Change-point detection for recursive Bayesian geoacoustic inversion

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Overview

• Motivation
• Review of recursive Bayesian estimation
• Change-point detection
• Simulation results
• Conclusion
Motivation

- Powerful / bulky source and large aperture arrays
- Low power / compact source, single receiver
- But methods [1,2] assume constant underlying model parameters.

Recursive Bayesian approach [2]

- Data model of \( l \)th pulse measurement
  - \( y_l = b_l(m) + w_l \) where
  - \( y_l \) = measured field
  - \( b_l \) = modeled/replica field
  - \( m \) = model parameters
  - \( w_l \) = Gaussian noise

Bayes rule  \( p(m|y) \propto p(y|m)p(m) \)

Consider \( y_{1:2} \) two measurement case

Posterior Probability Density (PPD)

\[
p(m|y_{1:2}) = \frac{p(y_2|y_1, m)p(m|y_1)}{p(y_2, y_1)}
= \frac{p(y_2|y_1, m)p(y_1|m)p(m)}{p(y_2, y_1)}
\propto p(y_2|m)p(y_1|m)p(m)
\]

Generalizing for L measurements

\[
p(m|y_{1:L}) \propto \prod_{l=1}^{L} p(y_l|m)p(m) \propto p(y_L|m)p(m|y_{1:L-1})
\]

Recursive Bayesian approach

- **Data model of l\textsuperscript{th} pulse measurement**
  - \( y_l = b_l(m) + w_l \) where
  - \( y_l \) = measured field
  - \( b_l \) = modeled/replica field
  - \( m \) = model parameters
  - \( w_l \) = Gaussian noise

Bayes rule  \( p(m | y) \propto p(y | m) p(m) \)

Consider \( y_{1:2} \) two measurement case

Posterior Probability Density (PPD)
\[
p(m | y_{1:2}) = p(m | y_1, y_2) = \frac{p(y_2 | y_1, m)p(m | y_1)}{p(y_2, y_1)} = \frac{p(y_2 | y_1, m)p(y_1 | m)p(m)}{p(y_2, y_1)} \propto p(y_2 | m)p(y_1 | m)p(m)
\]

Generalizing for \( L \) measurements
\[
p(m | y_{1:L}) \propto \prod_{l=1}^{L} p(y_l | m)p(m) \propto p(y_L | m)p(m | y_{1:L-1})
\]

Update PPD

Current Likelihood

Adaptive importance sampling

new measurement
Recursive Bayesian approach

- **Data model of** $l^{th}$ pulse measurement
  - $y_l = b_l(m) + w_l$ where
  - $y_l =$ measured field
  - $b_l =$ modeled/replica field
  - $m =$ model parameters
  - $w_l =$ Gaussian noise

Bayes rule $p(m | y) \propto p(y | m) p(m)$

Consider $y_{1:2}$ two measurement case

Posterior Probability Density (PPD)

$$p(m | y_{1:2}) = p(m | y_1, y_2) = \frac{p(y_2 | y_1, m)p(m | y_1)}{p(y_2, y_1)}$$

$$= \frac{p(y_2 | y_1, m)p(y_1 | m)p(m)}{p(y_2, y_1)}$$

$$\propto p(y_2 | m)p(y_1 | m)p(m)$$

Generalizing for L measurements

$$p(m | y_{1:L}) \propto \prod_{l=1}^{L} p(y_l | m)p(m) \propto p(y_L | m)p(m | y_{1:L-1})$$

Current Likelihood

Update PPD

Adaptive importance sampling

New measurement

ppd

$\text{Current posterior probability density (PPD)}$

$\text{Posterior prob. density (PPD)}$

$\text{Update PPD}$

$\text{Adaptive importance sampling}$

new measurement

PPD

$\text{Current likelihood}$

$\text{model parameters}$

$\text{m}$

$\text{m}_{\text{opt}}$

$l = 3$
Recursive Bayesian approach

- **Data model of** \( l \)th pulse measurement
  \[ y_l = b_l(m) + w_l \]  where
  - \( y_l = \) measured field
  - \( b_l = \) modeled/replica field
  - \( m = \) model parameters
  - \( w_l = \) Gaussian noise

Bayes rule \( p(m \mid y) \propto p(y \mid m)p(m) \)

Consider \( y_{1:2} \) two measurement case

Posterior Probability Density (PPD)
\[
p(m \mid y_{1:2}) = p(m \mid y_1, y_2)
\]
\[
= \frac{p(y_2 \mid y_1, m)p(m \mid y_1)}{p(y_2, y_1)}
\]
\[
= \frac{p(y_2 \mid y_1, m)p(y_1 \mid m)p(m)}{p(y_2, y_1)}
\]
\[
\propto p(y_2 \mid m)p(y_1 \mid m)p(m)
\]

Generalizing for \( L \) measurements
\[
p(m \mid y_{1:L}) \propto \prod_{l=1}^{L} p(y_l \mid m)p(m) \propto p(y_L \mid m)p(m \mid y_{1:L-1})
\]
Change-point detection for recursive Bayesian geoacoustic inversion [3]

- A key assumption for methods [1,2]
  - constant underlying model parameters
- Long-time coherent integration and source-receiver motion
- space-time environment changes likely
- Modeling the change parametrically is the best approach but also adversely increase the inversion search dimension. E.g. $v_s, a_s$
- A model parameter change-point detection method that detects abrupt or gradual change in model parameters is utilized.
- Change-point detection is well established see Ref. 1-6 in [3]
- The probability distributions (importance samples and weights) from recursive Bayesian inversion that are generated for model parameters estimations can also be used for inferences about the possible change-points.

Change-point detection

• Applications

• when tracking a ship with constant radial speed and detecting the point where it changes speed;

• when accumulating snapshots for beamforming weak targets, and the direction of arrival changes;

• when the underlying environmental parameters changes in geoacoustic inversion (the focus in this paper).
Change-point detection

• Consider a sequence of measurements \( y_l \)

• Where there is a change-point \( r \)
  
  • Pre-change-point measurements follow model \( m_1 \)
  
  • Post-change-point measurements follow model \( m_2 \)

\[
y_l = \begin{cases} 
  b_l(m_1) + w_l, & \text{if } l = 1, \ldots, r. \\
  b_l(m_2) + w_l, & \text{if } l = r + 1, \ldots, L.
\end{cases}
\]

ML estimate \( \hat{r} = \arg \max_r p(y_{1:r} | \hat{m}_{1,r})p(y_{r+1:L} | \hat{m}_{2,r}) \)

where

\[
\hat{m}_{1,r} = \arg \max_m p(m | y_{1:r}),
\]

and

\[
\hat{m}_{2,r} = \arg \max_m p(m | y_{r+1:L}).
\]
Change-point detection example

• For 8 measurements with true change-point \( r_{true} = 4 \)

\[
\hat{m}_{1,r} = \arg \max_m p(m|y_{1:r}) \\
\hat{m}_{2,r} = \arg \max_m p(m|y_{r+1:L})
\]

\[
p(y_{1:r} | \hat{m}_{1,r})p(y_{r+1:L} | \hat{m}_{2,r})
\]
Change-point detection example

- For 8 measurements with true change-point $r_{true} = 4$

$$\hat{m}_{1,r} = \arg \max_m p(m|y_{1:r})$$

$$\hat{m}_{2,r} = \arg \max_m p(m|y_{r+1:L})$$

$$p(y_{1:r} | \hat{m}_{1,r})p(y_{r+1:L} | \hat{m}_{2,r})$$

$\hat{m}_{1,r}$ and $\hat{m}_{2,r}$ are the estimated change-points.
Change-point detection example

- For 8 measurements with true change-point $r_{true} = 4$

\[
\hat{m}_{1,r} = \arg \max_m p(m | y_{1:r})
\]
\[
\hat{m}_{2,r} = \arg \max_m p(m | y_{r+1:L})
\]

\[
p(y_{1:r} | \hat{m}_{1,r}) p(y_{r+1:L} | \hat{m}_{2,r})
\]
### Change-point detection example

- For 8 measurements with true change-point $r_{true} = 4$

<table>
<thead>
<tr>
<th>$m_1$</th>
<th>$m_1$</th>
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<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_2$</th>
<th>$m_2$</th>
<th>$m_2$</th>
</tr>
</thead>
</table>

$r = 4$

\[ \hat{m}_{1,r} = \arg\max_m p(m | y_{1:r}) \]

\[ \hat{m}_{2,r} = \arg\max_m p(m | y_{r+1:L}) \]

\[ p(y_{1:r} | \hat{m}_{1,r}) p(y_{r+1:L} | \hat{m}_{2,r}) \]
Change-point detection example

- For 8 measurements with true change-point $r_{true} = 4$

\[
\hat{m}_{1,r} = \arg \max_m p(m | y_{1:r}) \]

\[
\hat{m}_{2,r} = \arg \max_m p(m | y_{r+1:L})
\]

\[
p(y_{1:r} | \hat{m}_{1,r}) p(y_{r+1:L} | \hat{m}_{2,r})
\]
Change-point detection example

• For 8 measurements with true change-point $r_{true} = 4$

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\hat{m}_{1,r} = \arg \max_{m} p(m | y_{1:r})
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\hat{m}_{2,r} = \arg \max_{m} p(m | y_{r+1:L})
\]

\[
p(y_{1:r} | \hat{m}_{1,r}) p(y_{r+1:L} | \hat{m}_{2,r})
\]
Change-point detection example

• For 8 measurements with true change-point \( r_{\text{true}} = 4 \)

\[
\begin{align*}
\hat{m}_{1,r} &= \arg \max_{m} p(m|y_{1:r}) \\
\hat{m}_{2,r} &= \arg \max_{m} p(m|y_{r+1:L}) \\
p(y_{1:r}|\hat{m}_{1,r})p(y_{r+1:L}|\hat{m}_{2,r})
\end{align*}
\]
Simulation: Abrupt change

- Density and attenuation
  - $c_1$, slope $s$, $c_2 = c_1 + sh_{sed}$ for sediment
  - $c_b$ for bottom

- Source at $(0, z_s)$ with velocity $v_s$
- Receiver at $(r_0, z_r)$ with velocity $v_r$
- Water sound speed $c(z)$

- $r_{01} = r_{02} = \ldots$
- $v_{s1} = v_{r1} = v_{s2} = v_{r2} = \ldots$

- Moving source

- Moving receiver

- $v_{s(l-1)}$, $v_{s(l)}$, $v_{s(l+1)}$, $v_{r(l-1)}$, $v_{r(l)}$, $v_{r(l+1)}$
Simulation: Abrupt change

When a change-point is detected, the current inversion concludes and a new inversion is started using post change-point measurements. Pre-change-point importance samples retained Post-change-point importance samples discarded

Reconstructed PPD
Only 95% HPD plotted
Simulation: Gradual change
Simulation: Gradual change

\[ \mathcal{L}(r) = 10 \log_{10} \frac{p(y_{1:r}|\hat{m}_{1,r})p(y_{r+1:L}|\hat{m}_{2,r})}{\max_r p(y_{1:r}|\hat{m}_{1,r})p(y_{r+1:L}|\hat{m}_{2,r})} \]

Change-point triggered
\[ r = 41 \quad L = 71 \]

No significant bias due to constant model approximation
Gradual change segmented into three constant models

Pre-change-point importance samples retained
Post-change-point importance samples discarded

Reconstructed PPD
Conclusions

- Combining change-point detection and recursive Bayesian inversion has enabled a data-driven verification of the constant model parameter assumption.
- Controlling the coherent integration time in recursive Bayesian inversion.

Acknowledgements

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