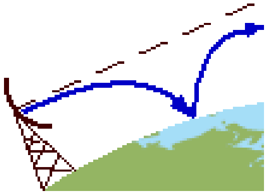
AOS SEMINAR I



RADAR CLUTTER INVERSION

USING BAYESIAN MONTE CARLO METHODS

April 29, 2004

Caglar YARDIM

OUTLINE

- Introduction
- Refractivity From Clutter Problem
- Implementation of the Inversion Problem
  - Bayesian Theory
  - Likelihood
  - Markov Chain Monte Carlo Methods
    - Metropolis Sampler
- Results
- Future Work & Conclusions

INTRODUCTION

- What is a EM Duct?

  - A decrease in the atmospheric index of refraction with increasing altitude will bend the EM wave downward, effectively trapping the signal within a layer called the "Duct".

$$n = c / v$$

where ***n*** is the index of refraction
c is the speed of light in vacuum
v is the speed of light in the medium

A typical value for n for the lower atmosphere is 1.000330. Since this is not very practical, the parts-per-million version is used, where

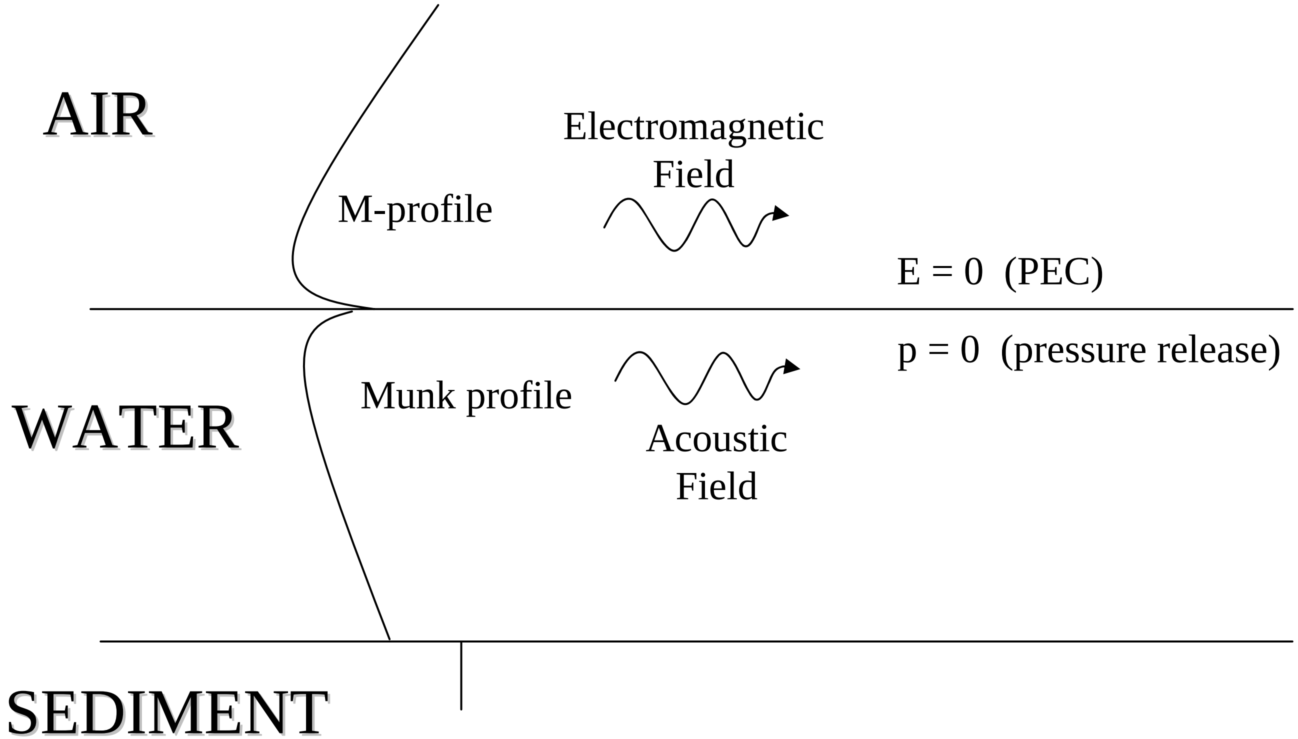
$$N = (n - 1).10^6$$

So N will be 330. However this is for the flat surface and after taking into account the curvature of the earth we end up with the currently used M-profile, where

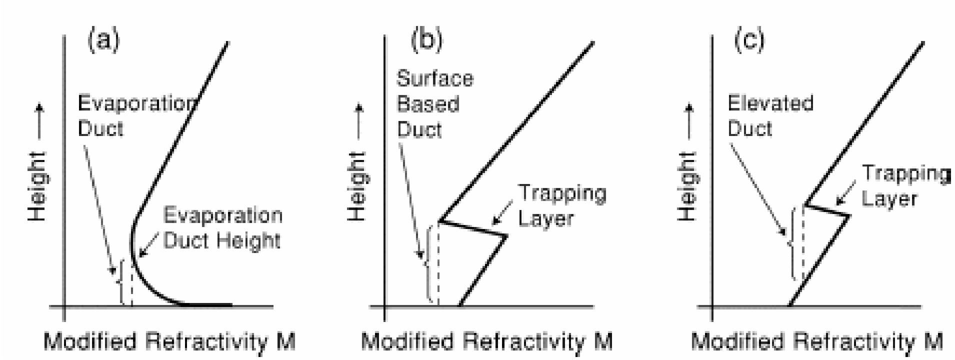
$$M = N + .157h$$

where ***h*** is the altitude

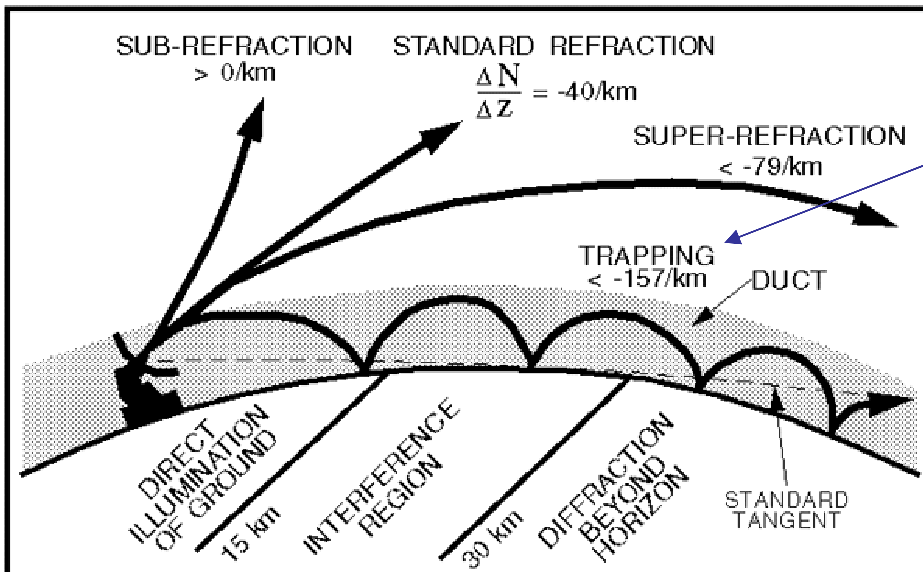
EM vs Acoustic Inversion



Possible Duct Profiles

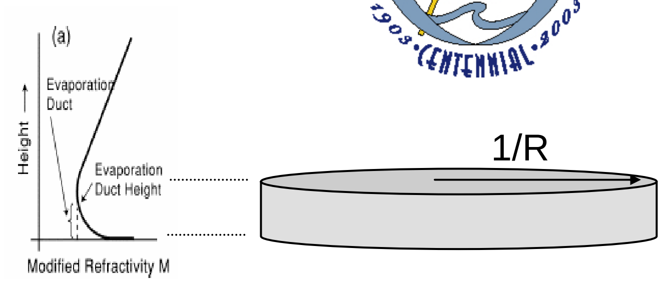


Three most common ducting profiles



$$\frac{\partial M}{\partial z} < 0$$

- Why does it occur? Where and when?



A decrease in M can happen if - Temperature increases w/ height
- Humidity decreases w/ height

where the effect of humidity are far larger that of the temperature.

Land Duct : Clear summer nights with moist ground. Relatively short lived.

Thunderstorm Duct : Caused by the cool air spreading out from the base of the thunderstorm. Short lived.

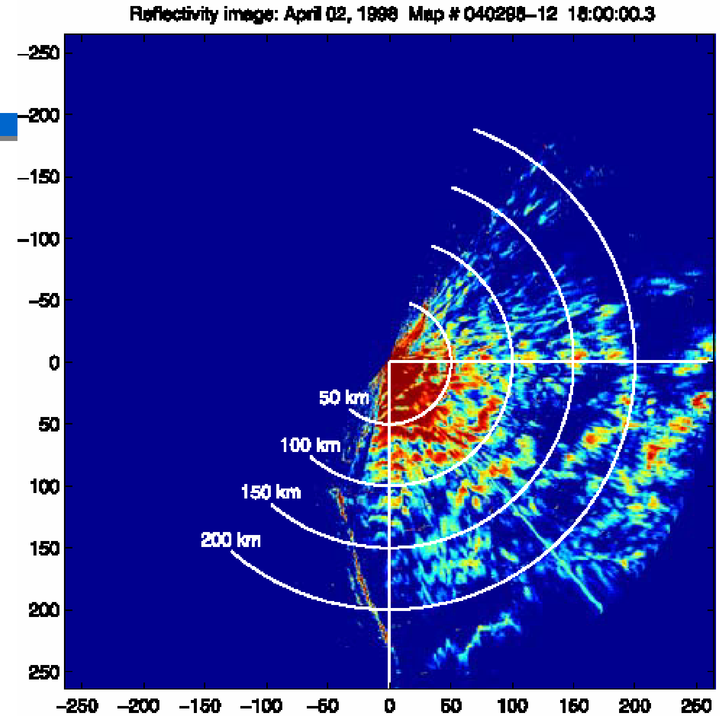
Sea Duct : Warm dry air from land over cooler bodies of water. Can last for long durations. Marine Boundary Layer.

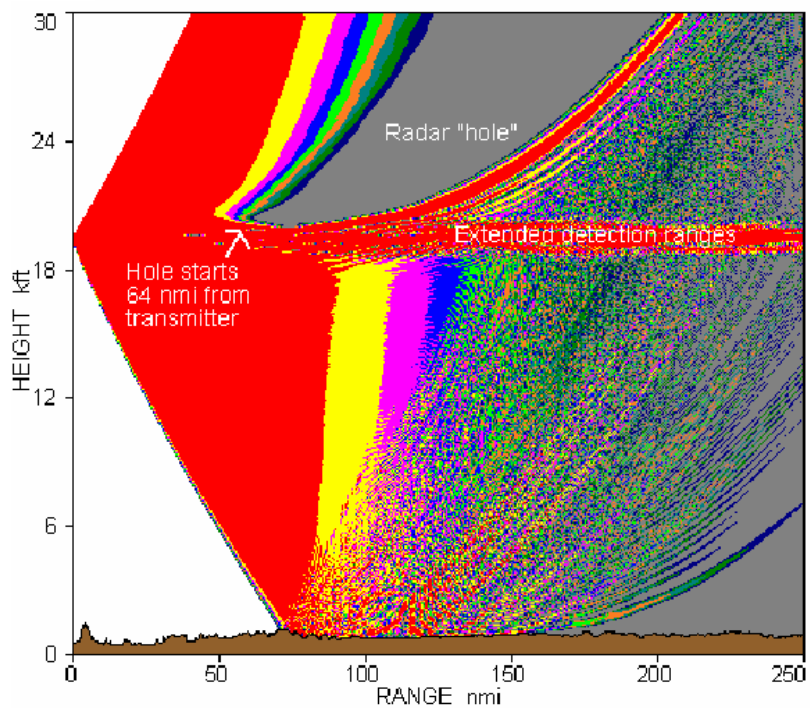
- Result in cylindrical spreading (1/R) instead of usual spherical EM spreading ($1/R^2$).

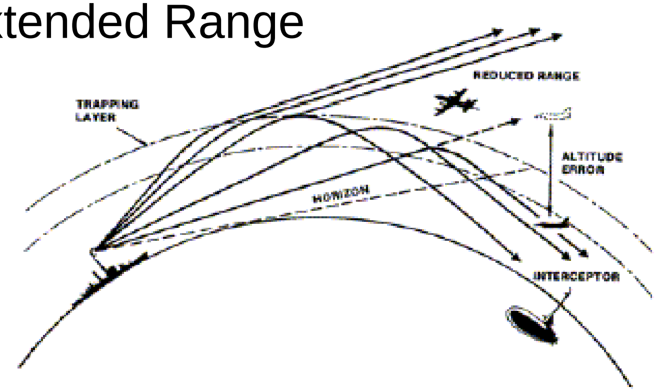
Effects of Ducting

- Why do we care about it?
  What are the effects on EM Propagation?

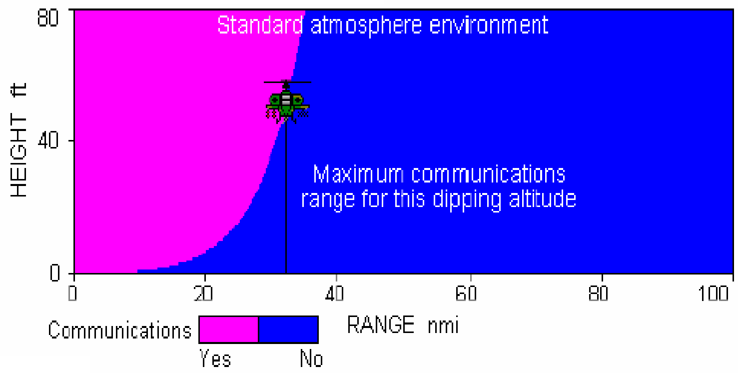
1. Blind Zones (Radar Holes)
2. Height Error for 3-D Radars
3. Clutter Rings
4. Extended Range



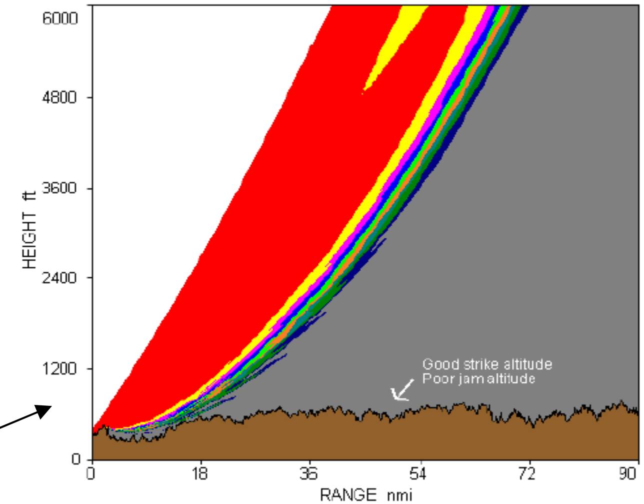




Effects of Ducting



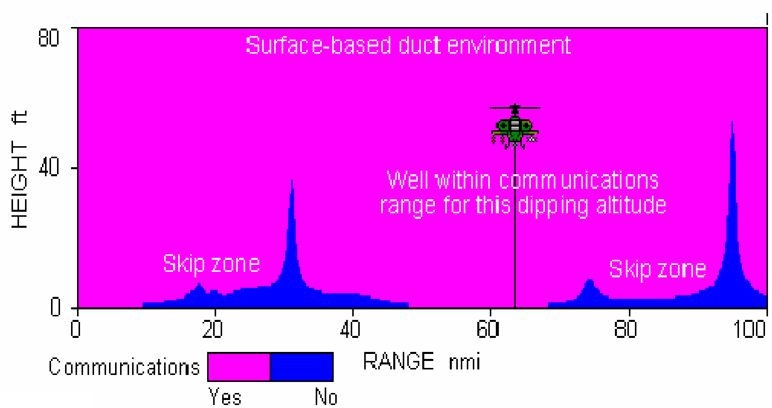
UHF communications under standard atmosphere conditions.



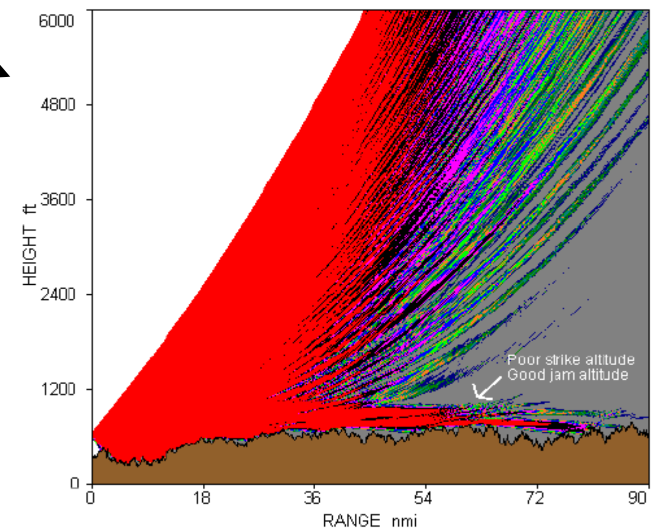
Coverage display for standard atmosphere.

No Ducting

Ducting



UHF communications under surface-based ducting conditions.



Coverage display for a surface-based duct atmosphere.

Estimation of the M-Profile

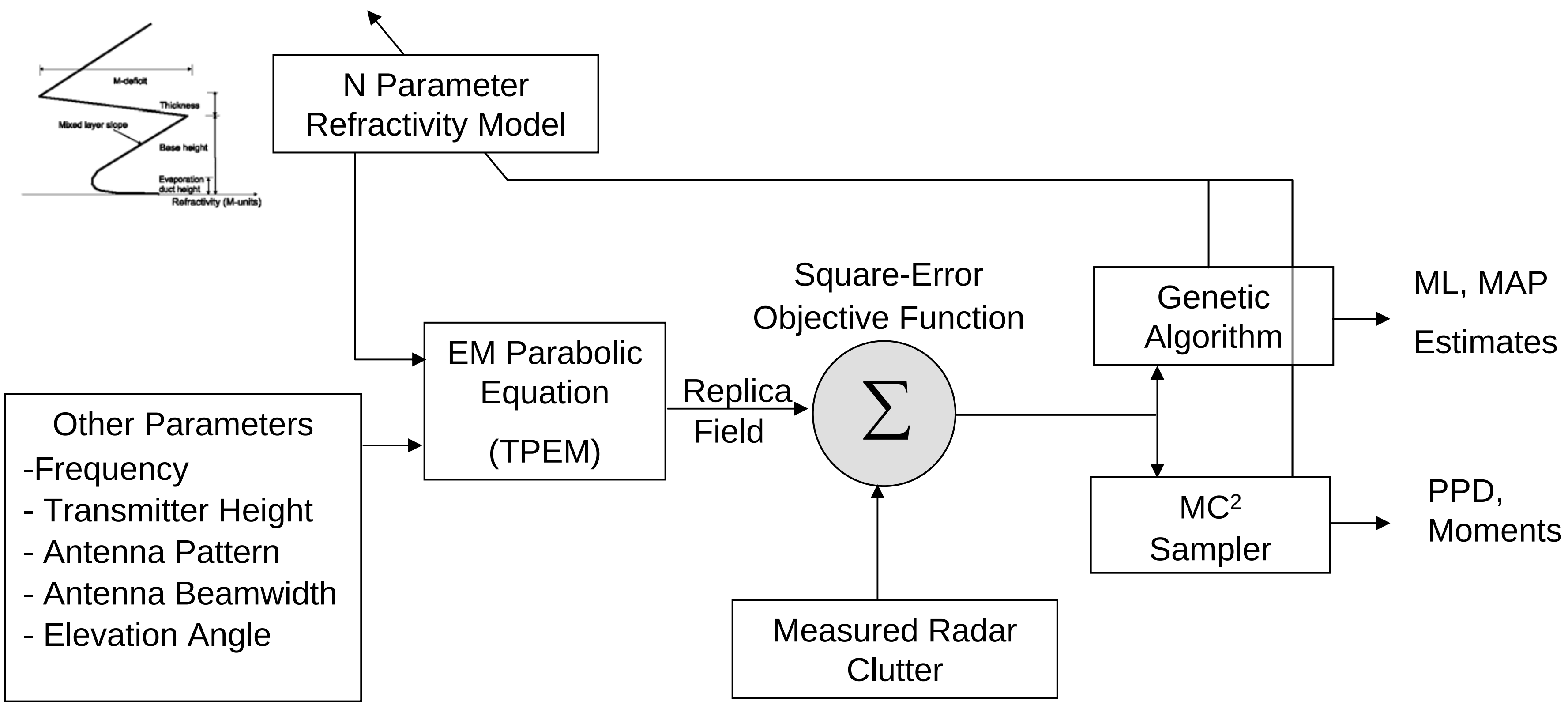
- Conventional Duct Measurement Techniques

  - Bulk Measurements (radiosonde, helicopter soundings, etc)
  - Numerical Weather Prediction Models

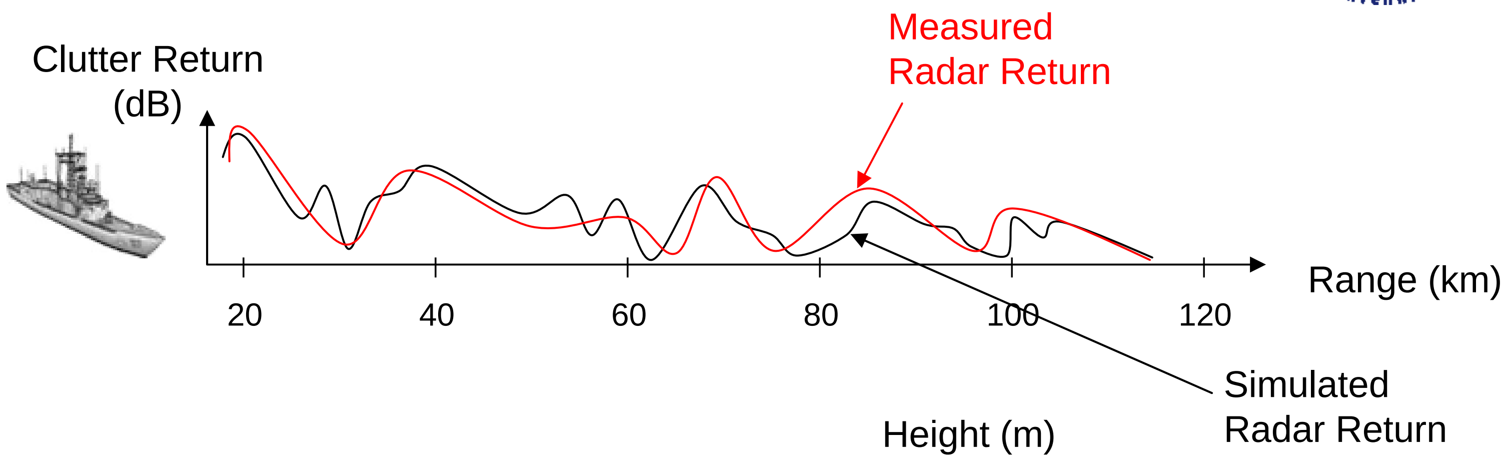
- Alternative Method

  - Refractivity From Clutter (RFC)
    1. No ship based equipment or measurement
    2. No additional signal, Inversion is performed the data acquired during the normal radar operation
    3. Near real time range dependent refractivity profile

RFC as an Inversion Problem



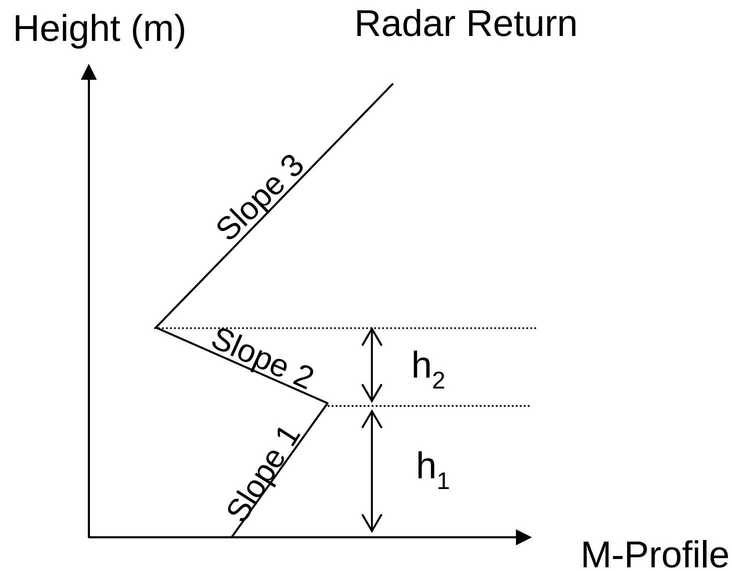
Inversion ….continued



Minimize Objective Function

$$Objective \quad Function \ = \sum_{range=R_{\min}}^{R_{\max}} \left[d^{obs} - d(\mathbf{m}) \right]^2$$

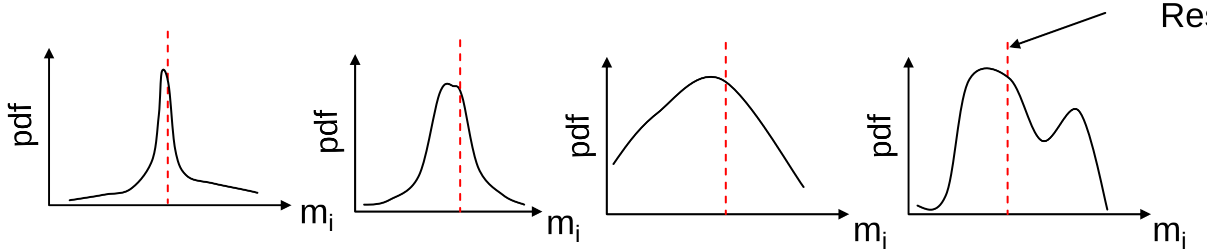
Is it enough?



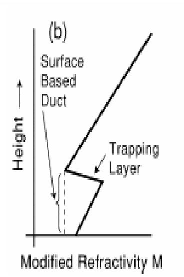
How to Implement the Inversion Problem?

What else do we want to find?

- We want to address the uncertainties in the estimated results.

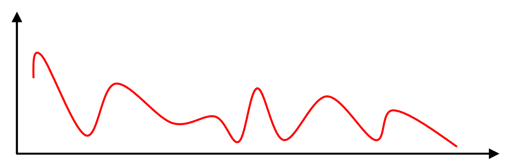


So, we are looking for the probability of a selected model given the data measured in the experiment.



m, *model*

m *: [m_1, m_2, m_3,…, m_N]*



d, *data*

We want :

1. p(*model*|*data*) = p(**m**|**d**)
2. Probability distribution of each parameter, pdf, p(m_i|**d**)
3. Means, variances, medians of each parameter

Desired Quantities

$$\mu_x = E[x] = \int x p(x) dx$$

$$\sigma_x^2 = E[(x-\mu_x)^2] = \int (x-\mu_x)^2 p(x) dx$$

N-Dimensional Posterior Probability Density

$$PPD \equiv p(\mathbf{m}|\mathbf{d})$$

$$p(m_i|\mathbf{d}) = \iiint \ldots \int_{m_j\ j\neq i} p(\mathbf{m}|\mathbf{d}) dm_1 dm_2 \ldots . dm_{i-1} dm_{i+1} \ldots . dm_N$$

Marginal Posterior Probability Density

$$\mu_i = \langle m_i \rangle = \int_{\mathbf{m}} m_i p(\mathbf{m}|\mathbf{d}) d\mathbf{m}$$

$$\sigma_i^2 = \int_{\mathbf{m}} (m_i - \mu_i)^2 p(\mathbf{m}|\mathbf{d}) d\mathbf{m}$$

How are we going to get $PPD,\ p(\mathbf{m}|\mathbf{d})$ → Bayesian Theory,

Markov Chain Monte Carlo Samplers (MC^2)

Bayesian Theory :

$$p(A,B)=p(A|B).p(B)=p(B|A).p(A)$$

Then, $$p(A|B)=\frac{p(B|A).p(A)}{p(B)}$$

$$p(B)=\int p(A,B)dA=\int p(B|A)\,p(A)dA$$

Hence, $$p(A|B)=\frac{p(B|A)\,p(A)}{\int p(B|A)\,p(A)dA}$$ Bayes' Thm.

Applying it to our case,

$$\underbrace{p(\mathbf{m}|\mathbf{d})}_{\text{POSTERIOR}}=\frac{p(\mathbf{d}|\mathbf{m})\,p(\mathbf{m})}{p(\mathbf{d})} = \frac{\overbrace{p(\mathbf{d}|\mathbf{m})}^{\text{LIKELIHOOD}}\,\overbrace{p(\mathbf{m})}^{\text{PRIOR}}}{\underbrace{\int p(\mathbf{d}|\mathbf{m})\,p(\mathbf{m})d\mathbf{m}}_{\text{EVIDENCE}}}$$

… continued

$$\text{Posterior} = \frac{\text{Prior x Likelihood}}{\text{Evidence}}$$

- Prior : p(***m***) , density before the experiment, usually taken as uniform.

- Evidence : p(**d**)

$$Evidence = p(\mathbf{d}) = \int p(\mathbf{d}|\mathbf{m})\, p(\mathbf{m})\, d\mathbf{m} \longrightarrow \text{constant}$$

Therefore, assuming uniform prior, p(**m**) :

$$p(\mathbf{m}|\mathbf{d}) \propto L(\mathbf{m}|\mathbf{d})$$

… continued

It is well known that, if the errors are assumed to be of Gaussian distribution w/ zero mean and uncorrelated at different ranges, the likelihood function will be:

$$L(\mathbf{m}|\mathbf{d}) = \frac{1}{\sqrt{(2\pi\sigma^2)^R}} \exp\left(\frac{-\sum_R \left(d^{obs} - d(\mathbf{m})\right)^2}{2\sigma^2} \right)$$

$$L(\mathbf{m}|\mathbf{d}) \propto e^{-[E(\mathbf{m})]}$$

$$where \quad E(\mathbf{m}) \equiv \frac{1}{2\sigma^2} \sum_R \left(d^{obs} - d(\mathbf{m})\right)^2$$

$$p(\mathbf{m}|\mathbf{d}) \propto e^{-[E(\mathbf{m})]}$$

… continued

$$p(\mathbf{m}|\mathbf{d}) \propto e^{-[E(\mathbf{m})]}$$

Just calculate $e^{-E(\mathbf{m})}$ for all m and obtain PPD. But it is not that easy!

• For N=10 and a discretization of 20 possible values per parameter:

Need 20^{10} forward model runs (Parabolic Equation in our case)

If we assume 10 runs/sec we need 30,000 years to calculate it!

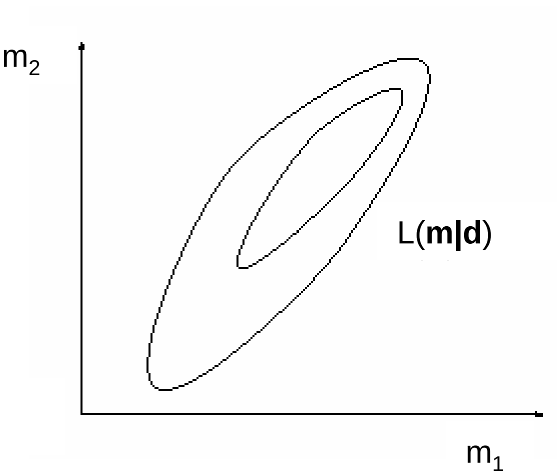
A clever sampling (Metropolis Algorithm) needs about 100k samples (3 hours).

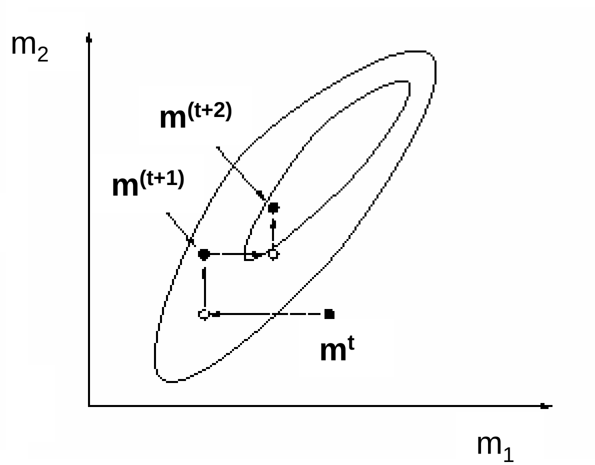
→ **USE CLEVER SAMPLING STRATEGY!**

Efficient Sampling Techniques – Markov Chain Monte Carlo

MC^2 are algorithms that are mathematically proven to sample the state space in such a way that PPD can be found using these few samples. (Metropolis – Hastings Algorithm, Gibbs Sampling, Slice Sampling,…)

Metropolis Algorithm :





$$a = \frac{p(m_{proposed})}{p(m^t)}$$

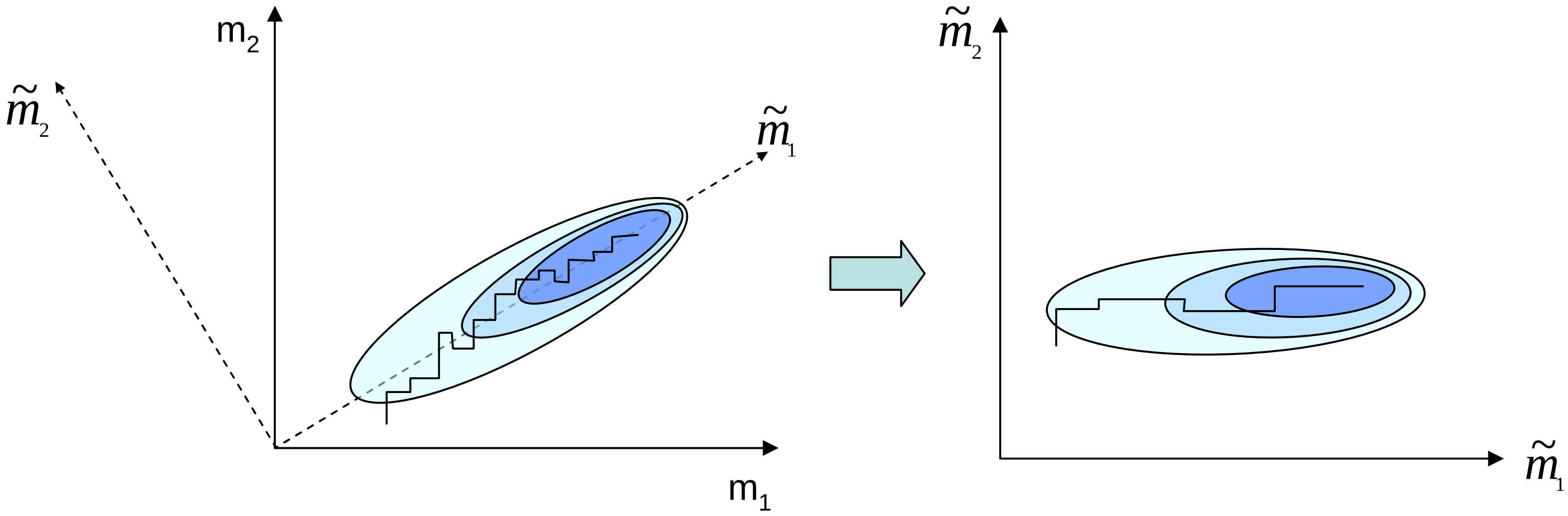
a > rand[0,1] accept

else reject proposal

Accept: $m^{t+1} = m_{proposed}$

Reject : $m^{t+1} = m^t$

Coordinate Rotation



Using the first couple of hundred samples :

$$C = U \Lambda U^T$$

$$\tilde{\mathbf{m}} = U^T \mathbf{m}$$

$$\mathbf{m} = U \tilde{\mathbf{m}}$$

C : Covariance matrix of the collected first samples

U : Rotation matrix (found by eigenvalue decomposition)

Λ : Eigenvalues

Summary of the Algorithm

1. Burn-in Phase to find a initial point to start sampling.

   Can be genetic algorithm, simulated annealing, etc.

2. Initial Sampling Phase

   - Takes samples to compute C.

   - Find the rotation matrix.

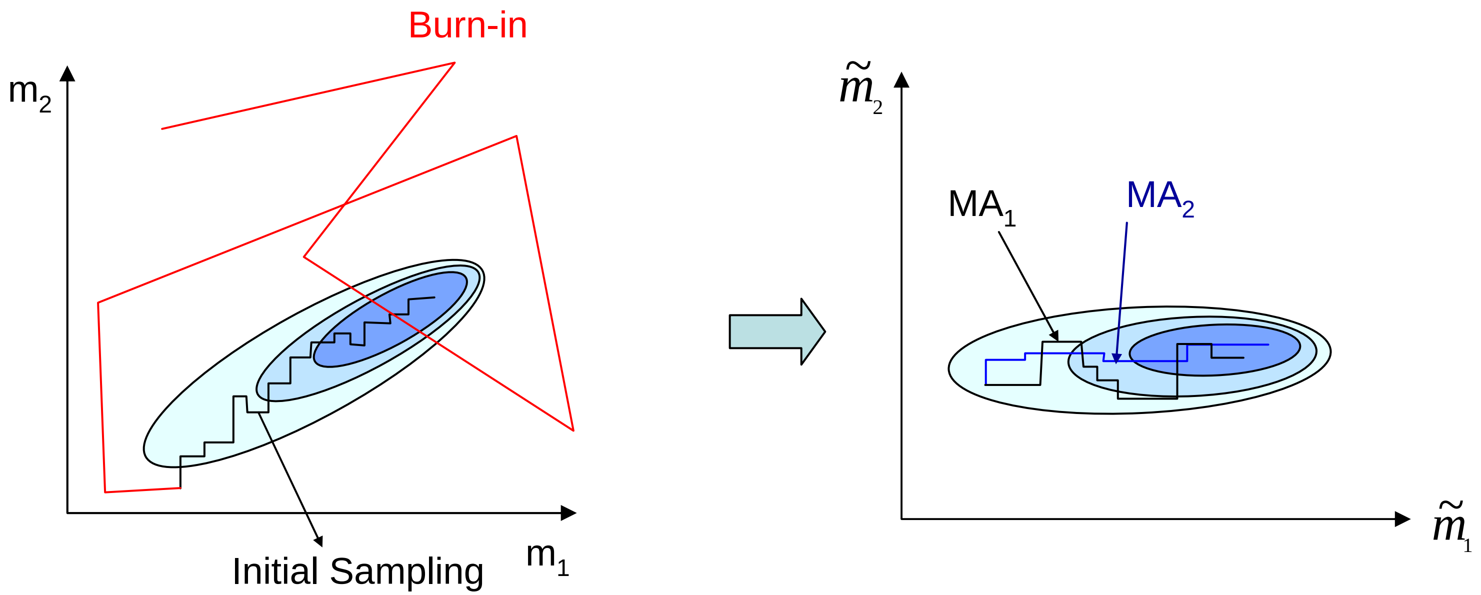
   - Rotate the space and create new rotated parameters.

3. Metropolis Phase

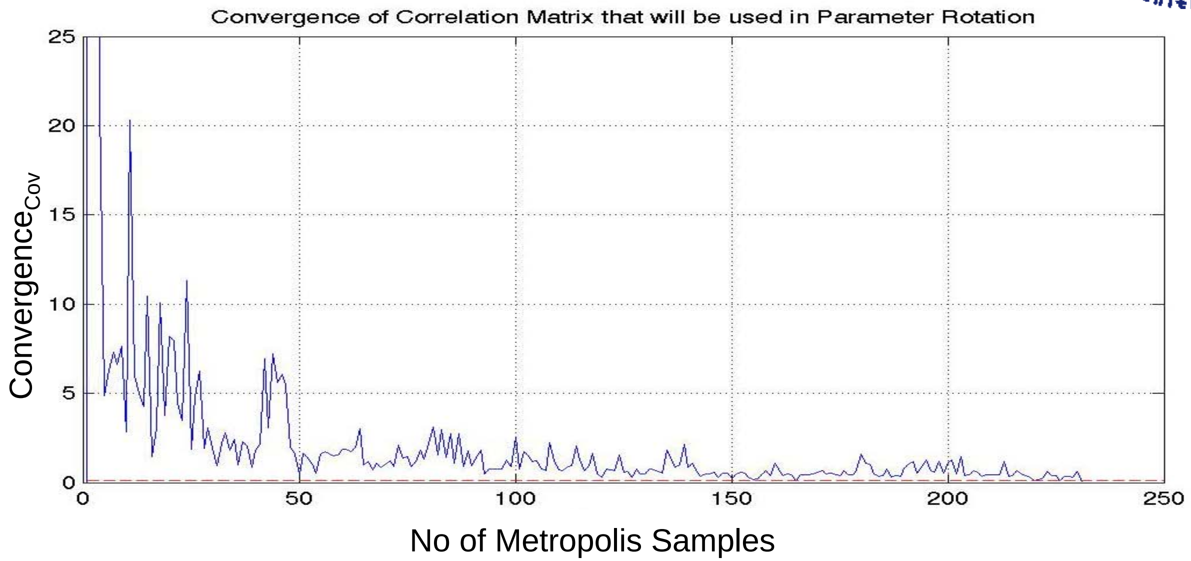
   - Run 2 independent parallel MA samplers in this new space.

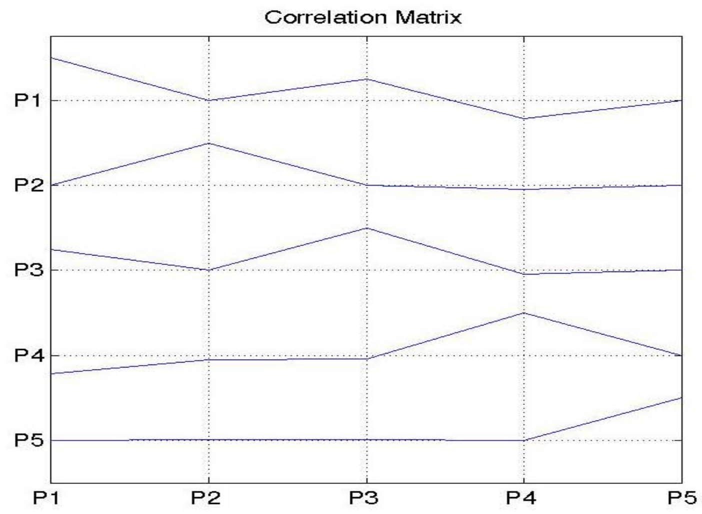
   - Quit when both independent runs histograms converges to the same distribution.

Illustration of How the Algorithm Works



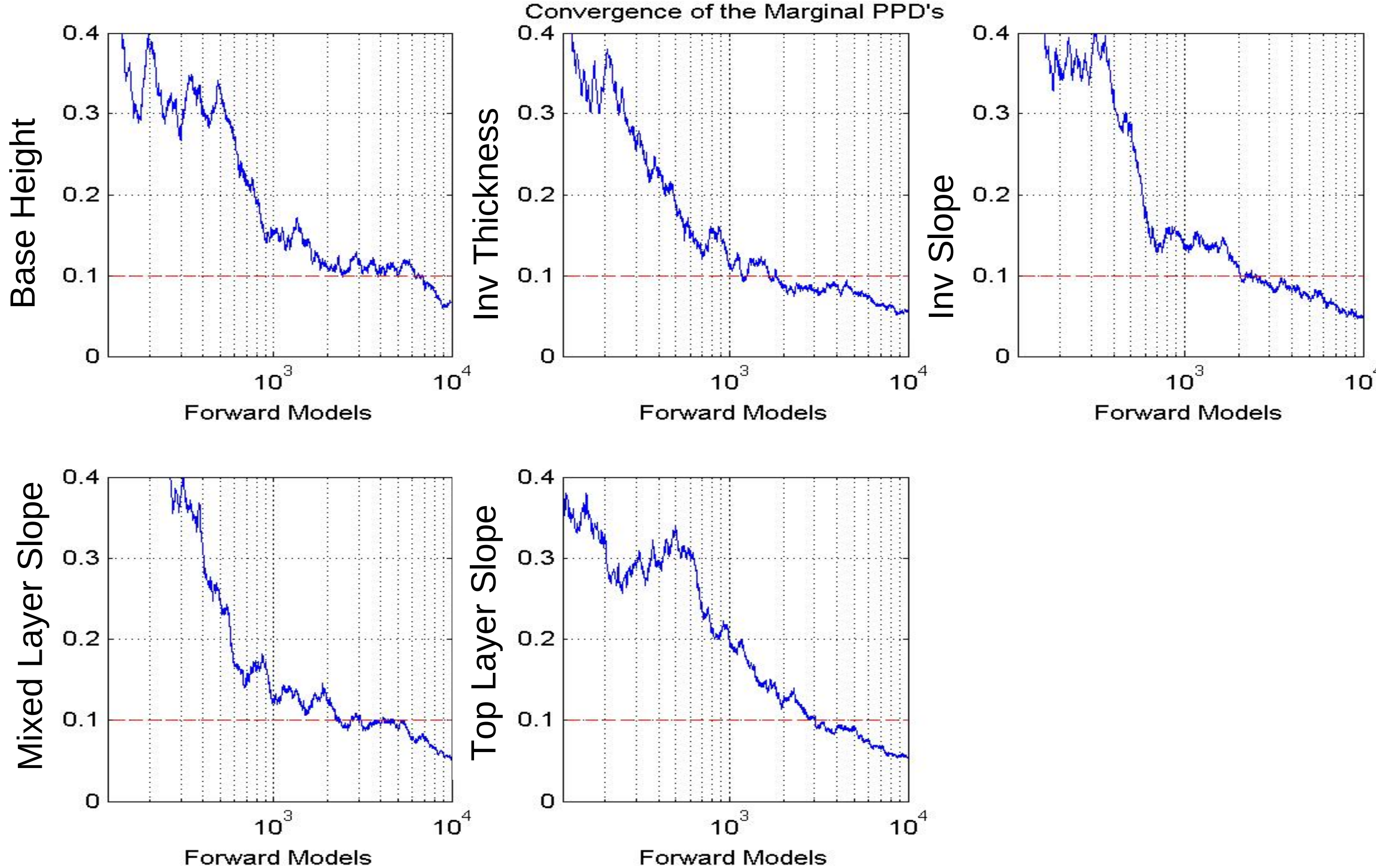
RESULTS



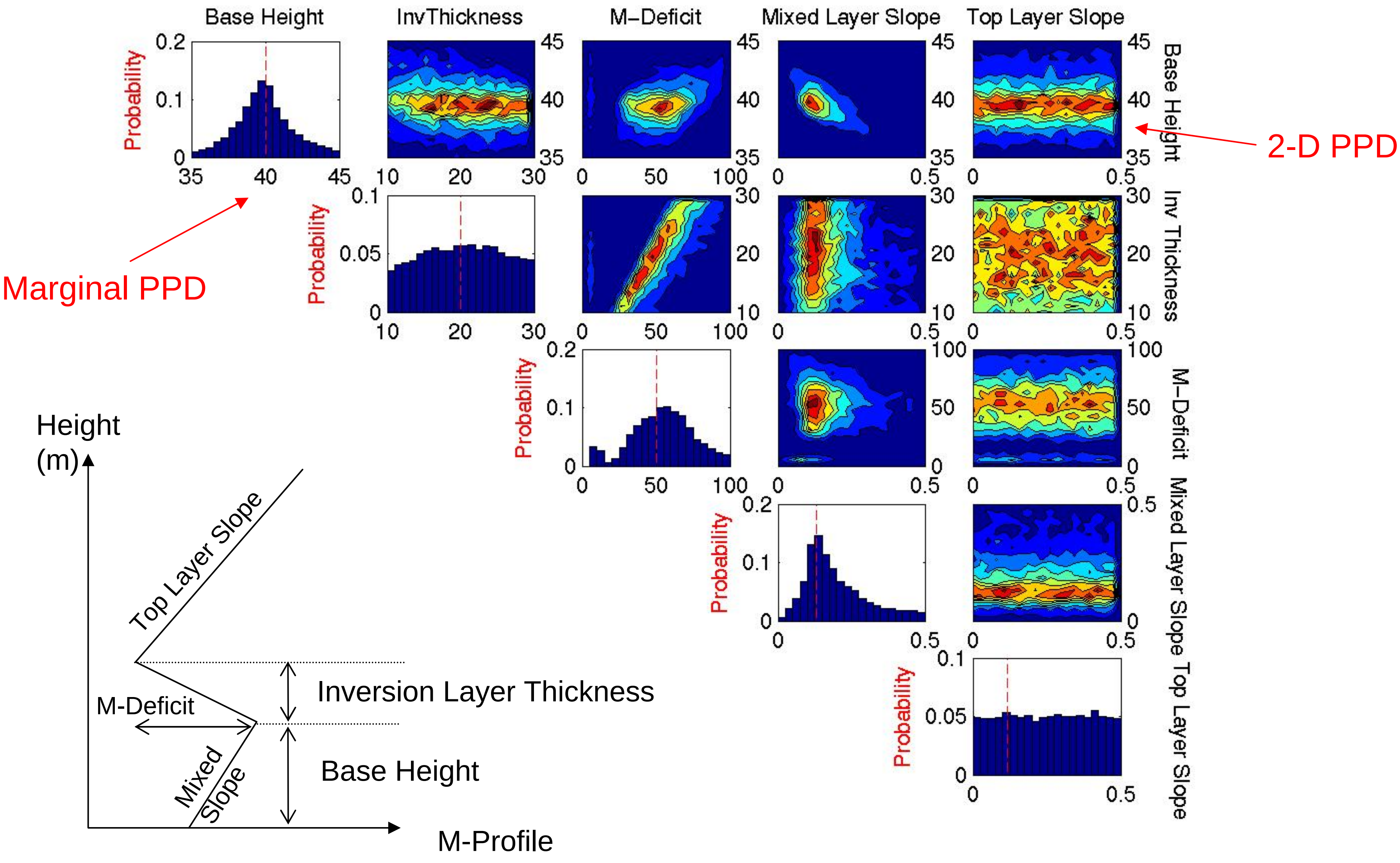


Correlation Matrix

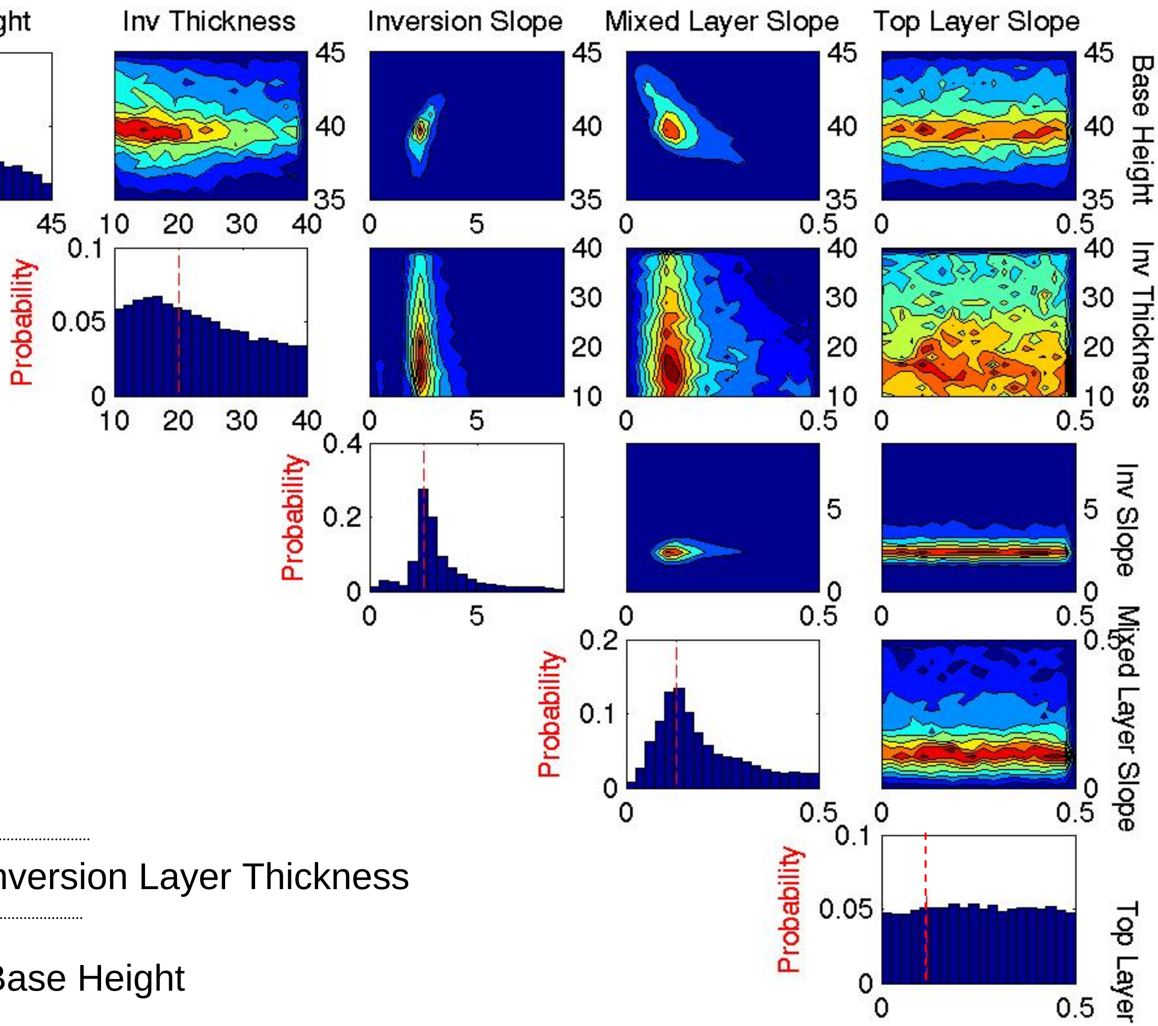
Convergence

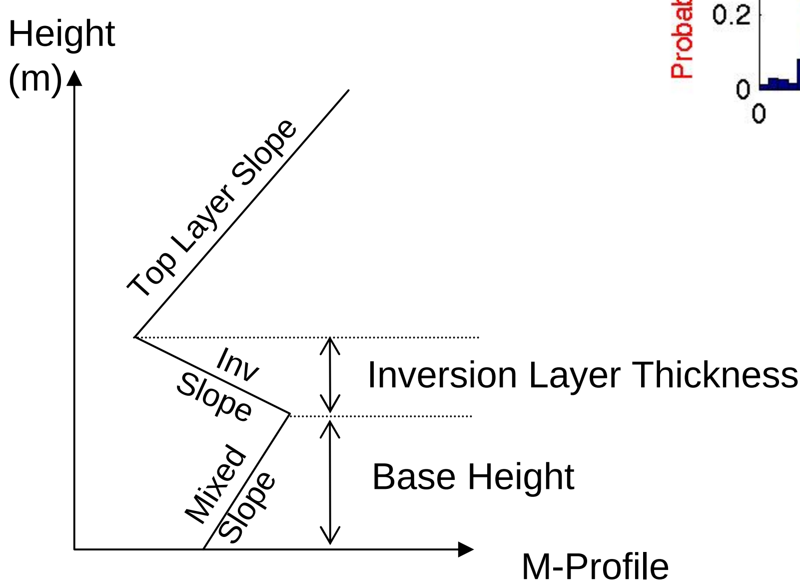


Final Results (PPD's) CASE I

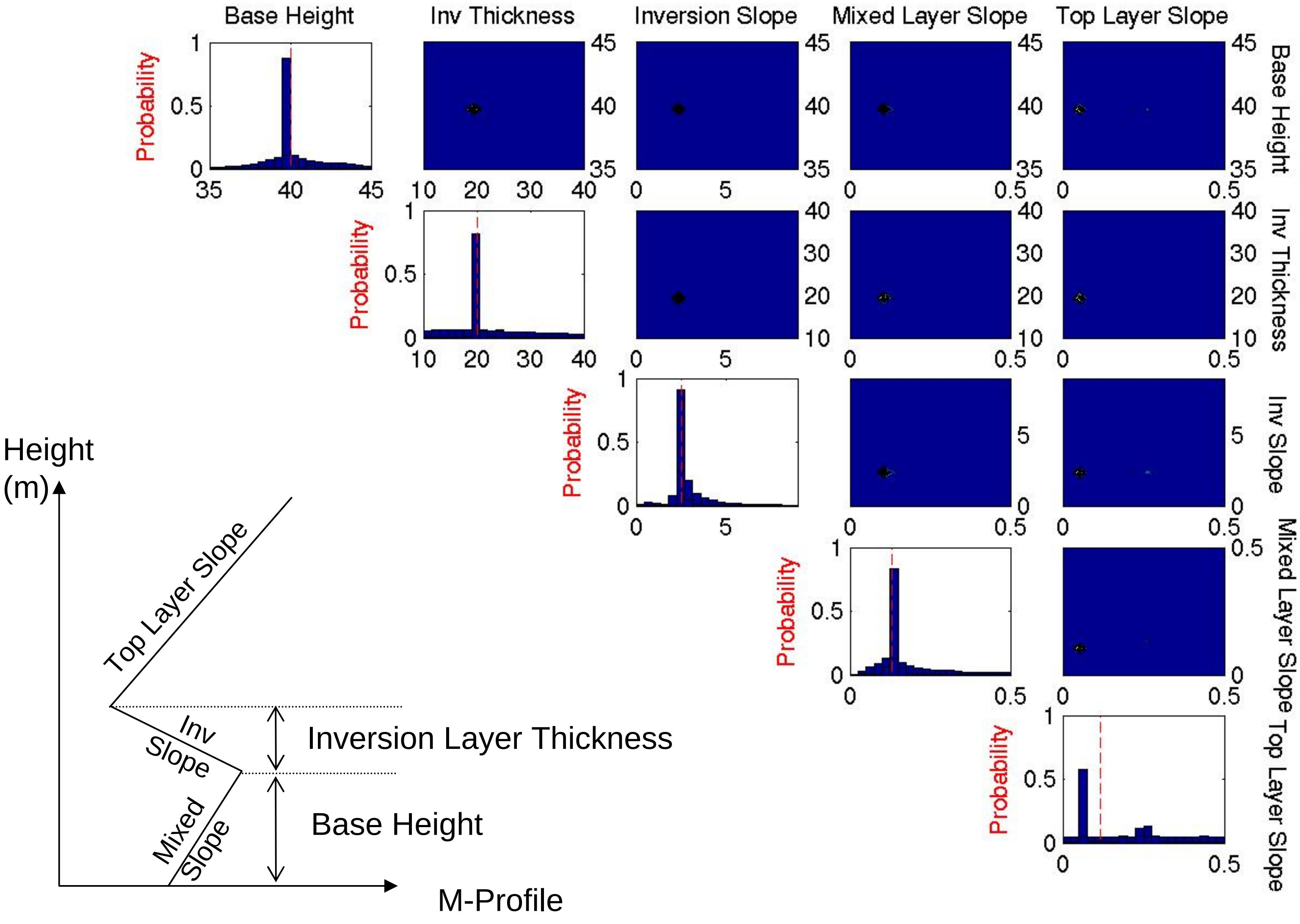


Final Results (PPD's) CASE II





Final Results (PPD's) CASE III - GA



Future Work

- Estimation of evidence for a few model shapes and reparameterization after a single inversion and usage of that model in the next inversions.
- Incorporation of our own electromagnetic Split-Step FFT Parabolic Equation. Testing it with Wide-Angle Pade PE and analyzing the differences.
- Addition of range dependence.
- Inclusion of grazing angle and range dependence of sea surface RCS (Radar Cross-Section).
- Comparison w/ PPD’s obtained by so-called biased samplers like Genetic Algorithm.

CONCLUSIONS

- An alternate way of measuring the duct properties has been introduced.
- The method provides us not only with the parameter estimates but also with their uncertainties, by providing probability distribution, mean and variance of each parameter.

Thanks…

Some of the figures are taken from AREPS user manual