Refractivity from Clutter (RFC) Estimation Using a Hybrid Genetic Algorithm - Markov Chain Monte Carlo Method

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Abstract

A hybrid Genetic Algorithm - Markov Chain Monte Carlo Sampler (GA-MCMC) is introduced for estimation of low altitude atmospheric radio refractivity. This is done by inverting for the environmental parameters using the returned radar clutter data. A classical Bayesian framework is used so that the solution can be described in terms of a posterior probability distribution (PPD). An electromagnetic split-step fast Fourier transform (FFT) parabolic equation is used as the forward propagation model. The problem is solved with five different optimizers/samplers including the exhaustive search, genetic algorithms, Metropolis-Hastings and Gibbs samplers, some of which were used in previous literature, as well as the new GA-MCMC hybrid based on the Nearest Neighborhood Algorithm (NN). The results show that the new hybrid method improves the speed of a conventional MCMC sampler by a factor of 10 or more while conserving the accuracy in estimating the probability distributions of the inverted parameters.

Introduction

In many maritime regions of the world, such as the Mediterranean, Persian Gulf, and California, atmospheric ducts are common occurrences and they can heavily affect the electromagnetic propagation. Hence, radar systems operating in these areas need to know the effects of the environment on their system performance. This requires the knowledge of the atmospheric radio refractivity, which is usually represented by the modified refractivity (M-profile) in the radar community.

The main purpose of the Refractivity from Clutter (RFC) method is finding the M-profile using only radar clutter return. This clutter return can easily be obtained during the radar operation, without requiring any additional measurement or hardware [1-3]. Moreover, if the inversion algorithm is fast, systems with near-real time updates on M-profiles are feasible. Here, a simple range independent tri-linear M-profile model is used (Fig.1) for validation purposes.

Selection of a Bayesian framework enables us to define these unknown model parameters as random variables so that the inversion results will be in terms of the means, variances and marginal, as well as the n-dimensional joint posterior probability distributions (PPD). This gives the user not only the ability of obtaining the maximum a posteriori (MAP) solution, but also the prospect of performing an uncertainty analysis on the inversion results. These probabilistic properties can be calculated by taking multi-dimensional integrals of the joint PPD. However, these calculations require an accurate estimate of the overall probability distribution. Therefore, although much slower, unbiased samplers like Markov Chain Monte Carlo (MCMC) will give more accurate results than point estimators such as GA.

Moreover, these multi-dimensional integrals can be taken using the Monte Carlo Integration, which can be easily implemented if sampler is MCMC.

Bayesian Implementation and Results

Assuming an IID, zero-mean Gaussian-distributed error term with a variance of ν, the likelihood can be written as [4]

$$\mathcal{L}(\mathbf{m}) \quad = \quad (2\pi\nu)^{-N_{\mathrm{R}}/2} \ \exp\left[-\frac{\phi(\mathbf{m})}{2\nu}\right], \tag{1}$$

where $\mathbf{m}$ is a $1 \times n$ vector containing values of the model parameters, N_{R} is the size of the measured data vector $\mathbf{d}$ (one measurement for each range for this case), and $\phi(\mathbf{m})$ is the objective function given by

$$\phi(\mathbf{m}) = |\mathbf{d} - f(\mathbf{m})|^2 = \sum_{i=1}^{N_{\mathrm{R}}} |d_i - f(m_i)|^2, \tag{2}$$

where $f(\mathbf{m})$ is the replica clutter obtained by the split-step FFT parabolic equation forward model. With a non-informative prior, the Bayes' formulation states that PPD is proportional to the likelihood function. Hence, obtaining the multi-dimensional PPD requires knowing the shape of $\mathcal{L}(\mathbf{m})$ [5].

The new GA-MCMC hybrid method combines the speed of GA with the accuracy of MCMC. It first runs a classical GA, minimizing $\phi(\mathbf{m})$, during which not only the fittest individuals but all the consecutive populations and their likelihood values are stored. When GA converges, an "equivalent PPD" is constructed using Voronoi cells and the NN method [6]. The n-dimensional parameter space is divided into tiny n-dimensional hypercubes, called voronoi cells. There is only one GA sample in each cell and there does not exit any other GA sample, which is closer to any point inside this hypercube. A constant likelihood value, which belongs to the only GA sample inside the cell, is assigned to any point in the cell. Therefore, in this equivalent PPD, the likelihood of any point anywhere in the entire search space is known and there is no need for any further forward model runs. A very fast MCMC, without any forward modelling, is used to sample this equivalent PPD. In most cases, including the RFC problem, the fast MCMC lasts but a fraction of the GA section due to the lack of forward model runs. The accuracy of the results depends mostly on the quality of the equivalent PPD, which means that, GA should gather at least a few samples from all over the n-dimensional search space to allow the NN algorithm to construct a good enough n-dimensional mesh, hence a good enough equivalent PPD.

Five different methods were compared for a synthetic case with the M-profile given in Fig.1. The results are given in terms of marginal PPD's of the model parameters (Fig.2) for exhaustive search, Metropolis-Hastings, Gibbs sampler, GA, and the GA-MCMC hybrid respectively. The vertical dashed lines represent the MAP values of the model parameters. It is evident that all of the methods have good MAP estimates. The number of forward model runs required for each method is given in Table 1. With a small enough grid size, exhaustive search will provide us with the true distribution although it is very slow. A good fit in the marginal

Table 1: A Comparison of Different Methods Used

Algorithm	Speed	MAP Solution	Joint PPD
Exhaustive Search	V. Slow (390k samples)	Good	True Distribution
Metropolis-Hastings Algorithm (MCMC)	Slow (80k samples)	Good but Less Accurate	V. Accurate
Gibbs Sampler (MCMC)	Slow (80k samples)	Good but Less Accurate	V. Accurate
Genetic Algorithm (GA)	V. Fast (5k samples)	Good	Not Accurate
Hybrid GA-MCMC Method	Fast (5k GA samples + a V. Fast MCMC Section)	Good	Accurate

distributions is achieved using two different MCMC samplers (Metropolis-Hastings and Gibbs samplers) and it is not surprising that they give very similar results in terms of both accuracy and speed. Although faster than exhaustive search, they still are far away from being suitable for a near-real time system. Both marginal and n-dimensional PPD's obtained by GA samples are not accurate although it gives a very good MAP estimate in a much shorter time than the others. This is expected due to the point estimator nature of GA designed to reach the MAP solution without delay. The hybrid improves the initial GA result considerably with only a slight increase in the overall runtime. The returned clutter power that is used in the inversion as data vector $\mathbf{d}$ and the resulting one-way propagation loss in terms of a coverage diagram obtained by propagating the MAP solution with a split-step FFT parabolic equation propagation model can be seen in Fig.3.

Conclusion

Five sampling methods has been compared for Bayesian analysis of the refractivity from clutter problem. The best method in terms of speed and accuracy was a new hybrid genetic algorithm - Markov Chain Monte Carlo sampler.

References

[1] Gerstoft, P., L.T. Rogers, J. Krolik, and W.S. Hodgkiss, "Inversion for refractivity parameters from radar sea clutter," *Radio Science, 38*, paper 18, pp 1-22, 2003.
[2] Gerstoft, P., W.S. Hodgkiss, L.T. Rogers, and M. Jablecki, "Probability distribution of low altitude propagation loss from radar sea-clutter data," *Radio Science, 39*, 2004.
[3] Rogers, L.T., M. Jablecki, and P. Gerstoft, "Posterior distribution of a statistic of propagation loss estimated from radar sea data," submitted to *Radio Science*, 2004.
[4] Gerstoft, P., and C.F. Mecklenbräuker, "Ocean acoustic inversion with estimation of *a posteriori* probability distributions," *J. Acoust. Soc. Am.* , *104*(2), 808–819, 1998.
[5] Tarantola A., Inverse Problem Theory and Methods for Model Parameter Estimation, Society for Industrial and Applied Mathematic, ISBN: 0898715725, 2004.
[6] Sambridge M., "Geophysical inversion with a neighborhood algorithm - I. Searching a parameter space," *Geophys. J. Int., 138*, pp 479-494, 1999.

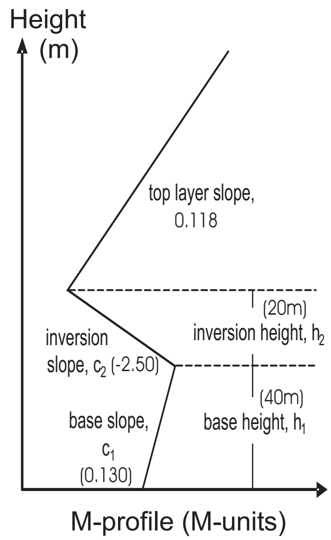

Fig.1 Four parameter tri-linear M-profile used in this work.

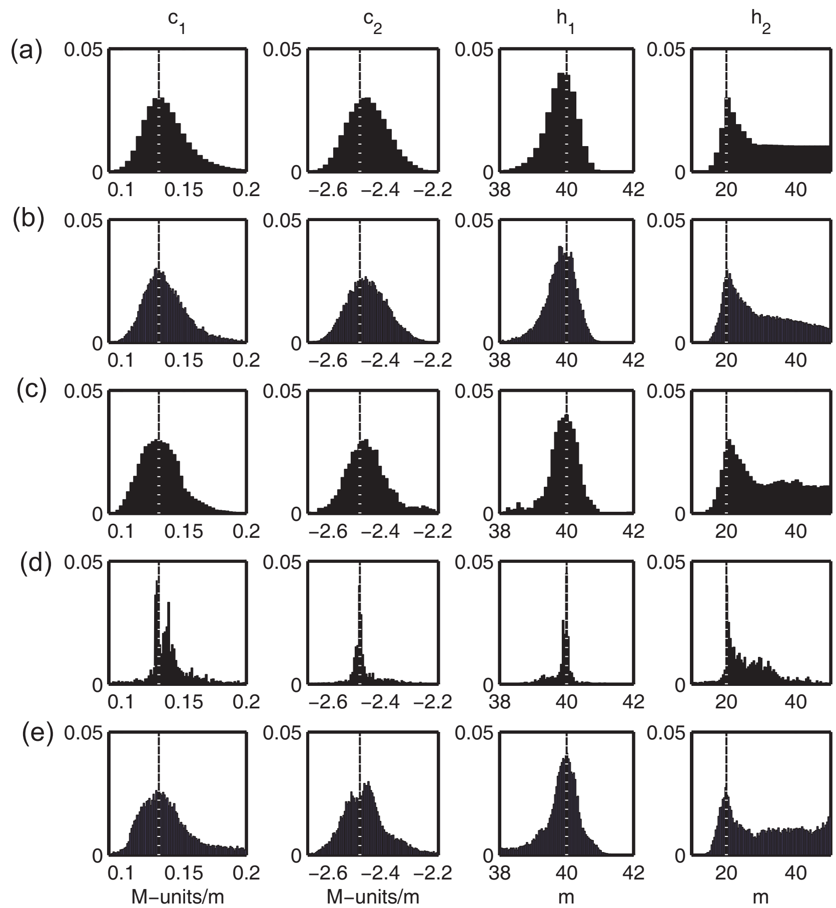

Fig.2 Marginal posterior probability distributions for the synthetic test case. Vertical lines show the MAP values of each method. (a) exhaustive search, (b) Metropolis-Hastings algorithm, (b) Gibbs algorithm, (d) genetic algorithm, and (e) GA-MCMC hybrid.

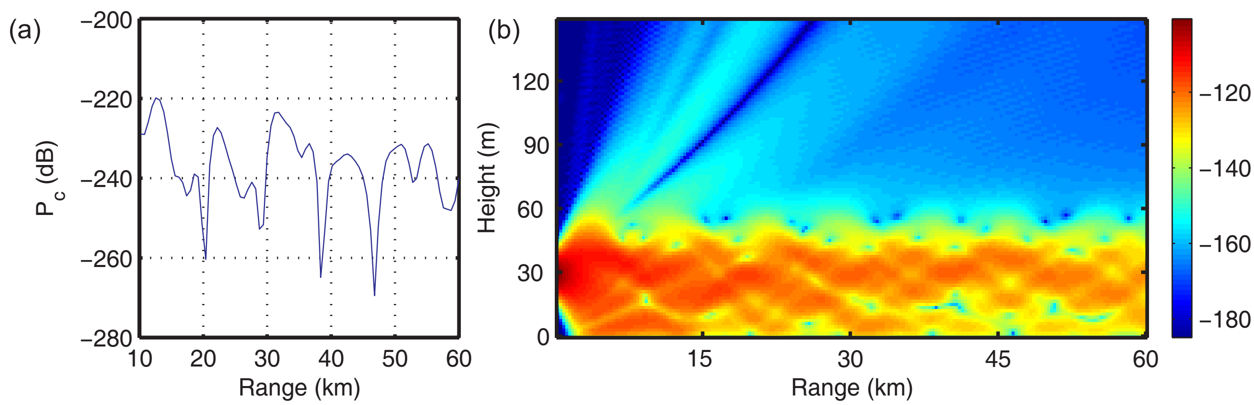

Fig.3 (a) The returned radar clutter used in inversion and (b) coverage diagram obtained by propagating the MAP solution with a split-step FFT parabolic equation propagation model