Effects of Sea Clutter on Atmospheric Refractivity Estimation

Caglar Yardim*, *Peter Gerstoft*, *and William S. Hodgkiss

Marine Physical Laboratory
University of California, San Diego
La Jolla, CA 92093–0238, USA
(email: cyardim@ucsd.edu, gerstoft@ucsd.edu, whodgkiss@ucsd.edu)

Abstract

This paper addresses the effects of sea clutter on the clutter-based atmospheric refractivity estimation techniques. It provides a mean square error (MSE) metric for refractivity estimators based on the available clutter-to-noise (CNR) ratio, the type of the sea clutter statistics (Rayleigh, K-distributed and log-normal) and the parameters forming these probability density functions (pdf) such as spikiness and mean-to-median ratios. It includes a discussion about the selection of appropriate statistics for refractivity estimation with very low grazing angles and low resolutions. The performance of an evaporation duct height estimator is provided as an example.

1 Introduction

Many of the lower atmospheric maritime radar systems have to operate under ducting conditions. Electromagnetic ducts result in non-standard electromagnetic propagation which can fundamentally change the performance of the system that is designed to operate under standard atmospheric conditions (a refractivity profile with a slope of 0.118 M-units/m). These include the change in the maximum operational radar range, creation of radar holes where the radar is effectively blind, target altitude errors and large sea surface clutter due to the increased interaction between the electromagnetic signal trapped within the duct and the sea surface. Refractivity from clutter (RFC) refers to techniques that estimate the atmospheric refractivity profile from radar clutter returns. The majority of the RFC techniques use a split-step fast Fourier transform (FFT) parabolic equation (PE) approximation to the wave equation to simulate propagation under ducting conditions. A typical RFC inversion would work by finding an environment whose FFT-PE simulated clutter pattern matches the radar measured one [1–4]. (See [3] for comparison between various other RFC algorithms) . In all of these previous studies a simple Gaussian pdf is used to represent the sea clutter statistics. This paper examines the effects of non-Gaussian clutter pdf on the RFC performance.

2 Clutter Statistics and CNR in RFC

To carry out a detailed RFC performance analysis, realistic clutter conditions must be incorporated into the simulations. The complex signal $y(r_k)$ received by the radar in non-standard propagation conditions can be computed by integrating the surface reflection over the radar illuminated sea surface as

$$y(r_k) = \int\int_{\Omega_k} G^2(\mathbf{r}_\mathrm{o}, \mathbf{r}')b(\mathbf{r}')d\Omega' + w(r_k), \tag{1}$$

where $b(\mathbf{r}')$ is the complex reflection coefficient of the sea surface at $\mathbf{r}' = (r', \theta', z') \in \Omega_k$, r_k and Ω_k are the range and the area of the sea surface illuminated by the radar (integration domain) at the kth radar range bin, $w(r_k)$ is the receiver noise, and $G(\mathbf{r}_\mathrm{o}, \mathbf{r}')$ is the Green's function defined as the signal on the ocean surface at $\mathbf{r}'$ for a point source at the radar location $\mathbf{r}_\mathrm{o} = (0, 0, z_\mathrm{a})$ with z_a being the antenna height. $G(\mathbf{r}_\mathrm{o}, \mathbf{r}')$ takes both the radar parameters and the ducting conditions into account. It can be separated using the propagation factor $F(\mathbf{r}')$, defined as the ratio of the electric field that would be measured in a complex environment to that of free space $G(\mathbf{r}_\mathrm{o}, \mathbf{r}') = F(\mathbf{r}')G_\mathrm{o}(\mathbf{r}_\mathrm{o}, \mathbf{r}')$, where $G_\mathrm{o}(\mathbf{r}_\mathrm{o}, \mathbf{r}')$ is the free space Green's function. The Propagation factor $F(\mathbf{r}')$ can be obtained from the split-step FFT-PE. Assuming that

r_k is large and $F(\mathbf{r}')$ is almost constant throughout the illuminated area Ω_k, the received radar signal can be written as

$$G(\mathbf{r}_\mathrm{o}, \mathbf{r}') = cF(\mathbf{r}')\frac{e^{-jk_\mathrm{o}|\mathbf{r}_\mathrm{o}-\mathbf{r}'|}}{4\pi|\mathbf{r}_\mathrm{o}-\mathbf{r}'|}, \qquad y(r_k) = c\frac{F^2(r_k)}{(4\pi r_k)^2}\int\int_{\Omega_k} e^{-j2k_\mathrm{o}|\mathbf{r}_\mathrm{o}-\mathbf{r}'|}b(\mathbf{r}')d\Omega' + w(r_k), \tag{2}$$

where c represents the collection of constant terms. The propagation loss factor F can be replaced with the propagation loss L using $L = \frac{L_{fs}}{F^2}$, where $L_{fs} = \frac{(4\pi r)^2}{\lambda^2}$ is the free space propagation loss, and λ is the wavelength. Sea clutter radar cross section (RCS) σ is given by

$$\sigma(r_k) = \left[\int\int_{\Omega_k} e^{-j2k_\mathrm{o}|\mathbf{r}_\mathrm{o}-\mathbf{r}'|}b(\mathbf{r}')d\Omega'\right]^2 = A_k\sigma^\mathrm{o}(r_k) = cr_k\sigma^\mathrm{o}(r_k), \tag{3}$$

where A_k is the illuminated area at the kth radar bin and σ^o is the surface reflectivity. Finally, the signal y_k received by the radar in a ducted environment is given by

$$y(r_k) = cr_k^{1/2}L^{-1}(r_k)\sqrt{\sigma^\mathrm{o}(r_k)} + w(r_k), \tag{4}$$

where $w(r_k)$ is the additive complex Gaussian noise, and $\sqrt{\sigma^\mathrm{o}(r_k)}$ represents the complex sea clutter amplitude. Selection of appropriate clutter statistics for RFC applications depends on various factors such as the grazing angle, surface wind speed and direction, polarization, and the size of the radar resolved area. The performance calculations used here are based on the peak available CNR within the selected inversion interval (which corresponds to the CNR at the inversion start range, taken as 10 km for this case). Henceforth, this value will simply be referred as the CNR. In general, RFC applications are:

1. Very low grazing angle applications typically less than 0.5^o. This results in complex scattering mechanisms such as multiple scattering and interference of scattered signals traveling in different directions, shadowing caused by the sea swells, and diffraction over the wave edges. This results in more spiky sea clutter.
2. Low radar resolution (radar resolution cell much larger than the sea swell wavelength) applications. Even if the system is high resolution, a RFC inversion (especially in surface-based and mixed ducts) algorithm will average a number of cells to decrease the CPU time, which is directly proportional to the number of range bins in split-step FFT PE. Typical range resolution values used in previous RFC applications run between 100–600 m. Due to the increased illuminated area of the low resolution radar there are more random scatterers contributing to the overall clutter reducing the spiky behavior.

To take these phenomena into account, three different sea clutter statistics are used and compared in this paper. Rayleigh statistics are suitable in applications with high grazing angle ($> 10^\mathrm{o}$) and low radar resolution. This pdf relies on the fact that there are many independent random scatterers in the resolution cell and the overall clutter is a summation of all these small reflections. Using the central limit theorem this summation can be shown to be a complex Gaussian pdf, which corresponds to Rayleigh amplitude statistics with $p(x) = \frac{2x}{X}\exp\left(\frac{-x^2}{X}\right)$, where x is the clutter amplitude and $\langle x^2\rangle = X$. The spikiness of the clutter can be given in terms of its clutter power mean-to-median ratio $\langle X\rangle/X_m$. The Rayleigh pdf has a fixed value of $\langle X\rangle/X_m = 1.6$ dB.

The log-normal pdf is used to describe spiky sea clutter. Although this pdf does not explain the underlying physics of the sea clutter, its pdf with a long tail fits well to spiky clutter measurements. The pdf of the log-normal distributed RCS is given by $p(X) = \frac{1}{X\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln(X)-\mu)^2}{2\sigma^2}\right)$, where μ and σ are the mean and the standard deviation of the Gaussian function in the logarithmic domain. The mean-to-median ratio in dB is given by $0.115\sigma^2_{\mathrm{dB}}$ with a high value corresponding to a spiky clutter.

The K-distribution was introduced as an effective means to represent the sea clutter amplitude distribution [5]. Later, its compound nature and the underlying physical mechanisms that enable the splitting

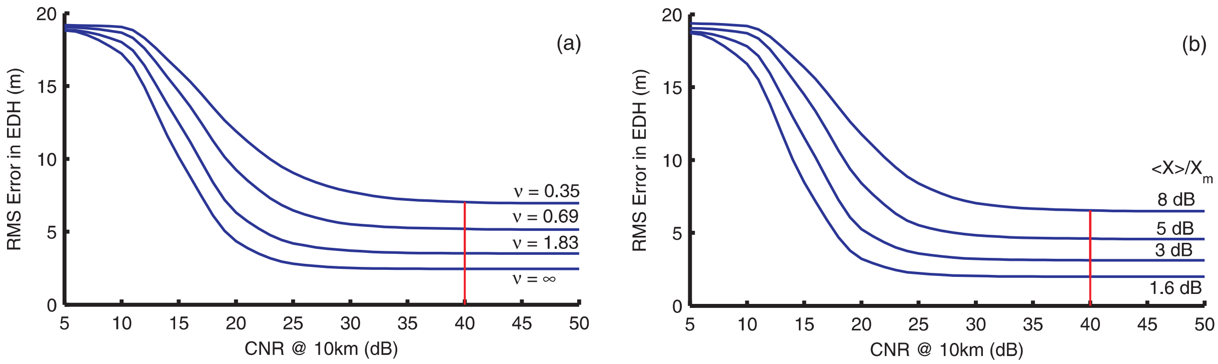

Figure 1: Performance plots: RMS evaporation duct height error for a 20 m high evaporation duct as a function of CNR for (a) K-distributed and (b) log-normal clutter with mean-to-median ratios of 1.6 (where K-distribution reduces to a Rayleigh), 3, 5, and 8 dB. Vertical lines represent the CNR SPANDAR attains (40 dB) at 10 km range.

of this complex pdf into two distinct components were demonstrated. It is extensively used in applications with low grazing angle and high radar resolution.

The K-distribution is a compound pdf where the gamma distributed local mean clutter power $p(X)$ is modulated by the quickly changing Rayleigh speckle. The gamma component represents the slowly varying long sea waves and swell structure. The clutter amplitude statistics $p(x)$ can be obtained as [6]

$$p(x) = \frac{4b^{1/2}}{\Gamma(\nu)}(bx)^{\nu} K_{\nu-1}(2x\sqrt{b}), \tag{5}$$

where $\Gamma(\nu)$ is the Gamma function, b and ν are the scale and shape factors with the mean clutter power $\langle X \rangle = \nu/b$, and $K_{\nu-1}$ is the modified Bessel function of the third kind of order $\nu - 1$. The shape parameter ranges from $0.1 \leq \nu \leq \infty$ and determines the spikiness of the pdf. A value less than 1 represents spiky clutter and $\nu = \infty$ reduces the K-distribution to a Rayleigh.

Depending on the radar and environmental parameters, all three distributions can be encountered for the RFC application. Therefore, RFC performance of an evaporation duct estimator given in [1] are computed along with the varying CNR using these three pdfs. The results are provided in Fig. 1. It shows the RMS error in the inversion of an evaporation duct with an EDH of 20 m for a CNR ranging from 5–50 dB at the inversion start range (taken as 10 km) and for clutter with different mean-to-median ratios. The ν values of the K-distribution are selected so that the pdf will give identical $\langle X \rangle / X_m$ and mean clutter power $\langle X \rangle$ as the log-normal simulations. The radar frequency and the antenna height corresponds those of SPANDAR (2.8 GHz and 31 m). The signal is constructed by averaging ten consecutive clutter returns ($n_{\text{avg}} = 10$). The RMS error at each CNR is computed by averaging 5000 simulations.

The plots show how low CNR can degrade the RFC performance. Below about 10 dB CNR, the signal is entirely noise-dominated and the performance quickly degrades. When the noise level is too high, the algorithm fails to correctly normalize the measured clutter, resulting in large errors in the RFC estimates. Similarly, above about 20–25 dB CNR, an infinite CNR assumption can be used. The received signal is purely clutter dominated and the performance rapidly converges to the infinite CNR case.

The spiky clutter requires a higher CNR to attain the same RMS error and the infinite CNR error values are larger compared to the less spiky clutter. Note that a transitional region exists between the noise-dominated and clutter-dominated CNR spectrum. This transitional region starts at a low CNR for clutter with a low mean-to-median ratio and is sharper. For highly spiky clutter with $\langle X \rangle / X_m = 8$ dB, the required CNR to reach the clutter-dominated region is as high as 35 dB. As shown in the figure, SPANDAR with a 40 dB CNR at 10 km range is minimally affected by the noise. However, most naval radar systems do

Table 1: K-distribution Shape Parameter for Different Scenarios

Scenario	A	B	C	D
Range (km)	10	20	15	20
Azimuth Beamwidth (^{o})	0.5	1.0	0.4	2.0
Range Resolution (m)	100	150	200	500
Grazing Angle (^{o})	0.5	0.2	0.2	0.5
Polarization Parameter	2.09	2.09	1.39	1.39
Swell Angle (^{o})	0	0	0	90
Shape Parameter (ν)	0.69	1.15	3.20	161

not have the antenna gain or the power of SPANDAR. A typical naval radar would attain a value of CNR around 20-25 dB [1]. For less spiky clutter the effects of this seem relatively small as at this CNR range the performance is close to that of the asymptotic infinite CNR case. However, for spiky clutter this CNR may not be enough. In those cases, a large number of averages may be needed, especially taking long correlation times of the gamma distributed component of the K-distribution into account.

Due to the low resolution nature of the RFC systems, very spiky clutter is less likely to be encountered in RFC applications. However a pure Rayleigh approach may be too optimistic in many cases. As shown in [7], even at low resolutions, the low grazing angle situations result in mildly spiky K-distributions. Typical values of ν that will be encountered are given in Table 1 using the empirical model developed for X-band [6]

$$\log(\nu) = \frac{2}{3}\log(\phi^o) + \frac{5}{8}\log(A_c) - k_{pol} - \frac{1}{3}\cos(2\theta_{sw}), \tag{6}$$

where ϕ^o, A_c, k_{pol}, and θ_{sw} are the grazing angle in degrees, radar resolved area, polarization parameter ($k_{pol} = 1.39$ for VV and $k_{pol} = 2.09$ for HH), and angle with respect to the swell direction. As can be seen from the table, the four scenarios show the spectrum of possible shape parameters from the spiky clutter of scenario A to almost Rayleigh clutter of scenario D.

3 Conclusion

In this paper, the effects of clutter-to-noise ratio (CNR), type and parameters of clutter statistics on the clutter-based atmospheric refractivity estimation algorithms were investigated.

4 Acknowledgment

This work was supported by the Office of Naval Research Code 32, under grant N00014-05-1-0369.

References

[1] L. T. Rogers, C. P. Hattan, and J. K. Stapleton, "Estimating evaporation duct heights from radar sea echo," *Radio Science*, vol. 35 (4), pp. 955–966, 2000, doi:10.1029/1999RS002275.

[2] C. Yardim, P. Gerstoft, and W. S. Hodgkiss, "Estimation of radio refractivity from radar clutter using Bayesian Monte Carlo analysis," *IEEE Trans. Antennas Propagat.*, vol. 54(4), pp. 1318–1327, 2006.

[3] ——, "Statistical maritime radar duct estimation using a hybrid genetic algorithm – Markov chain Monte Carlo method," *Radio Science*, vol. 42, 2007, doi:10.1029/2006RS003561.

[4] ——, "Tracking refractivity from clutter using Kalman and particle filters," *IEEE Trans. Antennas Propagat.*, to appear, 2008.

[5] E. Jakeman and P. N. Pusey, "A model for non-Rayleigh sea clutter," *IEEE Trans. Antennas Propagat.*, vol. 24, pp. 806–814, 1976.

[6] K. D. Ward, R. J. A. Tough, and S. Watts, *Sea Clutter: Scattering, the K Distribution and Radar Performance*. London, United Kingdom: The Institution of Engineering and Technology, 2006.

[7] H. C. Chan, "Radar sea-clutter at low grazing angles," *IEE Proceedings*, vol. 137 (F–2), pp. 102–112, 1990.