Change-point detection for recursive Bayesian geoacoustic inversion

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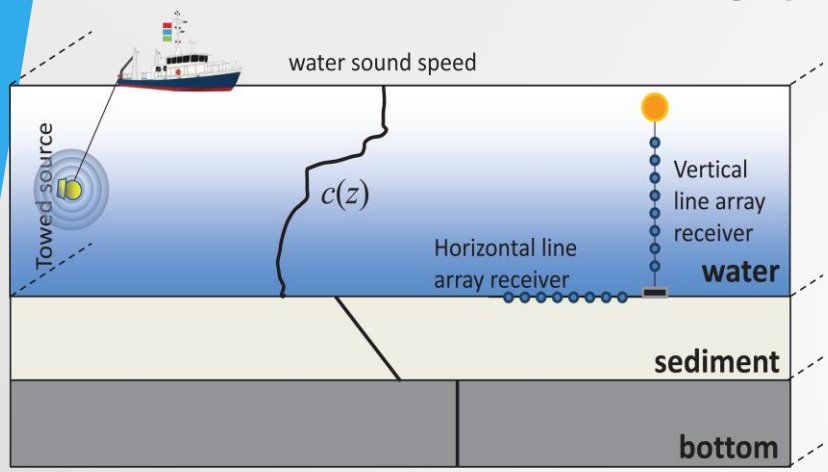
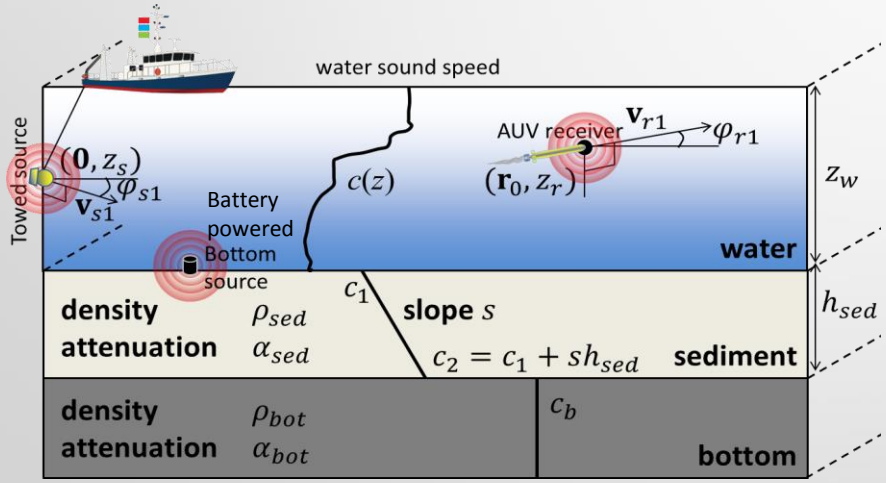
Overview

- Motivation
- Review of recursive Bayesian estimation
- Change-point detection
- Simulation results
- Conclusion

Motivation





- Powerful / bulky source and large aperture arrays

[1,2]

- Low power / compact source, single receiver

- But methods [1,2] assume constant underlying model parameters.

[1] B. A. Tan, et. al. "Broadband synthetic aperture geoacoustic inversion," *J. Acoust. Soc. Am.*, vol. 134, no. 1, pp. 312–322, Jul. 2013.

[2] B. A. Tan, et. al., "Recursive Bayesian synthetic aperture geoacoustic inversion in the presence of motion dynamics," *J. Acoust. Soc. Am.*, J. Acoust. Soc. Am., vol. 136, no. 3, pp. 1187-1198, 2014.

Recursive Bayesian approach [2]

- **Data model of** l^{th} pulse measurement
- $\mathbf{y}_l = \mathbf{b}_l(\mathbf{m}) + \mathbf{w}_l$ where
- $\mathbf{y}_l$ = measured field
- $\mathbf{b}_l$ = modeled/replica field
- $\mathbf{m}$ = model parameters
- $\mathbf{w}_l$ = Gaussian noise

Bayes rule $\quad p(\mathbf{m}\,|\,\mathbf{y}) \propto p(\mathbf{y}\,|\,\mathbf{m})\,p(\mathbf{m})$

Consider $\mathbf{y}_{1:2}$ two measurement case

Posterior Probability Density (PPD)

$$p(\mathbf{m}\,|\,\mathbf{y}_{1:2}) = p(\mathbf{m}\,|\,\mathbf{y}_1,\mathbf{y}_2)$$

$$= \frac{p(\mathbf{y}_2\,|\,\mathbf{y}_1,\mathbf{m})\,p(\mathbf{m}\,|\,\mathbf{y}_1)}{p(\mathbf{y}_2,\mathbf{y}_1)}$$

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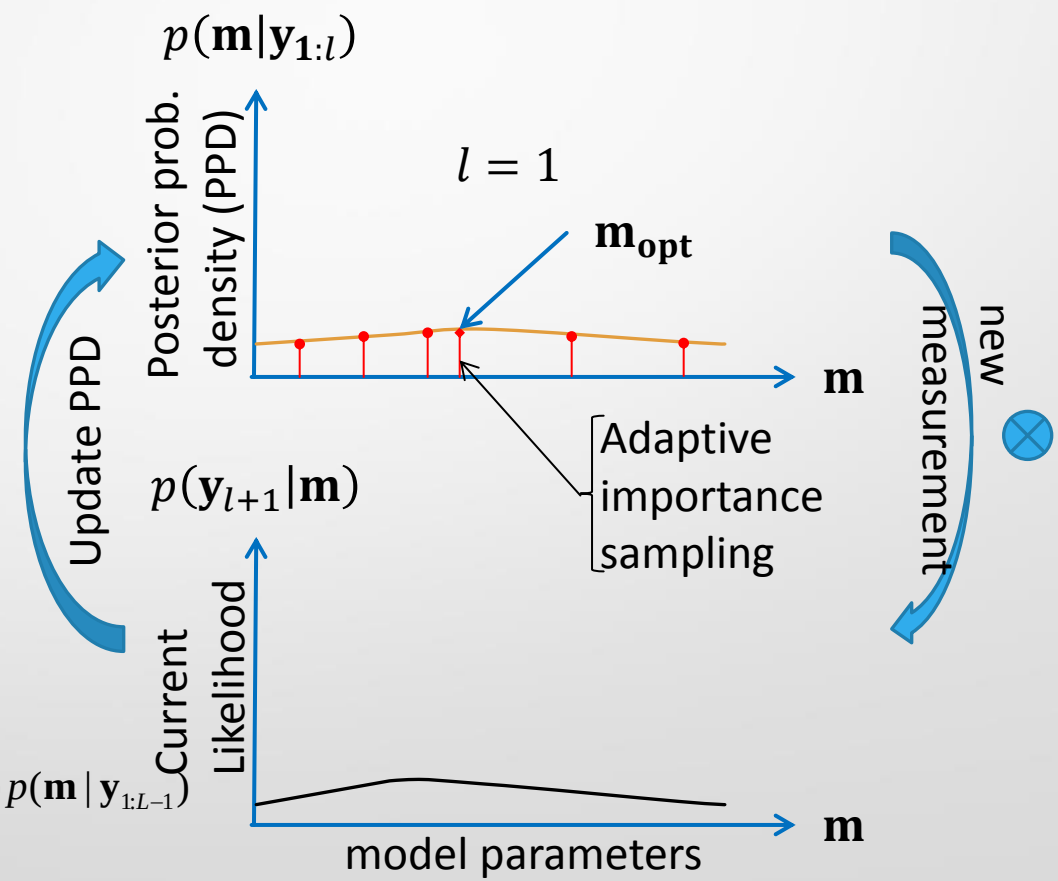
$$\propto p(\mathbf{y}_2\,|\,\mathbf{m})\,p(\mathbf{y}_1\,|\,\mathbf{m})\,p(\mathbf{m})$$

Generalizing for L measurements

$$p(\mathbf{m}\,|\,\mathbf{y}_{1:L}) \propto \prod_{l=1}^{L} p(\mathbf{y}_l\,|\,\mathbf{m})\,p(\mathbf{m}) \propto p(\mathbf{y}_L\,|\,\mathbf{m})\,p(\mathbf{m}\,|\,\mathbf{y}_{1:L-1})$$



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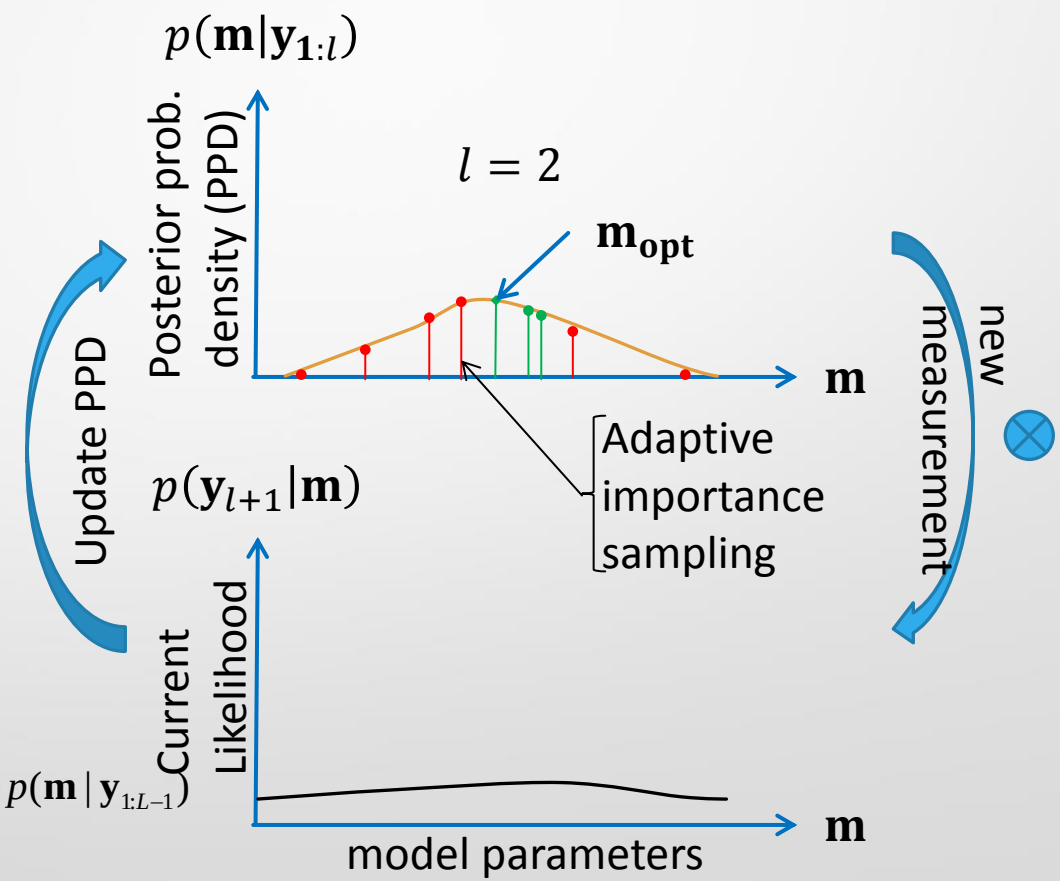

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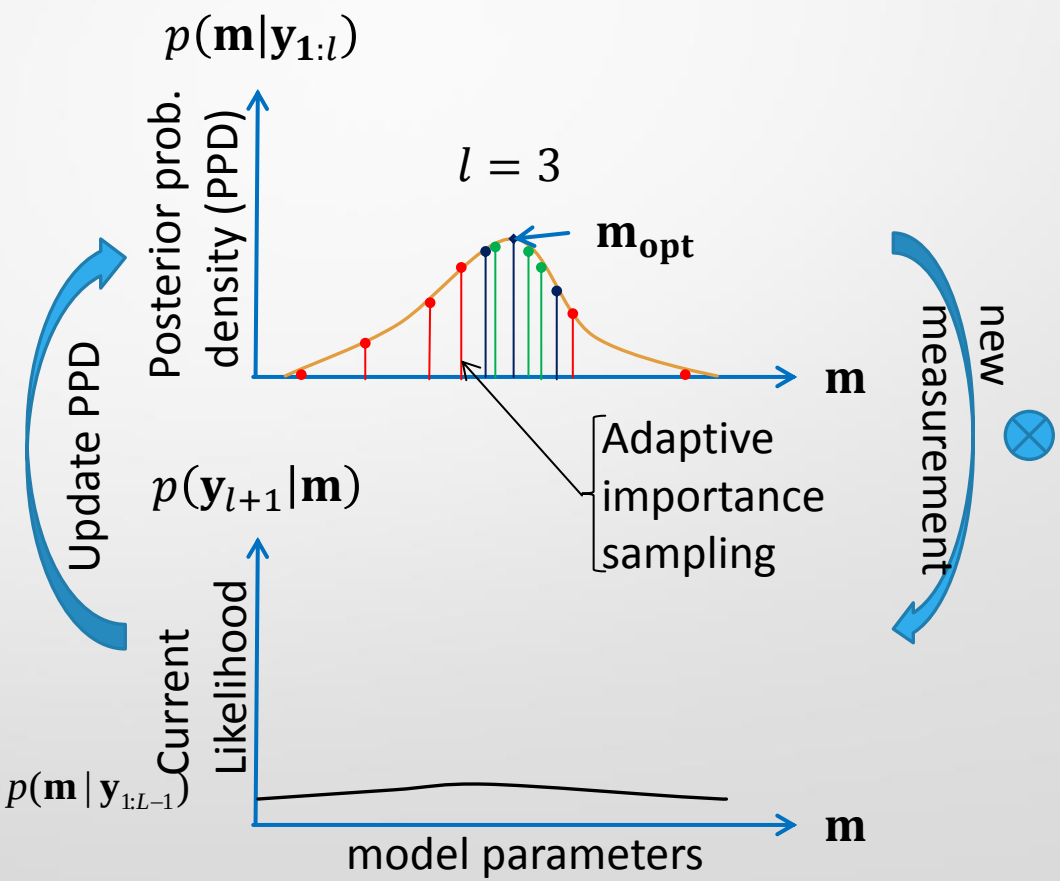

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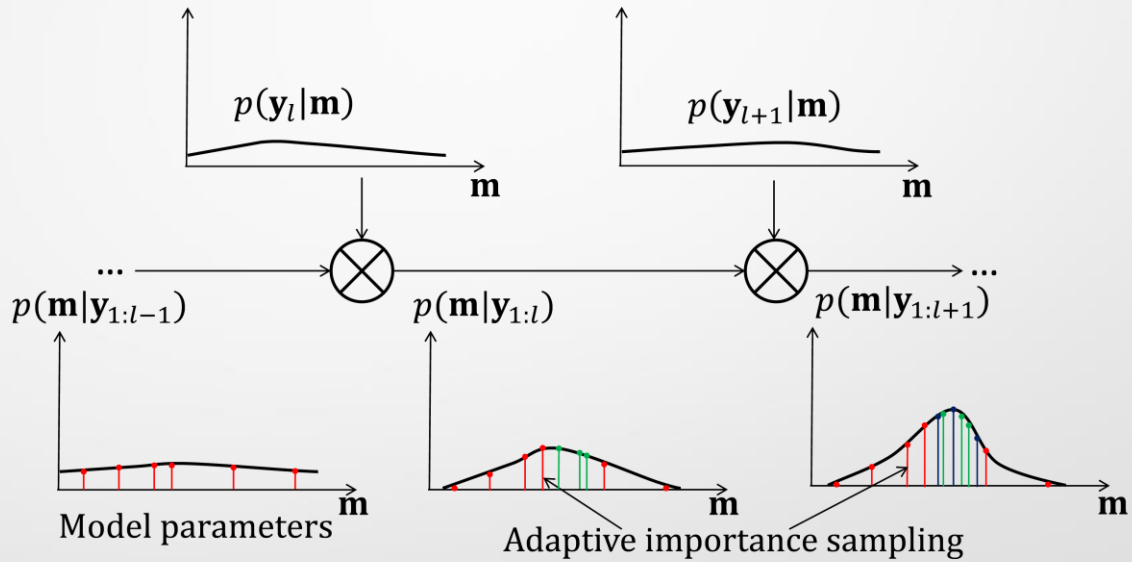

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Change-point detection for recursive Bayesian geoacoustic inversion [3]

- A key assumption for methods [1,2]
  - constant underlying model parameters
- Long-time coherent integration and source-receiver motion
- space-time environment changes likely
- Modeling the change parametrically is the best approach but also adversely increase the inversion search dimension. E.g. v_s, a_s
- A model parameter change-point detection method that detects abrupt or gradual change in model parameters is utilized.
- Change-point detection is well established see Ref. 1-6 in [3]
- The probability distributions (importance samples and weights) from recursive Bayesian inversion that are generated for model parameters estimations can also be used for inferences about the possible change-points.

[3] B. A. Tan, et. al., "Change-point detection for recursive Bayesian geoacoustic inversion," J. Acoust. Soc. Am., vol. 137, no. 4, pp. 1962-1970, 2015.

Change-point detection

- Applications
  - when tracking a ship with constant radial speed and detecting the point where it changes speed;
  - when accumulating snapshots for beamforming weak targets, and the direction of arrival changes;
  - when the underlying environmental parameters changes in geoacoustic inversion (the focus in this paper).

Change-point detection

- Consider a sequence of measurements $\mathbf{y}_l$
- Where there is a change-point r
  - Pre-change-point measurements follow model $\mathbf{m_1}$
  - Post-change-point measurements follow model $\mathbf{m_2}$

$$\mathbf{y}_l = \begin{cases} \mathbf{b}_l(\mathbf{m}_1) + \mathbf{w}_l, & \text{if } l = 1, \ldots, r. \\ \mathbf{b}_l(\mathbf{m}_2) + \mathbf{w}_l, & \text{if } l = r+1, \ldots, L. \end{cases}$$

ML estimate $\hat{r} = \arg\max_r p(\mathbf{y}_{1:r}|\hat{\mathbf{m}}_{1,r}) p(\mathbf{y}_{r+1:L}|\hat{\mathbf{m}}_{2,r})$

where

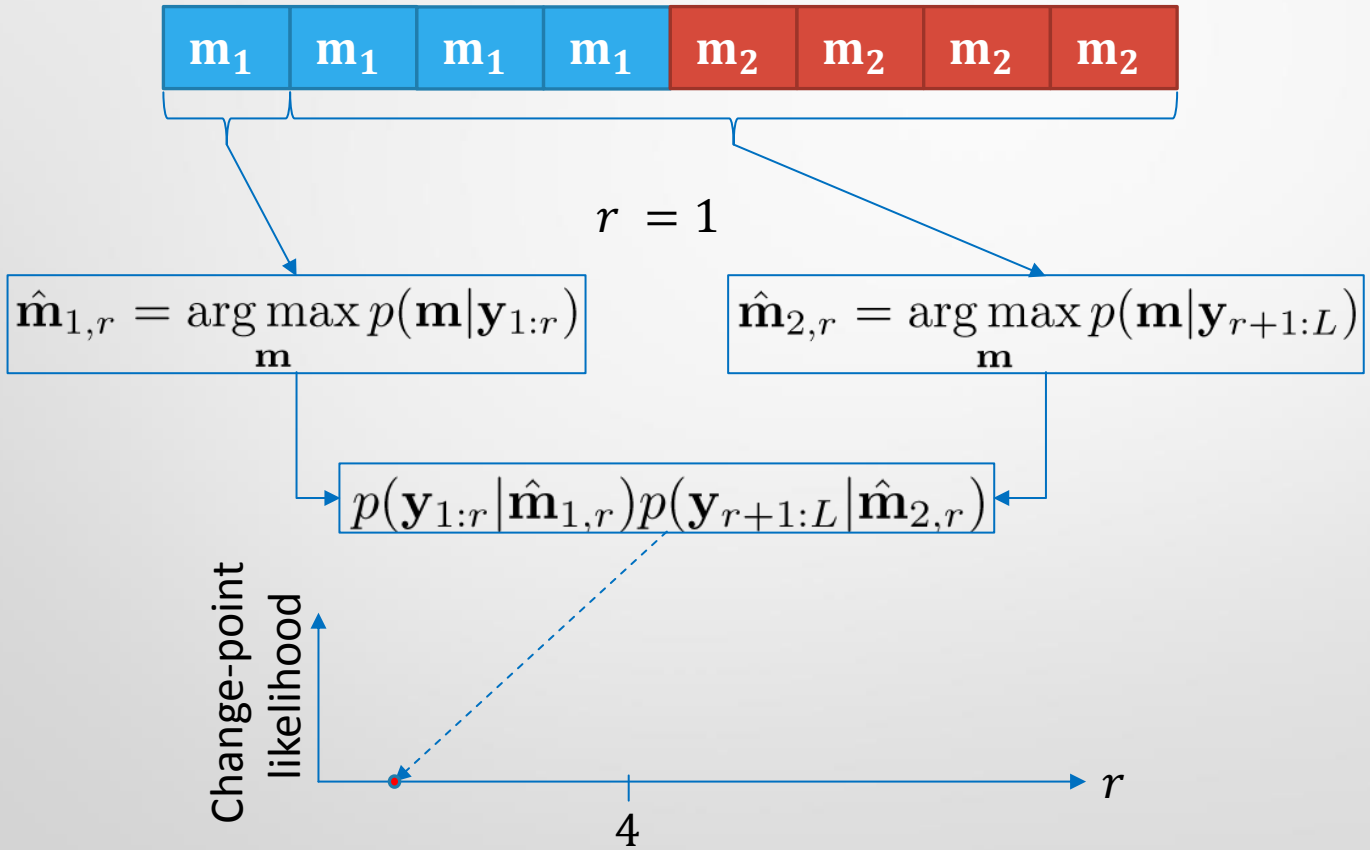
$$\hat{\mathbf{m}}_{1,r} = \arg\max_{\mathbf{m}} p(\mathbf{m}|\mathbf{y}_{1:r}),$$

and

$$\hat{\mathbf{m}}_{2,r} = \arg\max_{\mathbf{m}} p(\mathbf{m}|\mathbf{y}_{r+1:L}).$$

Change-point detection example

- For 8 measurements with true change-point $r_{true} = 4$



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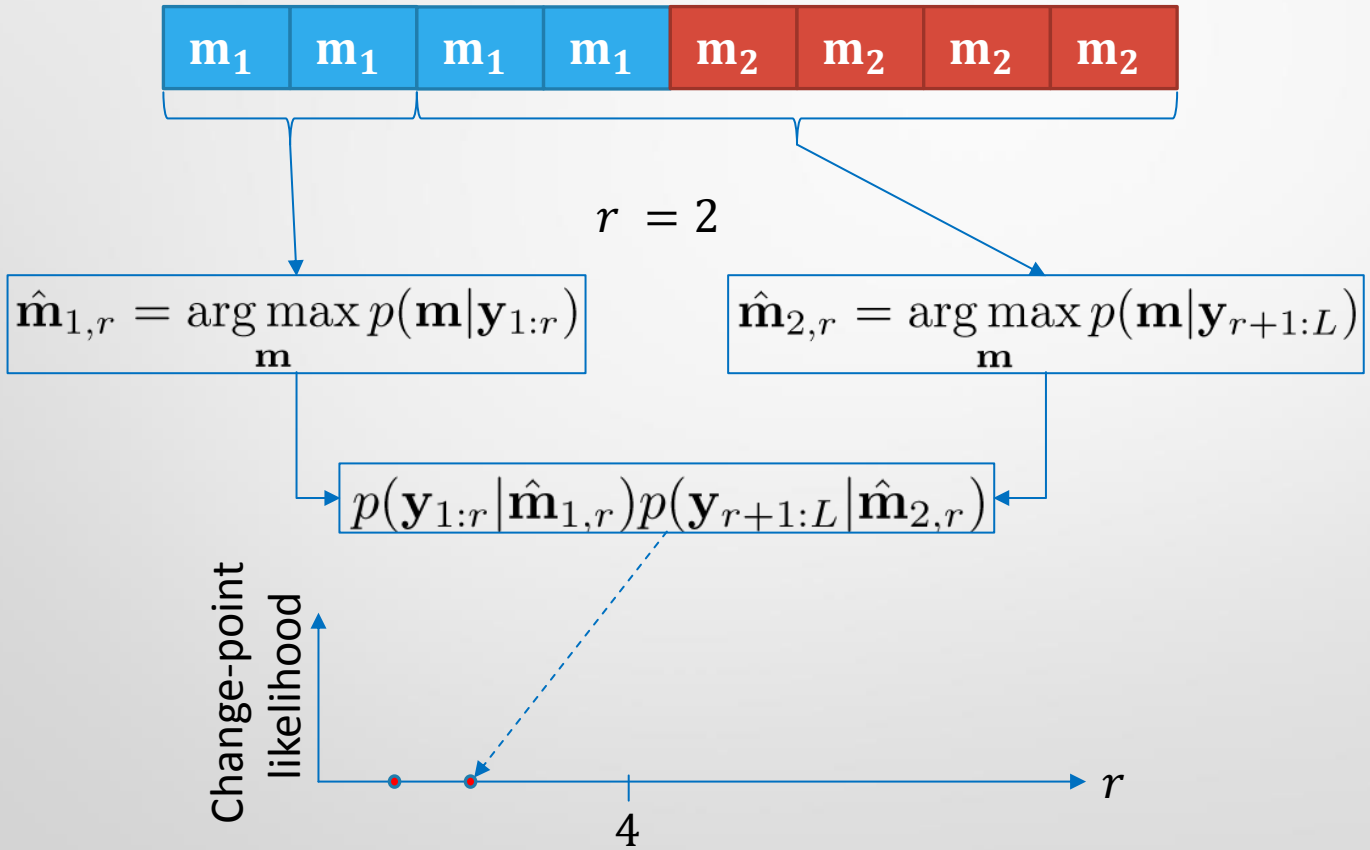

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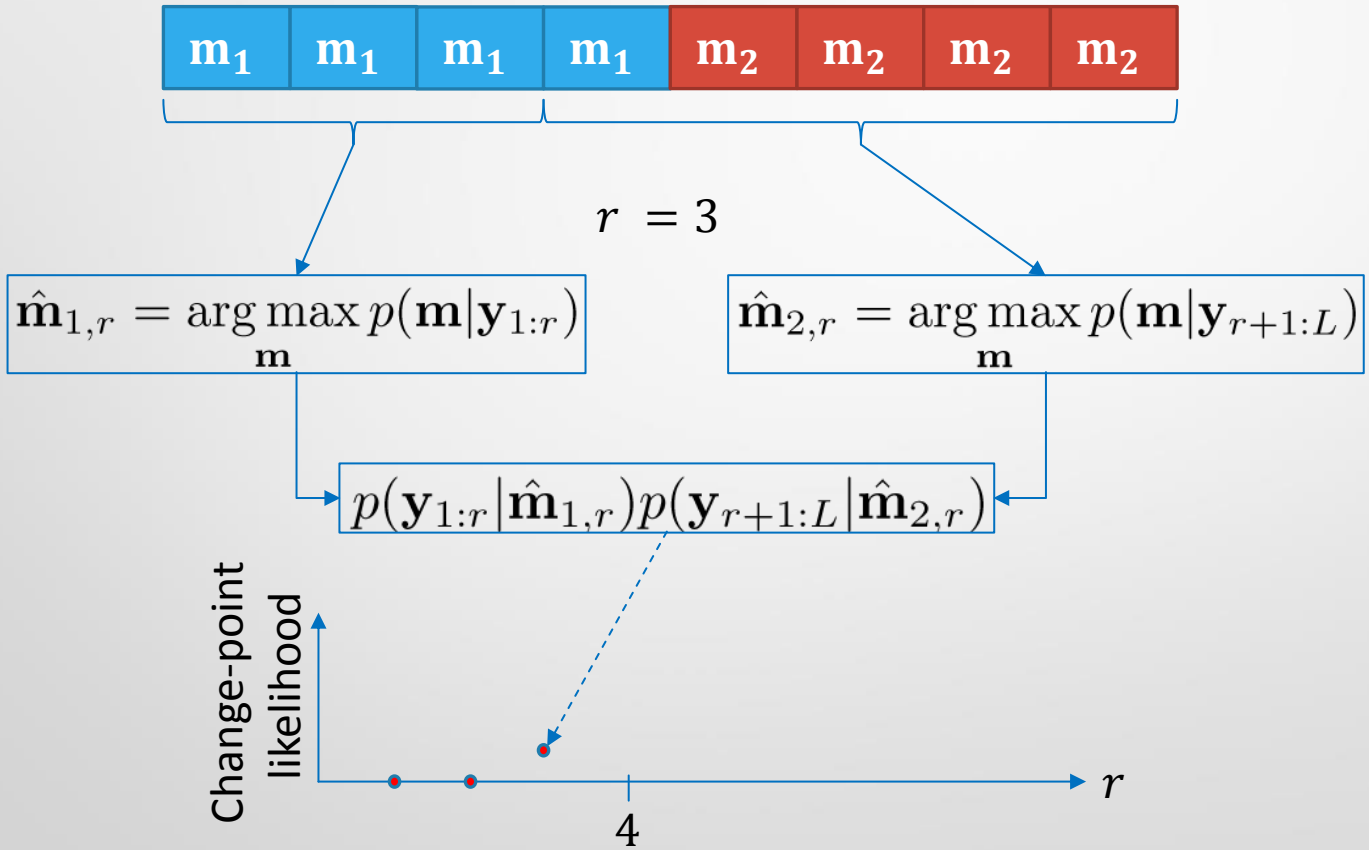

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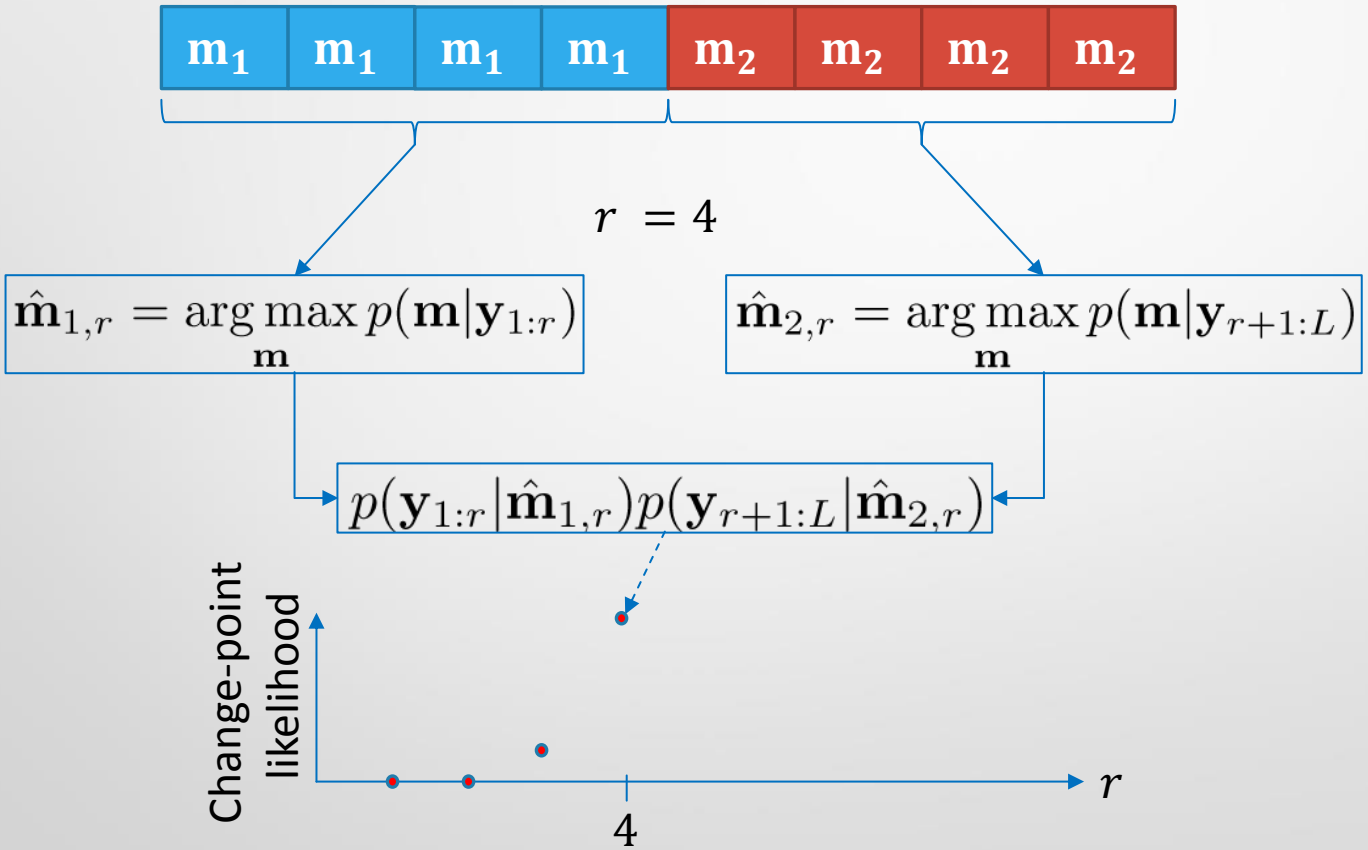

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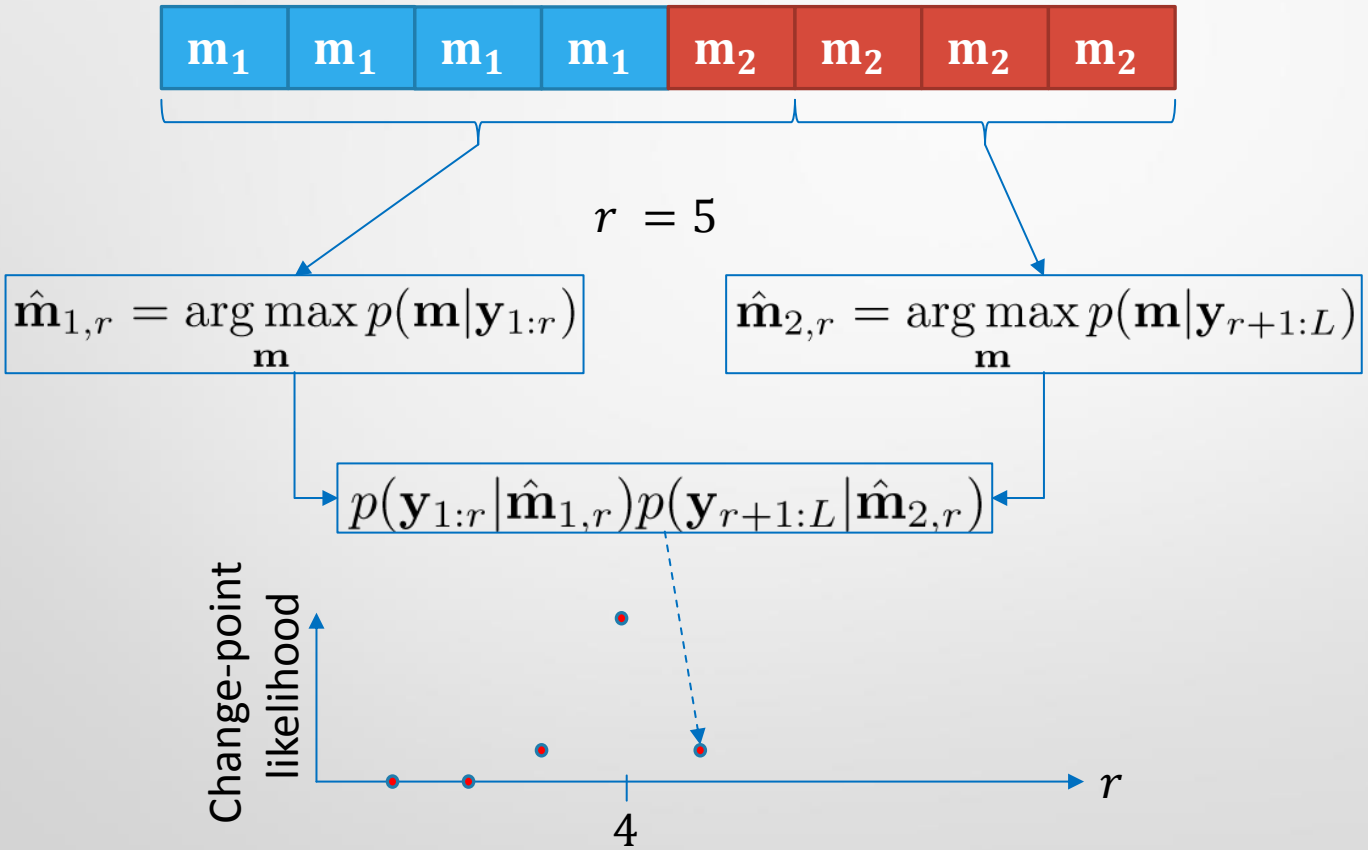

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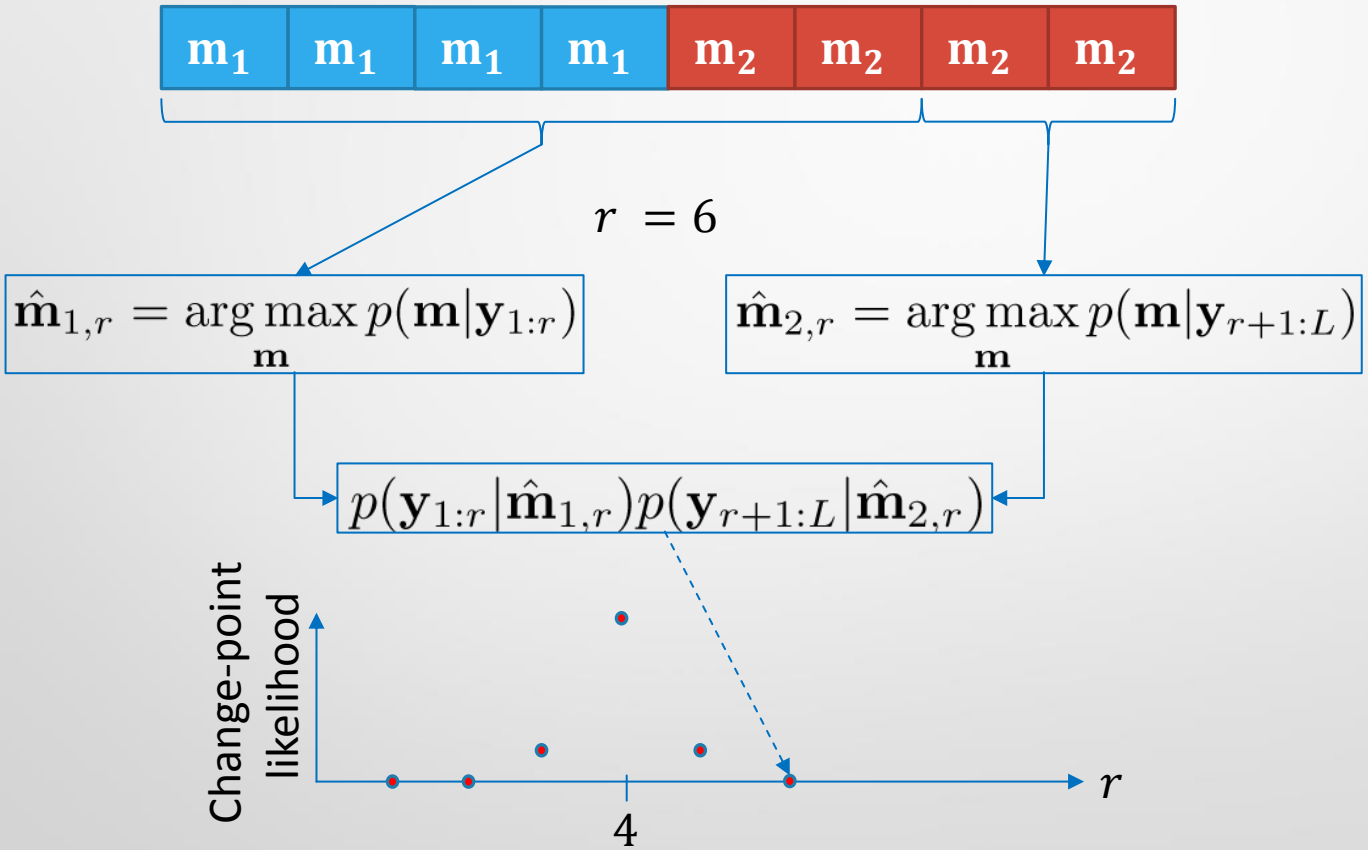

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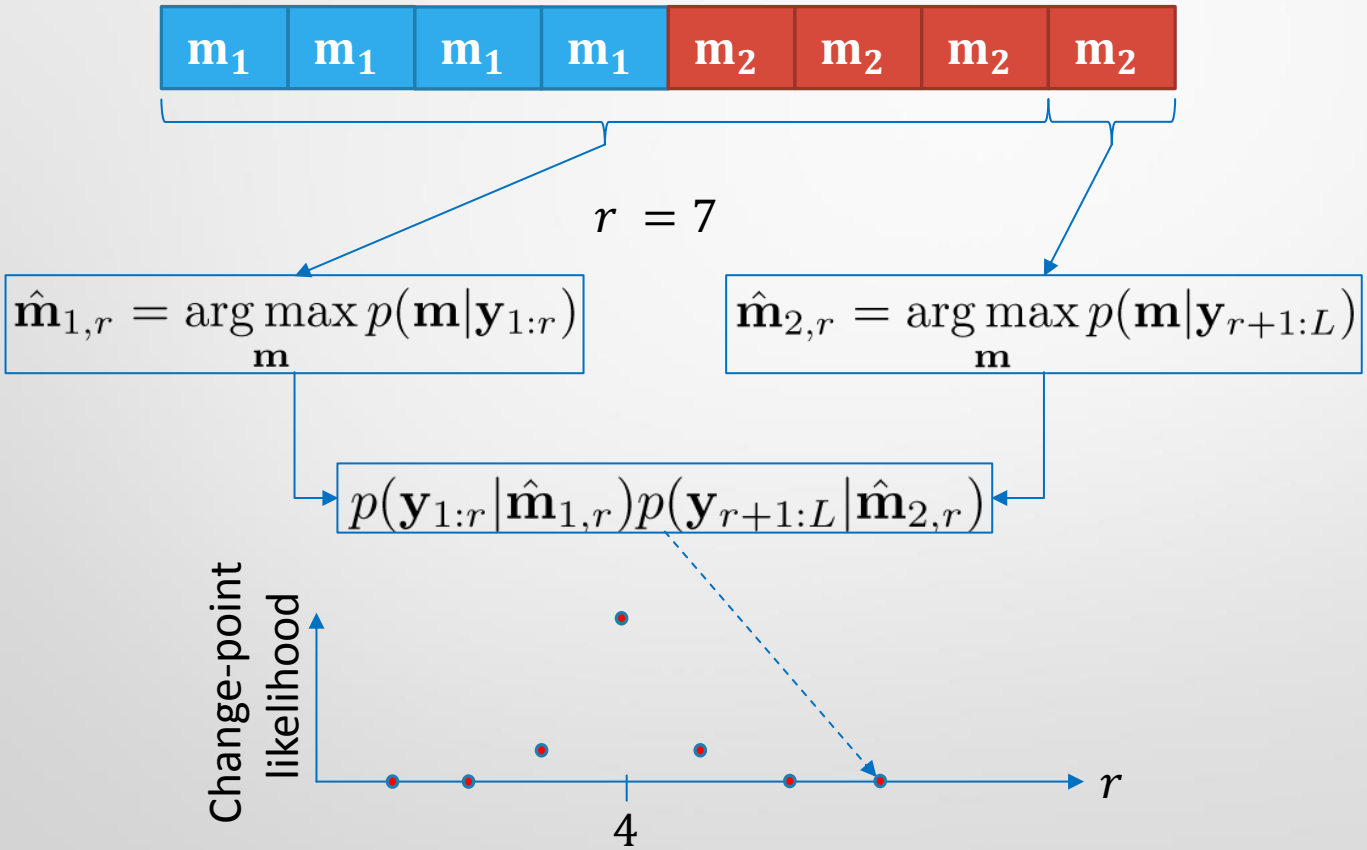
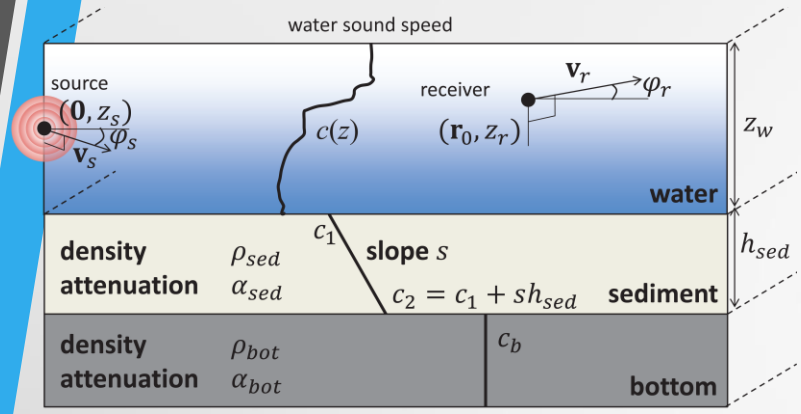
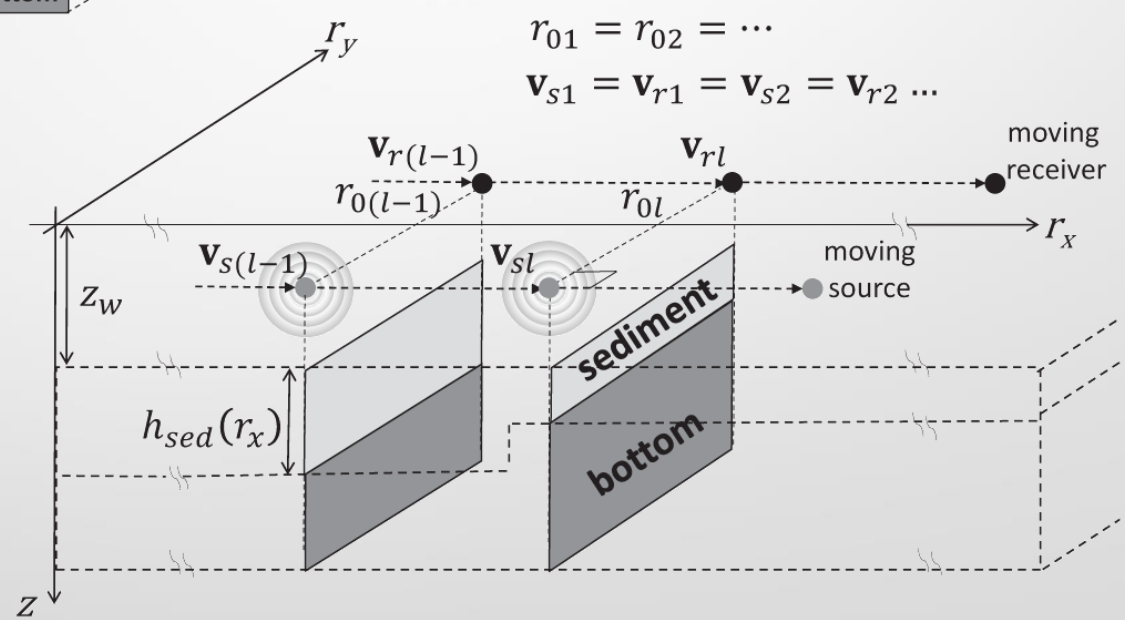


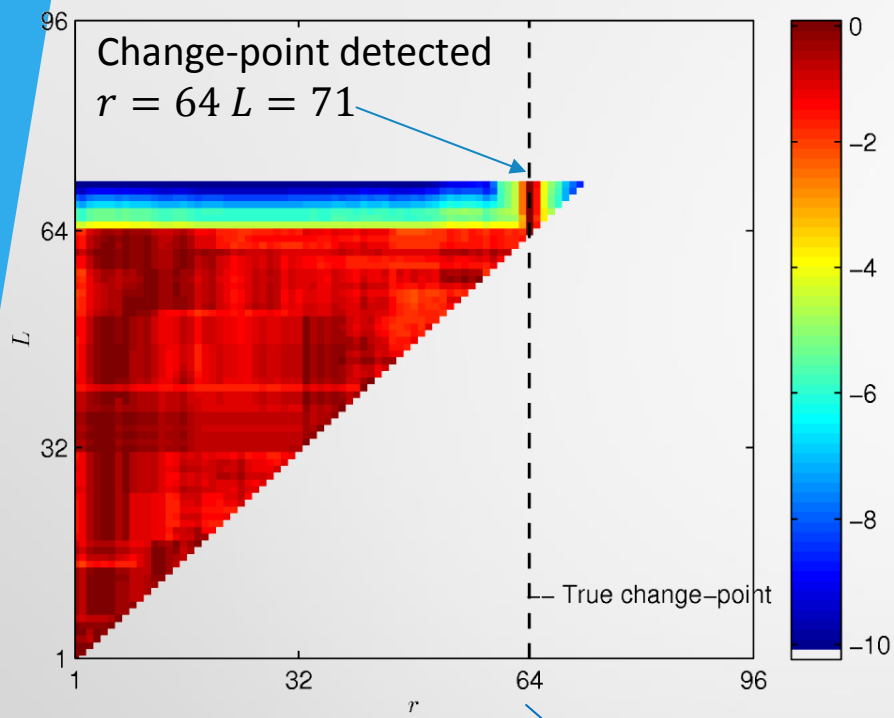
Simulation: Abrupt change

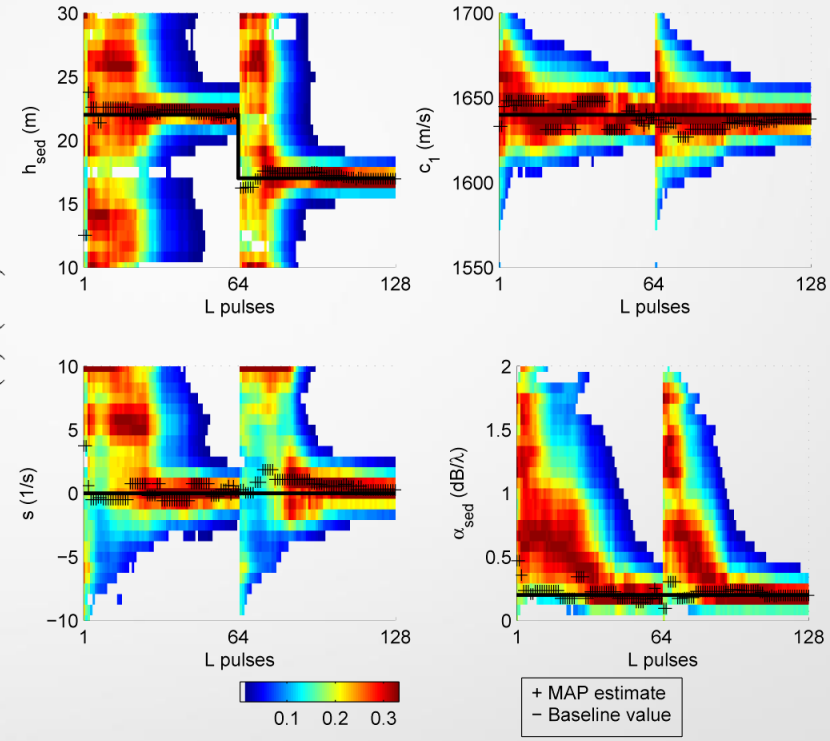




Simulation: Abrupt change

$$\mathcal{L}(r) = 10\log_{10}\frac{p(\mathbf{y}_{1:r}|\hat{\mathbf{m}}_{1,r})p(\mathbf{y}_{r+1:L}|\hat{\mathbf{m}}_{2,r})}{\max_r\, p(\mathbf{y}_{1:r}|\hat{\mathbf{m}}_{1,r})p(\mathbf{y}_{r+1:L}|\hat{\mathbf{m}}_{2,r})}$$



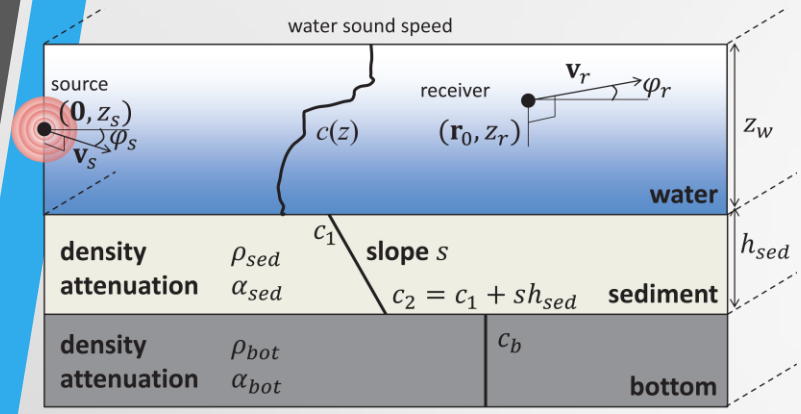


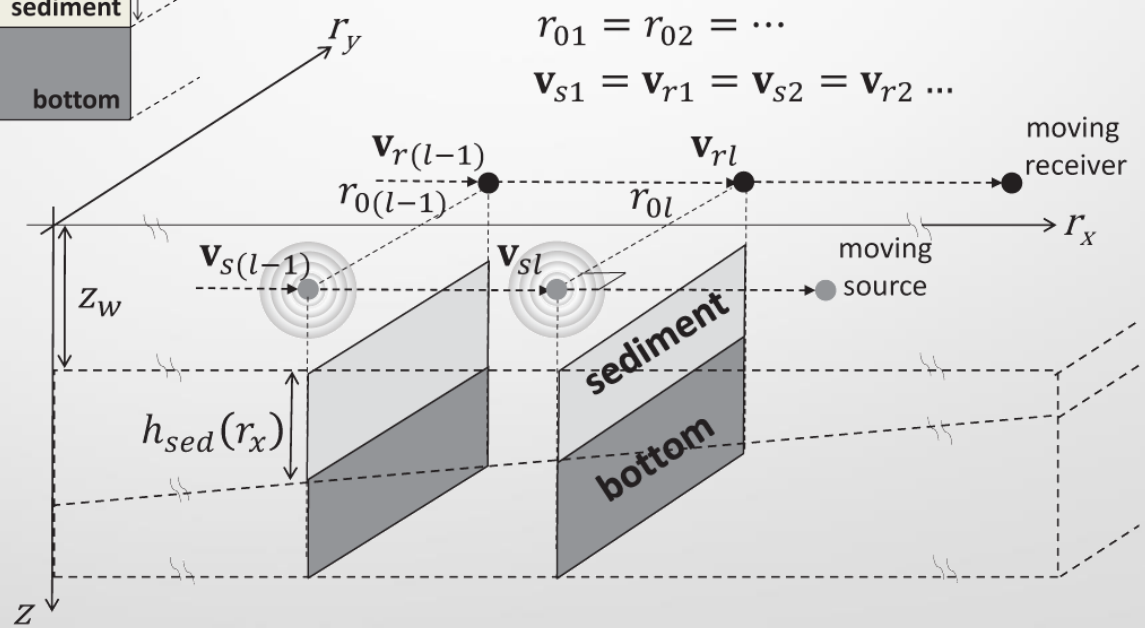
When a change-point is detected, the current inversion concludes and a new inversion is started using post change-point measurements.
Pre-change-point importance samples retained
Post-change-point importance samples discarded

Reconstructed PPD

Only 95% HPD plotted

Simulation: Gradual change





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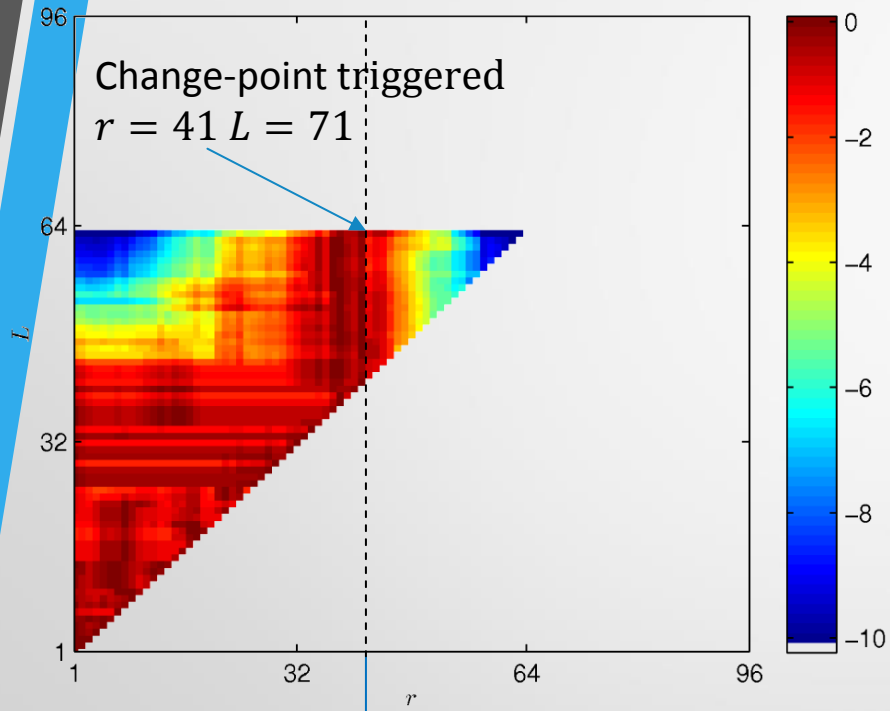
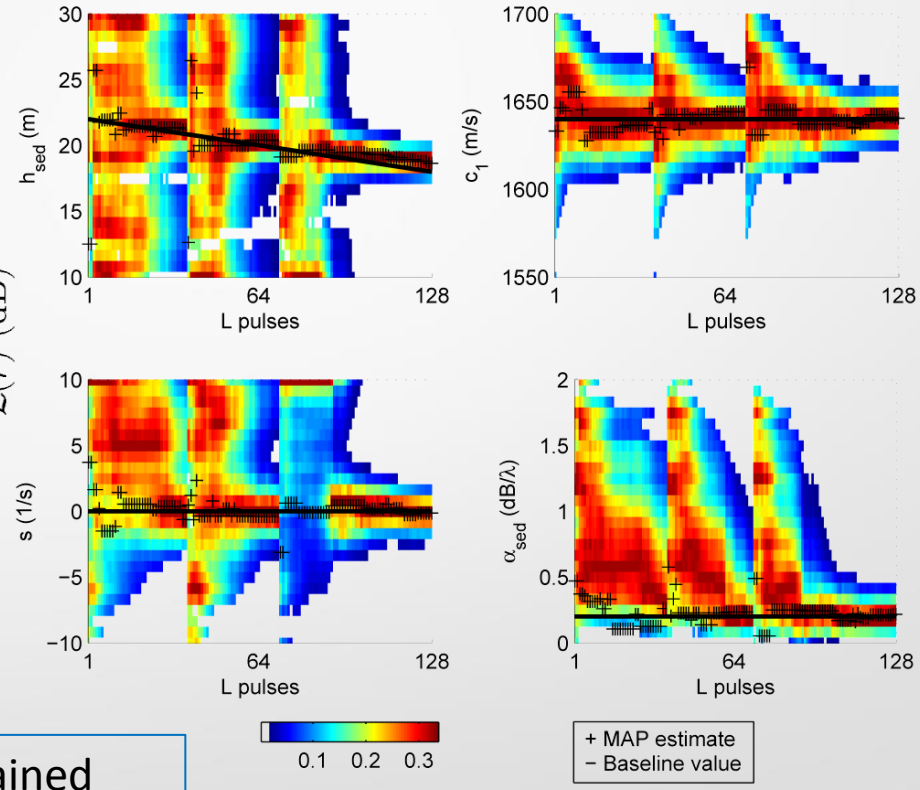
$$\mathcal{L}(r) = 10\log_{10}\frac{p(\mathbf{y}_{1:r}|\hat{\mathbf{m}}_{1,r})p(\mathbf{y}_{r+1:L}|\hat{\mathbf{m}}_{2,r})}{\max_r\, p(\mathbf{y}_{1:r}|\hat{\mathbf{m}}_{1,r})p(\mathbf{y}_{r+1:L}|\hat{\mathbf{m}}_{2,r})}$$

No significant bias due to constant model approximation
Gradual change segmented into three constant models





Pre-change-point importance samples retained
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Reconstructed PPD

Conclusions

- Combining change-point detection and recursive Bayesian inversion has enabled a data-driven verification of the constant model parameter assumption.
- Controlling the coherent integration time in recursive Bayesian inversion.

Acknowledgements

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