GLOBAL OPTIMIZATION IN MATCHED FIELD INVERSION

Peter Gerstoft[1] and Zoi-Heleni Michalopoulou[2]
[1]Marine Physical Laboratory, Scripps Institution of Oceanography, La Jolla, CA 92093-0701, USA
[2]Department of Mathematical Sciences and Center for Applied Mathematics and Statistics, New Jersey Institute of Technology, Newark, NJ 07102, USA

In recent years several approaches have been proposed for global optimization in matched-field inversion. Understanding the mechanics of these optimization methods is important in order to construct fast inversion schemes suitable for underwater propagation problems. It is the purpose of this paper to investigate the comparative performances of two global optimization techniques, genetic algorithms and simulated annealing, in underwater inversion problems. Our interest lies in accurate and simultaneously computationally efficient inversion for both environmental and geometrical parameters. The methods are evaluated in terms of their ability to obtain the global minimum, the speed of approaching that, and the variation in the inversion results.

INTRODUCTION

Matched-field processing describes a number of inverse problems in underwater acoustics. The term reflects the mechanics of the inversion process: unknown parameters are estimated by matching the true, received acoustic field to theoretically calculated acoustic fields (also called replicas) computed for likely values of the parameters. The match is quantified with an objective function, which expresses the similarity or “closeness” between real and replica fields. The optimum value of the objective function expresses the best match between real and replica fields that is achieved within the selected search range for the unknown parameters and, therefore, indicates those values of the parameters that appear to be the best candidates for having generated the observed acoustic field.

The global extremum of the objective function can be found by an exhaustive search over the range of all possible values of the unknown parameters. Grid-based implementations are ideal but feasible only if the search space has low dimension. Computations become prohibitively time consuming as the dimension of the search space increases, especially when resolution needs to be fine. In such cases, it is more efficient to discard grid-search methods and attempt to find the optimum with the help of an optimization technique. Conventional optimization methods applied to function maximization problems start with an initial set of values for the unknown parameters and "climb" toward the nearest peak of the function. The danger there is that often the search process gets trapped in the neighborhood of a local extremum and cannot converge to the global one, the ultimate goal of the search. Global optimization methods have the potential to overcome this difficulty. These methods have stochastic components that allow them to avoid "local traps" and are therefore more reliable when applied to a multimodal objective function.

In ocean acoustics, global optimization is mainly implemented with simulated annealing (SA) [1, 2, 3] and genetic algorithms (GA) [4, 5]. These optimization methods have been quite successful in several real data inversions for source localization and estimation of environmental parameters (GA was the most popular inversion method at the Benchmarking Workshop for Geoacoustic Inversion in Shallow Water [6]). It is the purpose of this paper to investigate the comparative performances of these two techniques in the inversion for both environmental and source location parameters from ocean acoustic measurements.

For good inversion results it is important to select a suitable objective function. In this work, we implement a simple, incoherent Bartlett estimator (attractive for both its simplicity and robust performance).

1. SIMULATED ANNEALING

In SA a random starting model $\mathbf{m}$ with energy $E(\mathbf{m})$ is selected. A neighbor $\mathbf{m}' \in \mathcal{N}(\mathbf{m})$ is then chosen, having energy $E(\mathbf{m}')$. Based on ΔE, the energy difference between the initial model and its neighbor, $\Delta E = E(\mathbf{m}') - E(\mathbf{m})$, the new state (model) is accepted or rejected. For $\Delta E < 0$ the new state is always accepted, whereas for $\Delta E >= 0$ the new state is accepted with probability $P = \exp(-\Delta E/T)$, where T is a temperature parameter. After acceptance or rejection of the new state, the temperature is lowered.

Three factors determine the performance of simulated annealing algorithms: cooling rate, which determines how temperature is lowered, move class which determines the size of the parameter changes at every iteration, and move direction, which determines the direction of the parameter perturbations.

Geman and Geman [7] showed that for a uniform distribution of the move class, convergence is obtained using a so-called logarithmic cooling rate, $T_k = T_0/ln(k)$, where T_k is the temperature in iteration k and T_0 is a sufficiently high initial temperature. Convergence is achieved with probability 1 after an infinite number of cooling steps [8]. No strict guidelines exist for the selection of a suitable T_0 in practice; experimentation and experience with a particular application usually indicate a reasonable value.

By selecting a multidimensional Cauchy probability distribution instead of a uniform one defining parameter moves, it can be shown [9] that a faster and more practical linear cooling schedule, $T_k = T_0/k$, can lead SA to convergence.

The logarithmic and linear cooling rates have been considered too slow for many applications and faster cooling schedules have been sought. Different cooling schemes are easy to derive, but for many of them convergence is not guaranteed (even on a theoretical level). Using faster cooling schemes that do not necessarily lead to convergence, SA becomes "simulated quenching", a suboptimal (in terms of convergence), accelerated form of the annealing process.

There have also been suggestions for fast cooling schedules that do not degrade the convergence properties of SA. It was shown in [10] that that the global minimum of an objective function can be statistically obtained using the cooling schedule $T_i^k = T_{0i} \exp\left(-c_i k^{1/M}\right)$, where for each parameter an initial temperature T_{0i} and a constant c_i must be specified. Thus, each parameter has its own temperature, and, just as for the classic Metropolis algorithm, this temperature is used to determine acceptance or rejection of a new state. Selecting values for as many temperatures and constants c_i as there are parameters is a complicated process. The term 'very fast simulated annealing' (VFSA) is used for optimization with the above cooling schedule.

In order to obtain a successful and very fast algorithm, it is important to give full attention to the move class—the breadth of the neighborhood surrounding the current model from which the future model will be selected. The move class is just as important as a proper cooling schedule. The optimization process might never explore important regions of the objective function if a narrow move class is selected or the process might oscillate between distant points of the object function not exploring information in between when the breadth of the class is excessive.

The direction of the new parameter vector after a perturbation is a third important issue. The new vector-model could be along the axis—term used to indicate generation of a new vector by perturbing a single parameter, or in an arbitrary direction, indicating perturbations of all parameters at the same time. Moving along the axis, the way perturbations are performed in the original Metropolis algorithm, is simple but not very efficient. In fact, it is suspected that a main issue in optimization performance comparison is not the specific algorithm, but the specifics of the move process (whether moves are done along the axis or in an arbitrary direction). Often, the main features are off the axes; exploration of these features is inefficient when parameters are perturbed along the axis.

2. GENETIC ALGORITHMS

Genetic algorithms start with the selection of a population of q member models. Models consist of bit strings for each uncertain/unknown parameters. (Parameters are, thus, discretized in GA, in contrast to SA methods, which usually work with continuous variables.) The "fitness" of each member is the value of the objective function for the particular model. Based on the fitness of the members, "parents" are selected and through a randomization a set of "children" is produced. These children replace the least fit of the original population and the process iterates to develop an overall fitter population.

The formation of child models is performed through the application of operators to the parents. The most important operators are those of crossover and mutation. Crossover uses a part of the string corresponding to a parameter from one parent and supplements it with a part of the string for the same parameter from the other parent. The operation is applied individually to every parameter string (multipoint crossover), resulting in all-direction parameter perturbations.

Mutation follows crossover and changes bit values in parameter strings in a random fashion. Bit changes occur with a low probability (usually 0.05). The small changes imposed on the new generation through the occasional bit changes assist the optimization process to escape from local extrema.

A more detailed description of genetic algorithms and their application to parameter estimation is given in [4].

3. THE GENERAL MISMATCH EXAMPLE

The general mismatch case described in [11] was selected for testing the optimization schemes discussed in this paper. The general mismatch data reflect a realistic data collection case involving several uncertain parameters. At the same time, however, these data are obtained from a controlled simulation experiment, allowing us to evaluate the results of the different optimization methods through a comparison of their estimates to the correct parameters that generated the “received” acoustic field, which are exactly known.

Figure 1 shows energy vs. iteration for a nine parameter inversion of the genlmis data. All methods (GA, SA, VFSA) converged to estimates close to the correct solution, but their efficiency and the degree of correlation achieved between data and model varied. Figure 1 highlights the benefit of perturbations in all directions over a single parameter perturbation per iteration. Concentrating on the top left figure, GA achieved a much lower energy state (higher correlation) than both SA and VFSA for on axis perturbations. The performances of SA and VFSA are similar, with that of VFSA being slightly superior. The energies for SA and VFSA drop faster when perturbations are performed in all directions and better matches for the unknown parameters are found more speedily (top right figure). It is interesting to note that SA with perturbations in all directions does best in the average sense for this particular case. However, looking at the bottom figures that illustrate the minimum energy achieved over 100 realizations, GA appear to have the best performance, acquiring the lowest energy of the three algorithms for the same number of iterations.

Global optimization methods are useful tools when the unknown parameter space has a high dimension. Exhaustive search is a better approach for a two dimensional parameter space. Nevertheless, for this study, we selected only two parameters from the general mismatch case to be unknown: bottom depth and source range. The depth varied between 100 and 130 m and range varied between 4 and 7 km. The two parameters, range and bottom depth, are highly correlated and result in objective functions with many local extrema. Limiting the search space dimension to two was done for demonstration purposes and resulted in the ambiguity surfaces of Figure 2, which illustrate the evolution of the estimation process vs. iteration number for GA, VFSA, and SA. The first set of four plots (left) outlines the progress in a GA application. Symbol “+” indicates the individuals in a generation, whereas symbol “o” indicates that an individual was also present in the previous generation. In each generation the least fit half of the population is replaced. As demonstrated in the figures, for the first generation the individuals are randomly spread

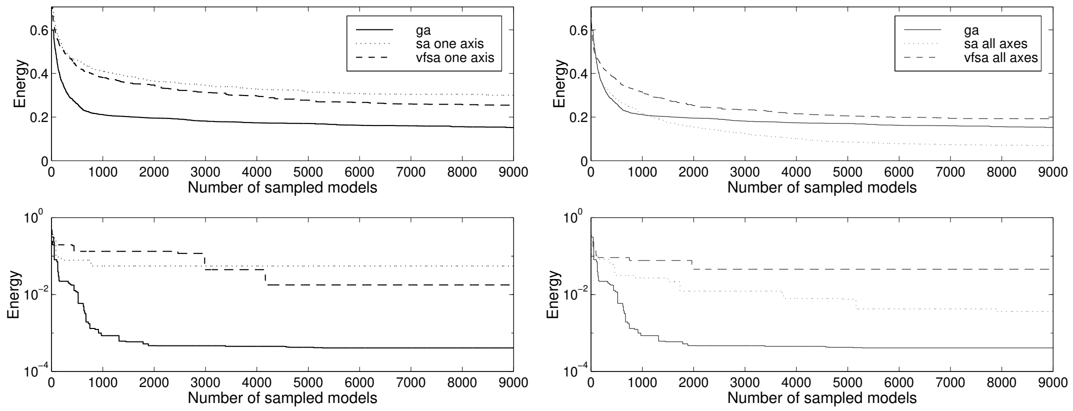

Figure 1: The performance of GA (solid line), FSA (dotted line), and VFSA (dashed line) for the genlmis case. Left: Only one parameter is perturbed in each iteration for SA and VFSA. Right: All parameters are perturbed in each iteration for SA and VFSA. The top figures show the mean energy averaged over 100 realizations. The bottom figures show the minimum energy of the 100 realizations.

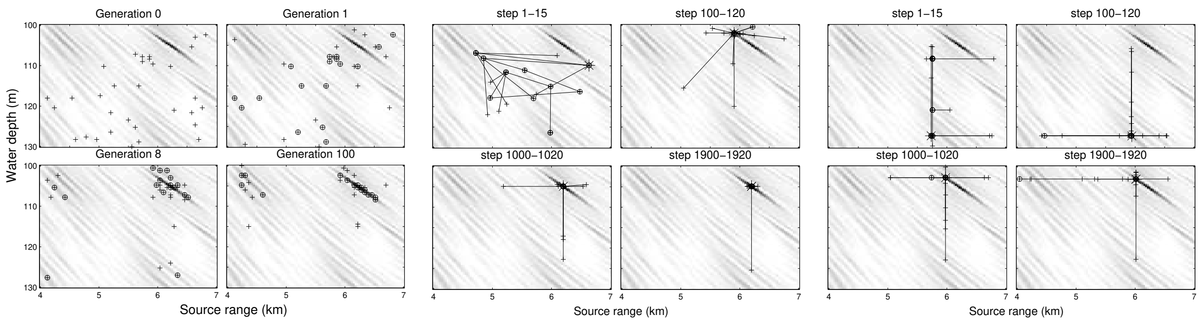

Figure 2: Estimates resulting from GA (left), VFSA (center), and SA (right) iterations.

out, but, as the optimization evolves the individuals focus on the main features. The second set of figures (center) illustrates the VFSA process with moves in any direction. The breadth of the moves changes as the iteration progresses. Symbol "+" indicates an attempted move, whereas symbol "o" indicates an accepted move. It can be seen that the method allows wide parameter moves in the start but it is later restricted to smaller moves and eventually focuses on the main features. The last set of four figures (right) shows the SA evolution. The size of the parameter perturbations remains the same with time and moves are on-axis. Symbols "+" and "o" are the same as in the VFSA figures. Symbol "*" indicates the best so far obtained point. The on-axis perturbations allow only horizontal or vertical moves. This restriction provides a qualitative explanation for the degraded performance resulting from on-axis perturbations (it takes longer to arrive at the main features than through a direct path.)

4. CONCLUSIONS

Genetic algorithms and simulated annealing are global optimization methods widely applied to underwater inversion problems. Both methods have been shown to lead to very good inversion results. For best performance, however, it is important to select carefully parameters that affect the optimization process, such as direction of parameter perturbations, perturbation steps, and cooling schedule. Lack of tuning of the optimization parameters can cause delays in the convergence and erroneous estimates. Especially important is the selection of the perturbation direction. It can be shown that, for most problems, perturbations in random directions in the parameter space can facilitate the search, making it faster and more accurate.

5. ACKNOWLEDGEMENTS

This work was supported by the Office of Naval Research.

REFERENCES

[1] M.D. Collins, W.A. Kuperman, and H. Schmidt,, "Nonlinear inversion for ocean-bottom properties," J. Acoust. Soc. Am. **92**, 2770–2783 (1992).

[2] S.E. Dosso, M.L. Yeremy, J.M. Ozard and N.R. Chapman, "Estimation of ocean bottom properties by matched-field inversion of acoustic field data," IEEE J. Oceanic Eng. **18**, 232–239 (1993).

[3] Z.H. Michalopoulou, M.B. Porter, *Focalization in the Gulf of Mexico*, Proceedings of ICASSP 1996, vol. 6, 3086–3089, Atlanta, May 1996.

[4] P. Gerstoft, "Inversion of seismoacoustic data using genetic algorithms and *a posteriori* probability distributions," J. Acoust. Soc. Am. **95**, 770–781 (1994).

[5] M.I. Taroudakis and M.G. Markaki, "On the use of matched-field processing and hybrid algorithms for vertical slice tomography," J. Acoust. Soc. Am. **102**, 885–895 (1997).

[6] A. Tolstoy, N.R. Chapman and G. Brooke, "Workshop97: Benchmarking for geoacoustic inversion in shallow water," J. Comp. Acoust., to appear, July (1998).

[7] S. Geman and D. Geman, "Stochastic relaxation, Gibbs distributions and the bayesian restoration of images," IEEE Trans. Pattern Anal. Machine Intell. Vol PAMI-6, p721–741 (1984).

[8] M.K. Sen and P.L. Stoffa, *Global optimization in geophysical inversion*, (Elsevier, Amsterdam, 1995).

[9] H. Szu and R. Hartley, "Fast simulated annealing," Phys. Lett. **122**, (1987).

[10] L. Ingber, "Very fast simulated reannealing," Math. Comput. Modelling, **12**, 967–973 (1989).

[11] M. Porter and A. Tolstoy, "The matched field processing benchmark problems" J. Comp. Acoust., **2**, 161–185 (1994).