SYNTHETIC APERTURE GEOACOUSTIC INVERSION IN THE PRESENCE OF RADIAL ACCELERATION DYNAMICS

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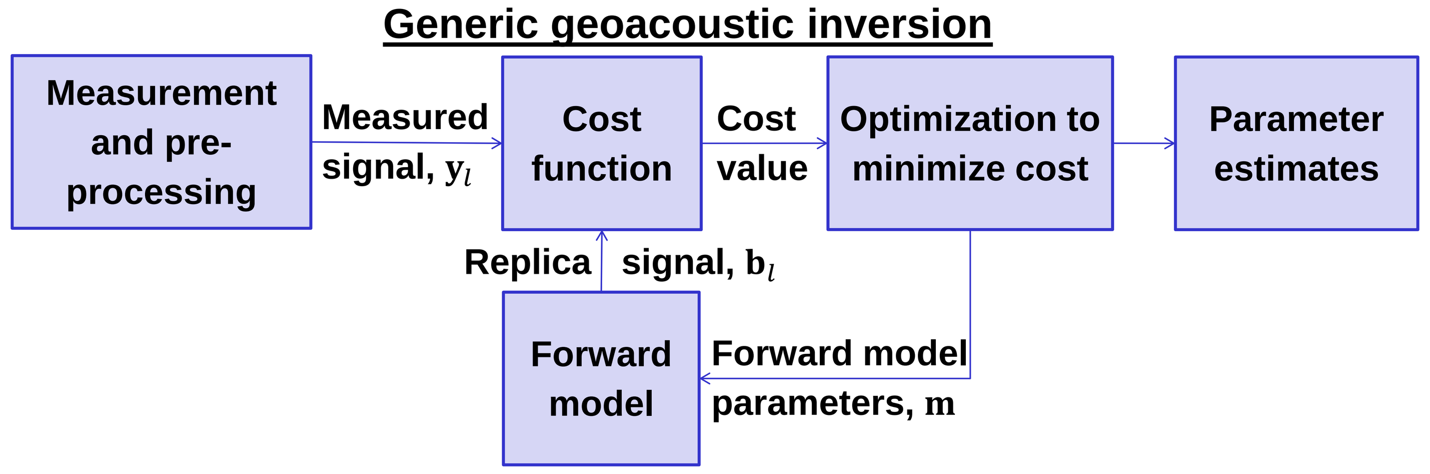
1. ABSTRACT

A low signal to noise ratio (SNR), single source/receiver, broadband, frequency coherent matched-field inversion procedure that exploits coherently repeated transmissions to improve estimation of the geoacoustic parameters recently has been proposed. The long observation time improves the SNR and creates a synthetic aperture due to relative source-receiver motion. However, the inversion performance degrades when source/receiver acceleration exists. In addition, coherently processing a train of pulses all-at-once is impractical as it prevents a recursive way to improve or assess the parameter estimation uncertainty as new data is made available. Hence, this paper proposed a recursive Bayesian estimation approach that coherently processed the data pulse-by-pulse and can perform online parameter uncertainty analysis. It also approximates source/receiver acceleration by assuming piecewise constant but linearly changing velocities pulse-to-pulse. When the source/receiver acceleration exists, it is demonstrated that modeling acceleration is critical for correct inversion. This is done in simulation and real data analysis of low SNR, 100–900 Hz LFM pulses from the Shallow Water 2006 experiment.

2. BACKGROUND & OBJECTIVES

- Assuming the signal measured at the receiver is some distance away from a source, the general idea in geoacoustic inversion is to optimize the forward model parameters by minimizing the difference between the measured and replica acoustic signal.

Generic geoacoustic inversion



- In doing this, we estimate sediment properties without resorting to costly direct measurements such as sediment coring.
- Knowing bottom sediment acoustic properties has impacts on naval applications such as performance prediction and operation of low and mid frequency anti-submarine warfare (ASW) sonars in shallow water and support on the classification for buried mines in underwater mine detections.
- However, matched field inversion typically uses large aperture arrays and powerful transmissions to reduce parameter estimation uncertainty.
- In contrast, broadband synthetic aperture geoacoustic inversion has two advantages [1].
  - **Operationally attractive**
    - This bistatic mobile single source/receiver method computes the waveguide Doppler trajectory field due to a moving source and hydrophone, instead of an approximation with a static point field [1,2].

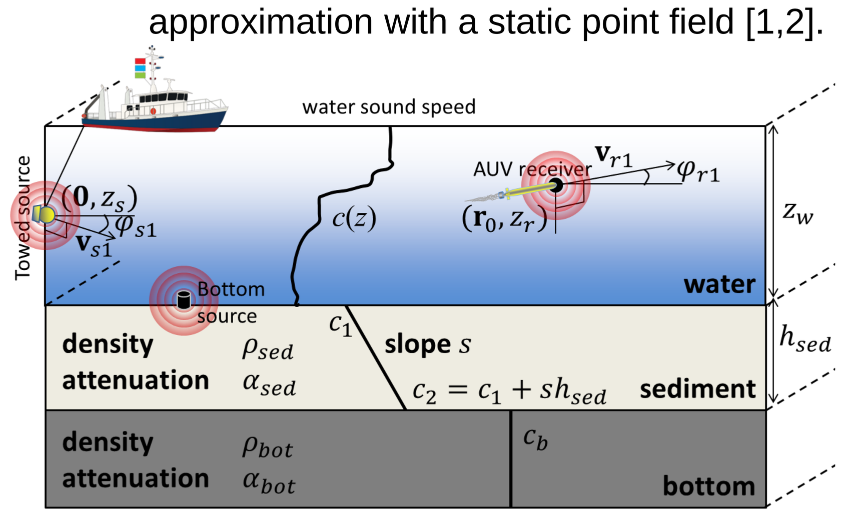


TABLE I. Baseline model parameters.

Model parameters	value
Source range at $t=0$, r_0 (m)	600
Source depth, z_{s1} (m)	30
Receiver depth, z_{r1} (m)	45
Source initial radial velocity, v_{s1} (m/s)	1.9
Receiver initial radial velocity, v_{r1} (m/s)	0
Source radial acceleration, a_s (m/s^2)	-0.006
Receiver radial acceleration, a_r (m/s^2)	0
Water depth, z_w (m)	80
Sediment depth, h_{sed} (m)	25
Sediment density, ρ_{sed} (g/cm^3)	1.8
Sediment attenuation., α_{sed} (dB/λ)	0.2
Sediment top velocity, c_1 (m/s)	1640
Sediment velocity slope, s (1/s)	0
Bottom density, ρ_{bot} (g/cm^3)	2.1
Bottom attenuation., α_{bot} (dB/λ)	0.2
Bottom velocity, c_b (m/s)	1800

    - Method is well suited for rapid environment assessment by autonomous underwater vehicles (AUV).
    - The source or receiver may be towed horizontally by a ship or an AUV.
    - Alternatively, battery powered acoustic source may be dropped onto the ocean bottom to aid AUV-based geoacoustic inversion.
  - **Works in low SNR or low source power condition**
    - Coherently exploits repeated transmissions to reduce parameter uncertainty.
- Though successful, the all-pulses-at-once coherent approach in [1] is impractical and limited to constant source/receiver radial velocities.
  - Alternatively, a coherent pulse-by-pulse processing using the Bayesian approach [3] will circumvent these constraints thereby allowing:
    - **An approximation to acceleration by assuming piecewise constant but linearly changing velocities pulse-to-pulse.**
    - **Uncertainty analysis of the Bayesian statistical parameter estimations.**

3. PROBLEM FORMULATION & SIMULATIONS

Waveguide Doppler normal mode approximation to accelerations a_s and a_r

- l^{th} pulse acoustic field

$$\psi_l(\mathbf{r}, z, \omega_r) \approx \frac{ie^{-i\frac{\pi}{4}}}{\sqrt{8\pi}\rho(z_s)} \sum_{n=1}^{N} \exp(i\omega_r(l-1)T_r) S_c(\omega_r - k_{nl}(v_{sl} - v_{rl})) \Psi_n(z_s;\omega_r)\Psi_n(z_r;\omega_r) \frac{e^{ik_{nl}r_{0l}}}{\sqrt{k_{nl}r_{0l}}}$$

- where $k_{nl} \approx \frac{k_{rn}}{\left(1-\frac{v_{rl}}{u_{rn}}\right)}$ is the n^{th} mode and l^{th} pulse propagating wavenumber. Wavenumber k_{rn} and group velocity u_{rn} is evaluated at ω_r receiver frequencies. $S_c(\omega)$ is the source spectrum. Ψ_n is the modal function. And T_r is the pulse repetition interval.
- and piecewise constant velocities $v_{sl} = v_{s1} + (l-1)T_r a_s$ and $v_{rl} = v_{r1} + (l-1)T_r a_r$
- Source-receiver separation at pulse transmission $r_{0l} = \begin{cases} r_0 & \text{, if } l = 1 \\ r_0 + \sum_{j=1}^{l-1} T_r(v_{rj} - v_{sj}) & \text{, if } l = 2, \dots, L \end{cases}$

Data Model

- $\mathbf{y}_l = \mathbf{b}_l(\mathbf{m}) + \mathbf{w}_l$ where $\mathbf{y}_l$ = measured field, $\mathbf{b}_l$ = replica field, $\mathbf{m}$ = model parameters, $\mathbf{w}_l$ = Gaussian noise, of the l^{th} pulse measurement and $\mathbf{b}_l(\mathbf{m}) = [\psi_l(\omega_{r1}, \mathbf{m}) \quad \dots \quad \psi_l(\omega_{rJ}, \mathbf{m})]^{\text{T}}$.

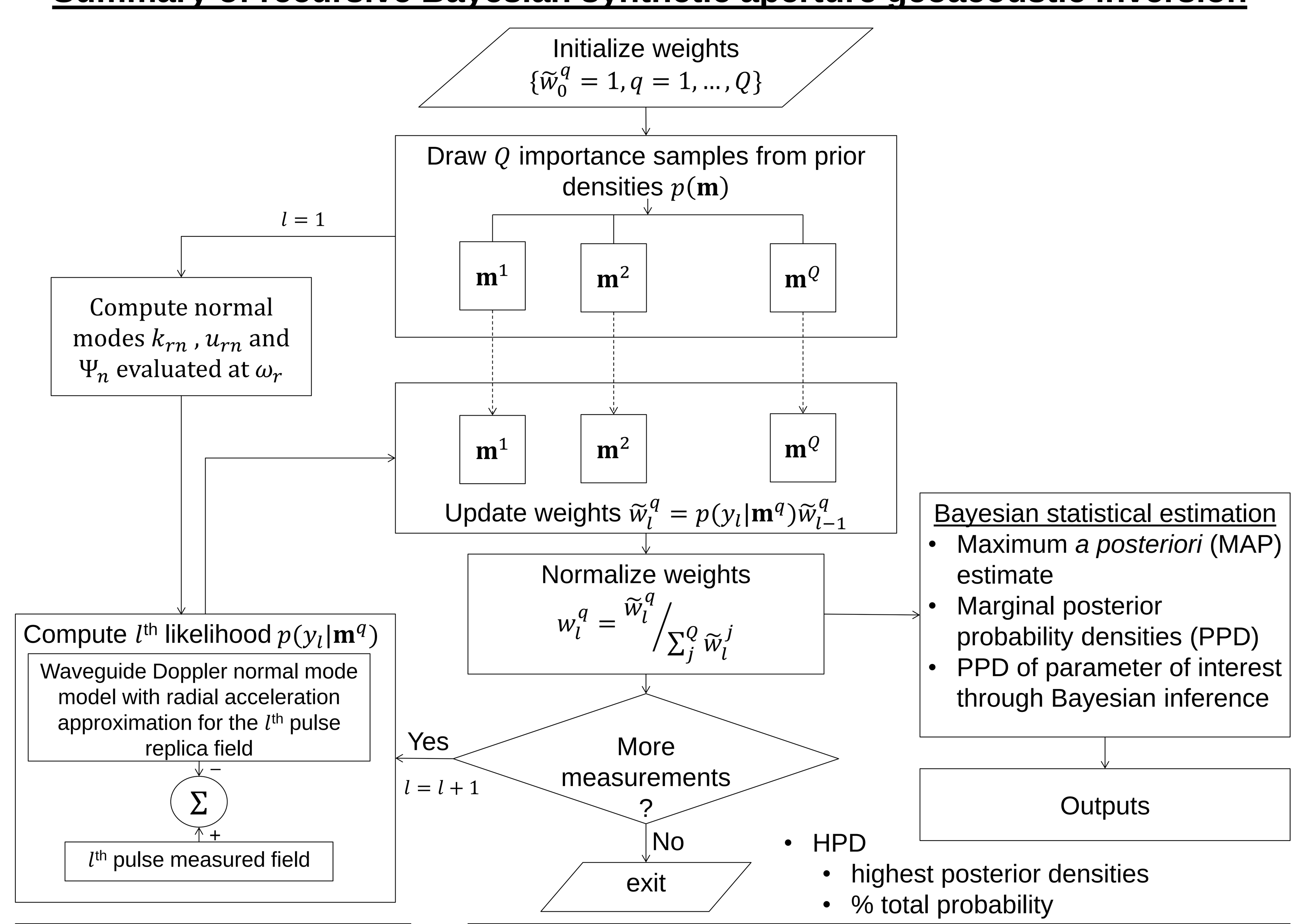
Recursive Bayesian estimation

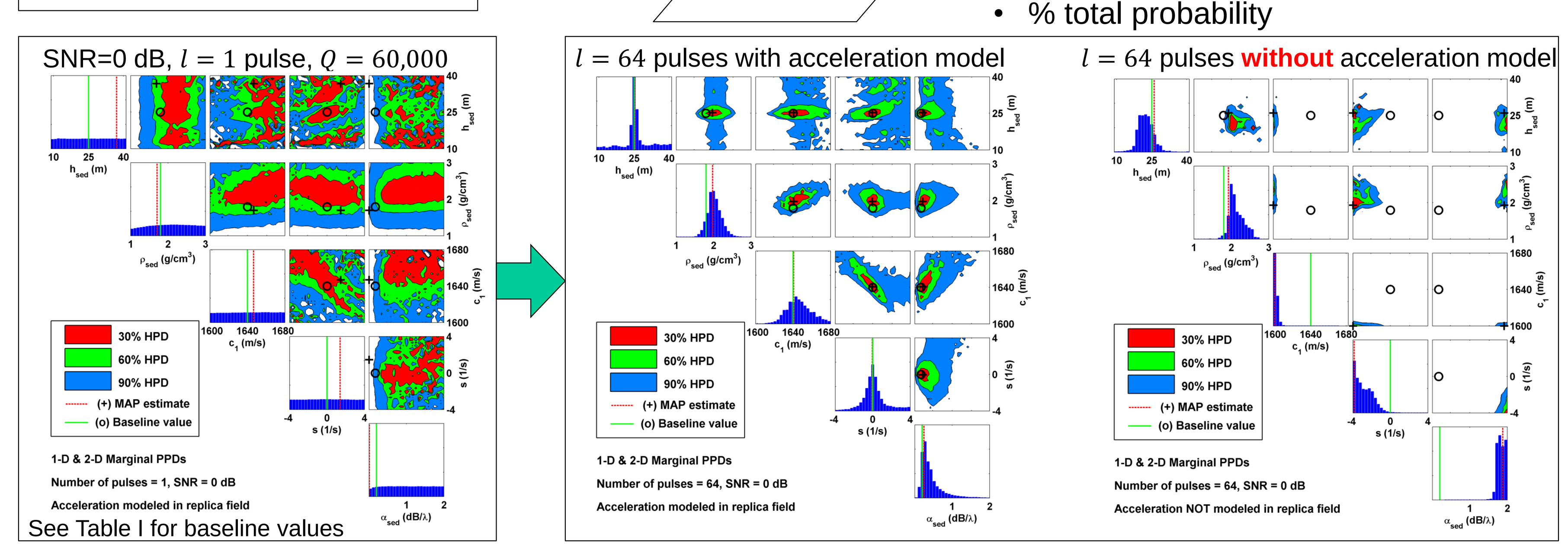
- Joint posterior prob. density (PPD), $p(\mathbf{m}|\mathbf{Y}_l) = \frac{p(\mathbf{y}_l|\mathbf{m})p(\mathbf{m}|\mathbf{Y}_{l-1})}{\int p(\mathbf{y}_l|\mathbf{m})p(\mathbf{m}|\mathbf{Y}_{l-1})\mathrm{d}\mathbf{m}}$ where $\mathbf{Y}_l = [\mathbf{y}_1^{\text{T}} \quad \dots \quad \mathbf{y}_l^{\text{T}}]^{\text{T}}$.

Recursive Monte Carlo integration and importance sampling

- $\int g(\mathbf{m})p(\mathbf{m}|\mathbf{Y}_l)\,\mathrm{d}\mathbf{m} = \sum_{q=1}^{Q} g(\mathbf{m}^q)w_l^q$ where $\tilde{w}_l^q = p(y_l|\mathbf{m}^q)\tilde{w}_{l-1}^q$ (weights recursion) and $w_l^q = \tilde{w}_l^q / \sum_j^Q \tilde{w}_l^j$ (normalization)
- $\{\mathbf{m}^q;\, w_l^q\}_{q=1}^{Q}$ is a set of support points with associated weights that characterize the joint PPD
- $g(\mathbf{m})$ is a function for different desired metrics, e.g. posterior mean, variances…etc.
- Joint PPD $p(\mathbf{m}|\mathbf{Y}_l) \approx \sum_{q=1}^{Q} \delta(\mathbf{m} - \mathbf{m}^q)w_l^q$

Summary of recursive Bayesian synthetic aperture geoacoustic inversion



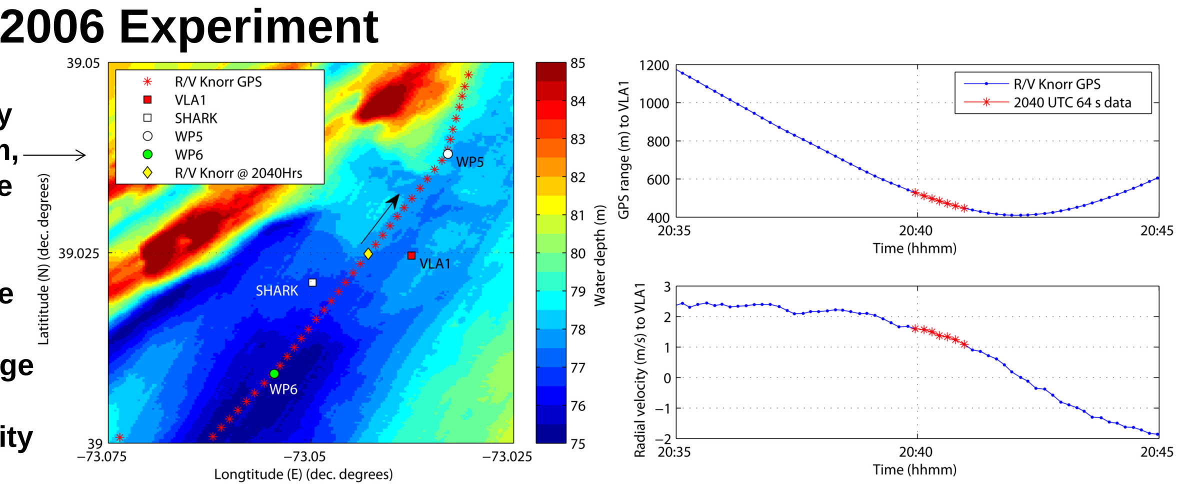


See Table I for baseline values

4. EXPERIMENTAL RESULTS

Shallow Water 2006 Experiment

- **JD238 2040 UTC**
- **Source: J-15 towed by R/V Knorr, Depth 30 m, LFM 100–900 Hz pulse width=1 s Repetition interval =1 s,**
- **Receiver: Hydrophone 8 of VLA, 44.6 m**
- **Source – receiver range ~600m**
- **Initial src radial velocity**
  - **~ 1.6 m/s**
- **Src radial acceleration**
  - **~ – 0.006 m/s²**
- **SNR ~ 0 dB**
- **SSPs measured using SHARK thermistors**



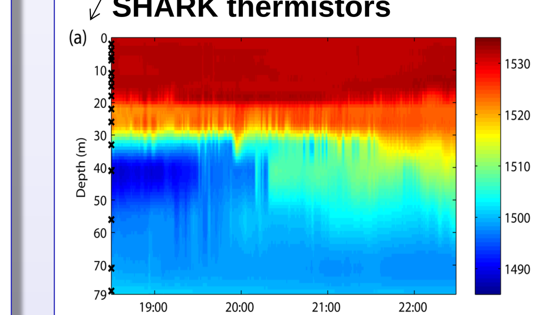


TABLE II. SW06 data inversion parameters prior bounds and MAP results for $L = 64$.

Model parameters	Lower bound	Upper bound	with a_s opt.	without a_s opt.
Src range at $t=0$, r_0 (m)	590	610	598	594
Src depth, z_{s1} (m)	27	33	30.3	30.0
Rcv depth, z_{r1} (m)	41	45	43.2	43.7
Timing error, ξ (msec)	−50	50	−22	−20
Src rad. vel., v_{s1} (m/s)	1.8	2.0	1.93	1.83
Src rad. accel., a_s (mm/s^2)	−7	−5	−6	n/a
EOF1 coef.	0	50	33.8	44.6
EOF2 coef.	0	20	3.5	14.6
EOF3 coef.	0	8	3.9	1.1
EOF4 coef.	−8	0	−3.6	−6.1
Sed. thickness, h_{sed} (m)	10	40	22.8	20.7
Sed. dens., ρ_{sed} (g/cm^3)	1	3	1.7	2.0
Sed. attn., α_{sed} (dB/λ)	0.001	2	1.4	1.2
Sed. top. vel., c_1 (m/s)	1600	1680	1632	1608
Sed. vel. slope, s (1/s)	−10	10	3	5
Water depth, z_w (m)	75	79	76.8	77.0
Src level, α	0	1	0.4	0.2
Noise level, γ	0	2	1.5	1.7

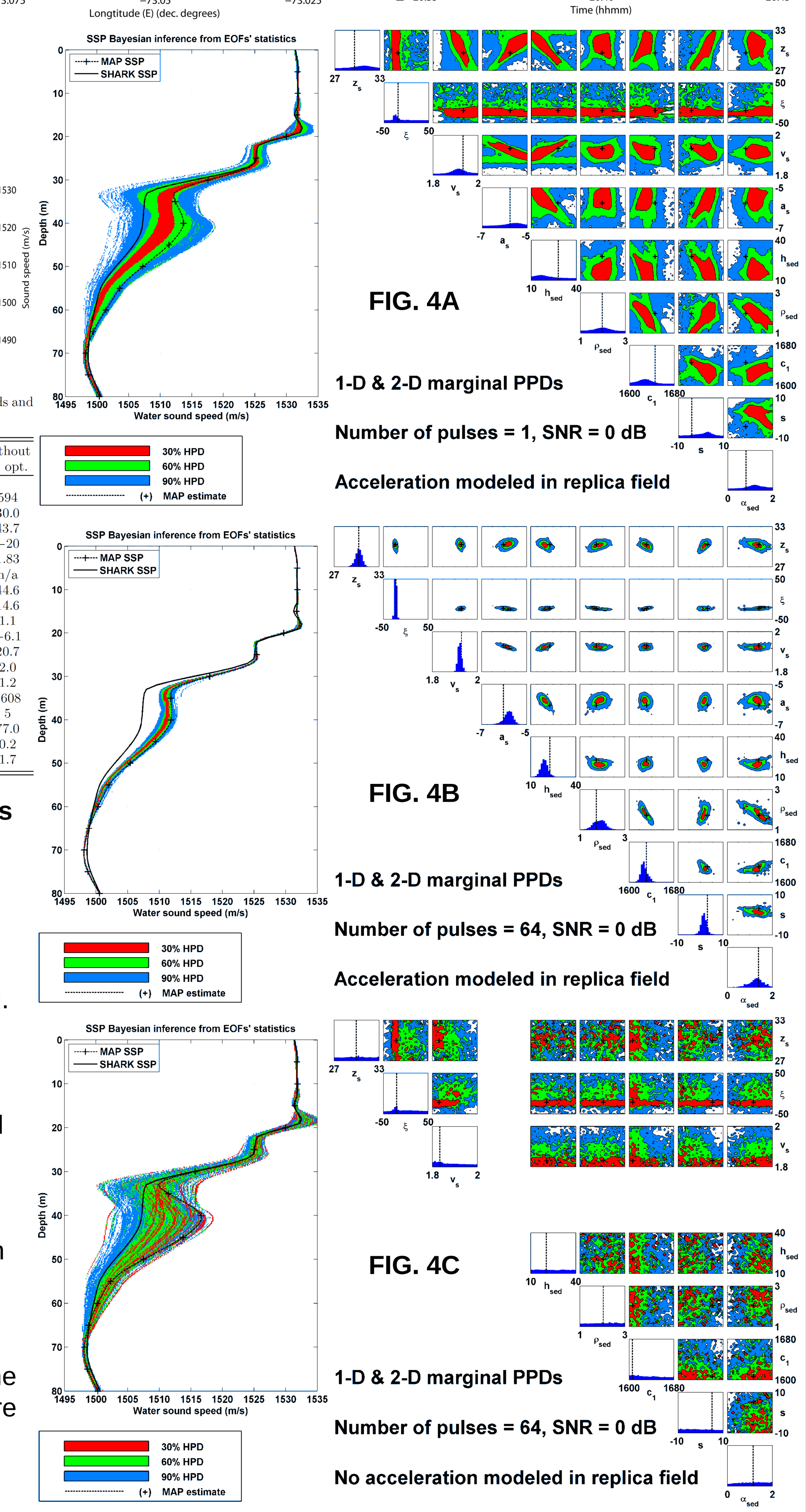


FIG. 4A

1-D & 2-D marginal PPDs

Number of pulses = 1, SNR = 0 dB

Acceleration modeled in replica field

FIG. 4B

1-D & 2-D marginal PPDs

Number of pulses = 64, SNR = 0 dB

Acceleration modeled in replica field

FIG. 4C

1-D & 2-D marginal PPDs

Number of pulses = 64, SNR = 0 dB

No acceleration modeled in replica field

Inversion Results/Analysis

- Inversion MAP results consistent with other published result at the same site [3] (Table II)
- Only 13 relevant parameters PPDs plotted. (Fig. 4A & 4B)
- Reduction of parameter uncertainties or HPD regions between one and 64 pulses is remarkably good (Fig. 4A & 4B)
- SSP uncertainty greatest at 40m which agrees with SSP measurements at SHARK (Fig. 4A & 4B)
- If no acceleration is modeled in the replica, the inversion will fail and there is little or no PPD information. (Fig. 4C)

5. CONCLUSIONS & REFERENCES

A broadband, frequency coherent matched-field inversion procedure for a moving source and receiver at low SNR using the recursive Bayesian pulse-by-pulse approach is proposed. This allowed online uncertainty analysis of the model parameters and an approximation for a horizontally accelerated source and receiver. Through simulation and real data analysis of the Shallow Water 2006 experiment, it is demonstrated that (1) via online uncertainty analysis, parameter uncertainty reduces with increasing number of pulses (2) acceleration should be modeled appropriately in the replica field for correct inversion.

[1] B. A. Tan, P. Gerstoft, C. Yardim, and W. S. Hodgkiss, "Broadband synthetic aperture geoacoustic inversion," *J. Acoust. Soc. Am.*, vol. 134, no. 1, pp. 312–322, Jul. 2013.

[2] H. Schmidt and W. A. Kuperman, "Spectral and modal representations of the Doppler-shifted field in ocean waveguides," *J. Acoust. Soc. Am.*, vol. 96, no. 1, pp. 386–395, 1994.

[3] B. A. Tan, P. Gerstoft, C. Yardim, and W. S. Hodgkiss, "Recursive Bayesian synthetic aperture geoacoustic inversion in the presence of motion dynamics," *J. Acoust. Soc. Am.*, to be submitted.

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