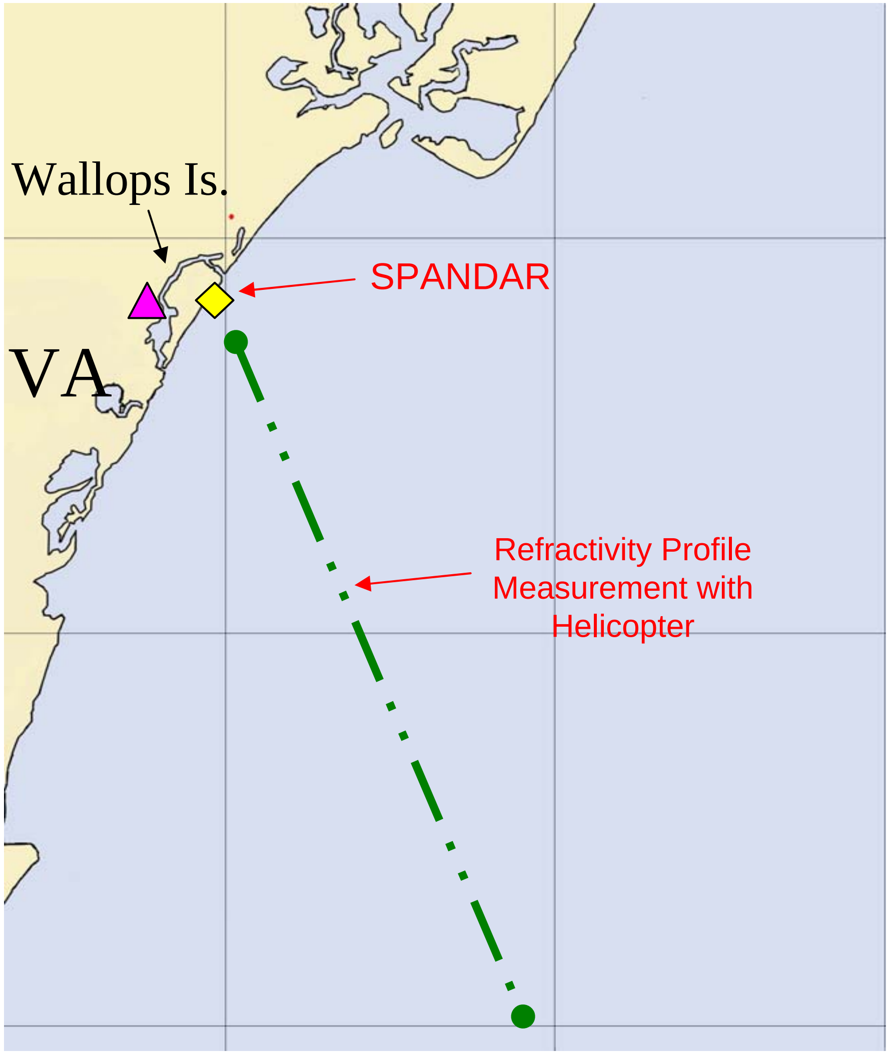
Hybrid

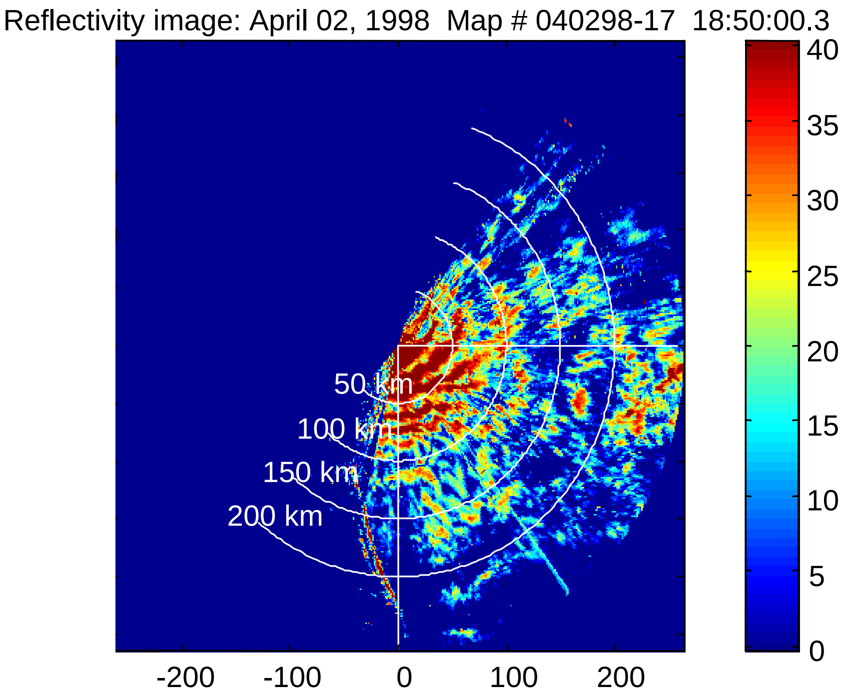
Comparison

	Speed	ML Solution	Probability Distributions
Exhaustive Search	Extremely Slow (390k forward model runs for a 25 points/parameter grid)	Yes	Very Accurate
GA	V. Fast (5k forward model runs)	Yes	Not Accurate
MCMC (Metropolis and Gibbs)	Slow (80k forward model runs)	Yes, but not main purpose	Accurate
GA-MCMC Hybrid	Fast (5k forward model runs followed by an MCMC with no forward model calculation)	Yes	Accurate

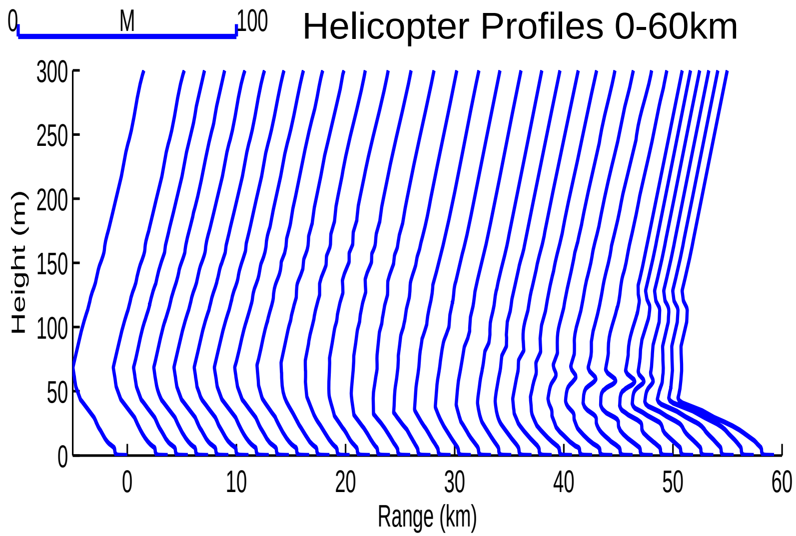
CASE III EXPERIMENTAL RESULTS

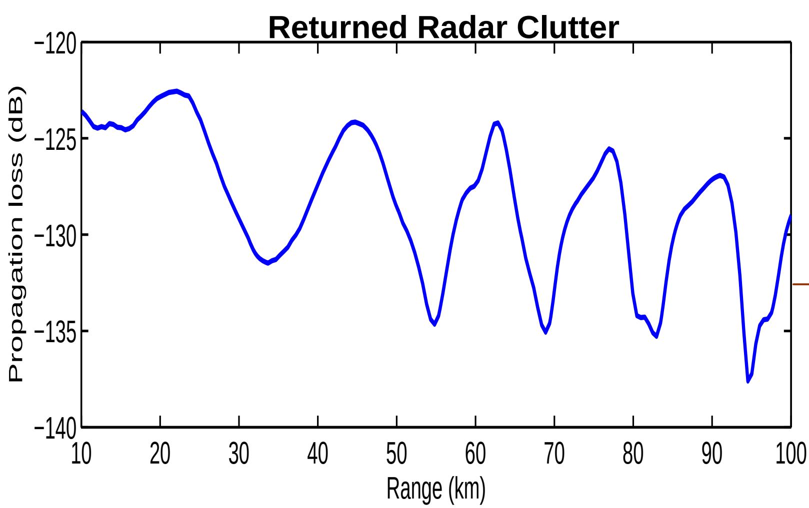
Wallops Island Experiment, Apr 1998





EXPERIMENTAL RESULTS

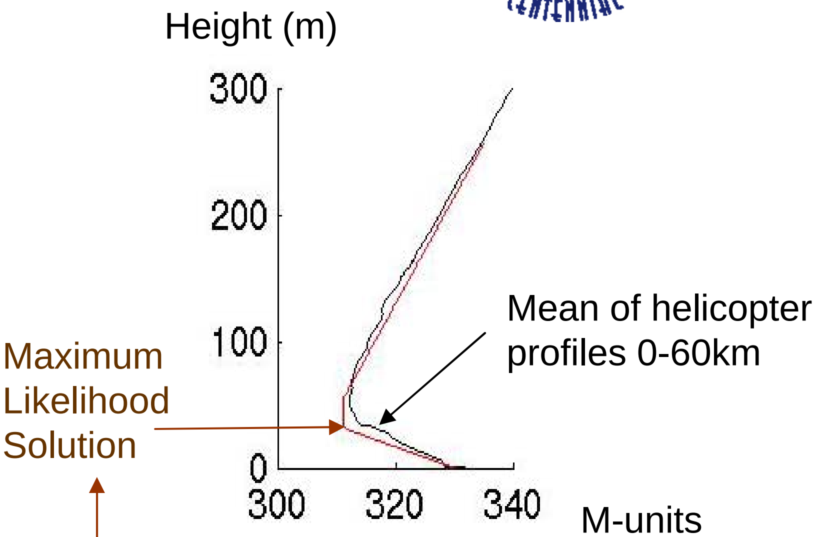




Bayesian

RFC

Inversion



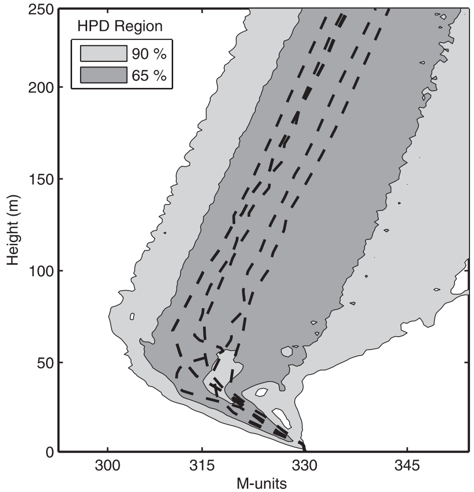
Post - Processing

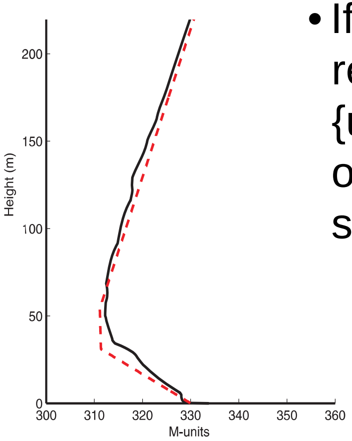
Obtaining other parameters-of-interest

N-Dimensional p(**m**|**d**)

$u = f(\mathbf{m})$

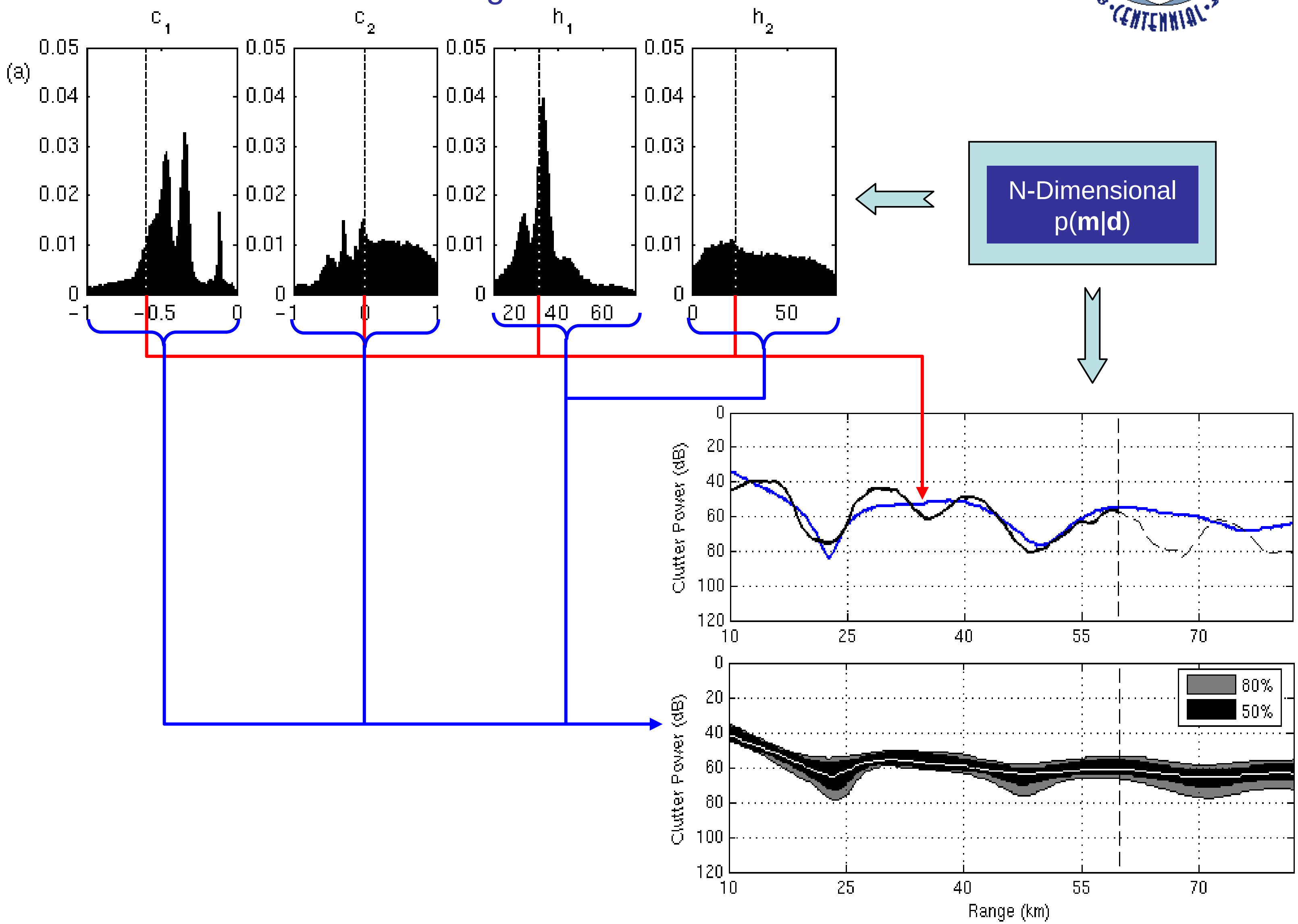
- Draw a large enough no of samples $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \ldots, \mathbf{m}_k\}$ from its own distribution, the n-D PPD.
- Using $u_i = f(\mathbf{m}_i)$ obtain the set $\{u_1, u_2, u_3, \ldots, u_k\}$.
- If the set $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \ldots, \mathbf{m}_k\}$ represents the PPD, the $\{u_1, u_2, u_3, \ldots, u_k\}$ can be used to obtain PPD_u and/or any other statistic of u.





Experimental Data - 4 Parameters

Marginal Distributions



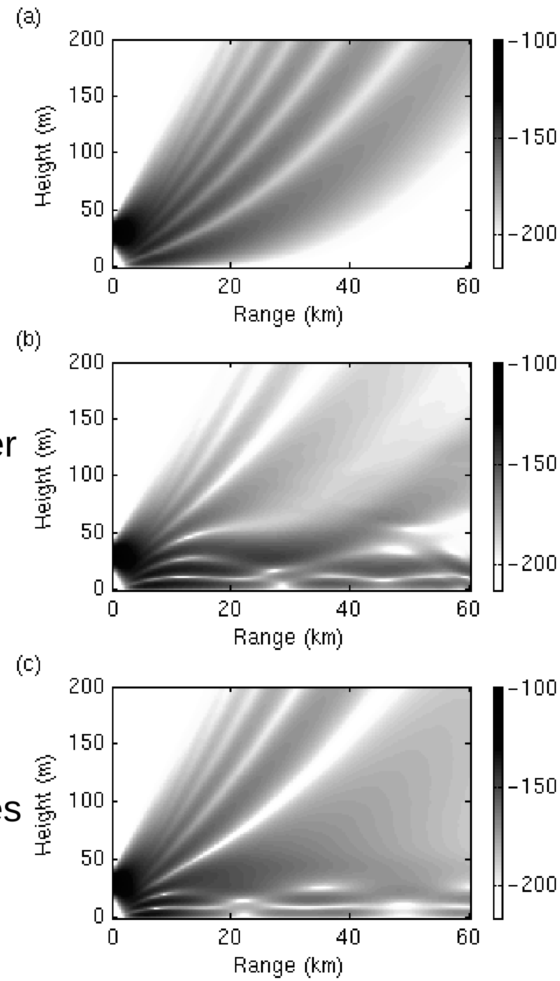
Coverage Diagrams

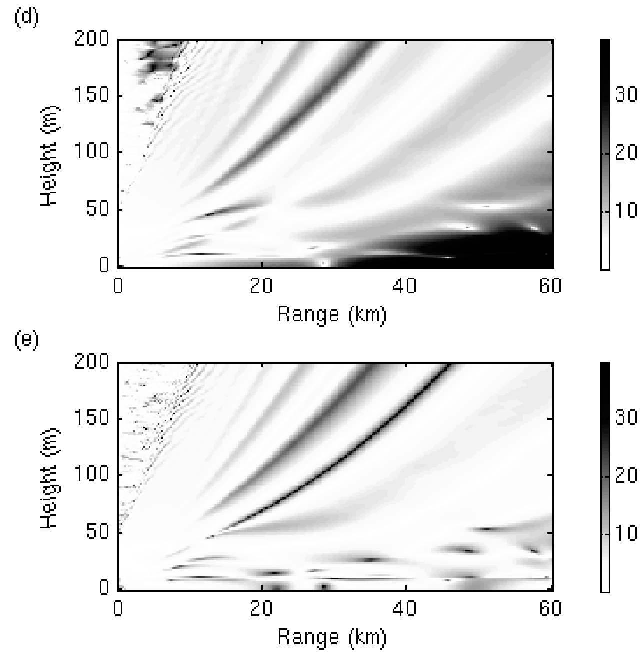
Difference Plots

Using Standard Atmosphere

Using Helicopter Profiles

Using the Inverted Profiles





L_{dB} (a) – L_{dB} (b)

L_{dB} (c) – L_{dB} (b)

CONCLUSIONS

- RFC is an alternate way of measuring the duct properties. It provides us not only with the parameter estimates but also with the *n*-dimensional posterior probability density (PPD).
- This PPD can be used to analyze uncertainties in the parameter estimates, by providing marginal probability distribution, mean and variance of each parameter.
- The GA-MCMC Hybrid method gives high accuracy while being at least 10 times faster than the classical MCMC.

Future Work :

- Accuracy analysis of the hybrid method.
- Simulations with higher number of unknowns, especially to include range dependence.

THANKS…

Some of the figures are taken from AREPS user manual