Tracking Atmospheric Ducts Using Radar Clutter:
I. Evaporation Duct Tracking Using Kalman Filters

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Introduction

This paper addresses the problem of tracking evaporation ducts in marine and coastal environments. The method tracks the evolution of the range and height-dependent index of refraction using the radar sea clutter. A split-step fast Fourier transform (FFT) based parabolic equation (PE) approximation to the wave equation is used to compute the clutter return in complex environments with varying index of refraction. Tracking is obtained using an extended Kalman filter (EKF).

Non-standard electromagnetic propagation due to formation of lower atmospheric sea ducts is a common occurrence in maritime radar applications. Under these conditions, some fundamental system parameters of a sea-borne radar can significantly deviate from their original values specified assuming standard-air (0.118 M-units/m) conditions. These include the variation in the maximum operational range, creation of regions where the radar is practically blind (radar holes), and increased sea surface clutter. Therefore, it is important to predict the real-time 3-D environment the radar is operating in so that the radar operator will at least know the true system limitations and in some cases even compensate for it.

The environment is characterized by the modified refractivity profile (M-profile) in the radar community and there are many techniques that measure or predict the lower atmospheric index of refraction. However, it is also possible to predict the duct properties using the radar itself. When launched at a low elevation angle, the electromagnetic signal will be trapped within the duct which can be taken as a range-dependent leaky waveguide bounded from below by the sea surface. This will result in multiple reflections and strong interaction with the surface which in turn will result in an increase in the sea clutter, forming clutter rings. This normally unwanted portion of the received signal can then be used to infer the environment that would give such a clutter structure. These techniques can be classified as refractivity-from-clutter (RFC) techniques [1–3]. Look into [3] for more details.

This paper is a natural extension to these previous RFC methods which compute the 2-D range and height-dependent M-profile for a given azimuth direction. Instead of trying to invert the environmental parameters for a given path and time, the emphasis is on tracking both the temporal and spatial evolutions of duct parameters.

Kalman Filter Formulation and Results for a Case Study

Two equations are necessary to fully characterize the dynamic system; one that describes the evolution of the lower atmosphere and another that governs the propagation of the electromagnetic signal in this environment.

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{v}_{k-1} \qquad \mathrm{E}\{\mathbf{v}_k\mathbf{v}_i^{\mathrm{T}}\} = \mathbf{Q}_k\delta_{ki} \qquad \mathrm{E}\{\mathbf{w}_k\mathbf{w}_i^{\mathrm{T}}\} = \mathbf{R}_k\delta_{ki} \tag{1}$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{w}_k \qquad \mathrm{E}\{\mathbf{v}_k\mathbf{w}_i^{\mathrm{T}}\} = \mathbf{0} \quad \forall\, i, k. \tag{2}$$

where $\mathbf{F}$ is a linear function of the state vector $\mathbf{x}_k$, $\mathbf{h}(\cdot)$ is a nonlinear function of the measurement vector $\mathbf{y}_k$, $\mathbf{v}_k$ and $\mathbf{w}_k$ are the process and the measurement noise vectors, respectively. The state vector $\mathbf{x}_k$ is composed of the n_x parameters such as the duct height that describe the complex environment at the step index k. The process noise $\mathbf{v}_k$ is taken as a zero-mean additive Gaussian pdf.

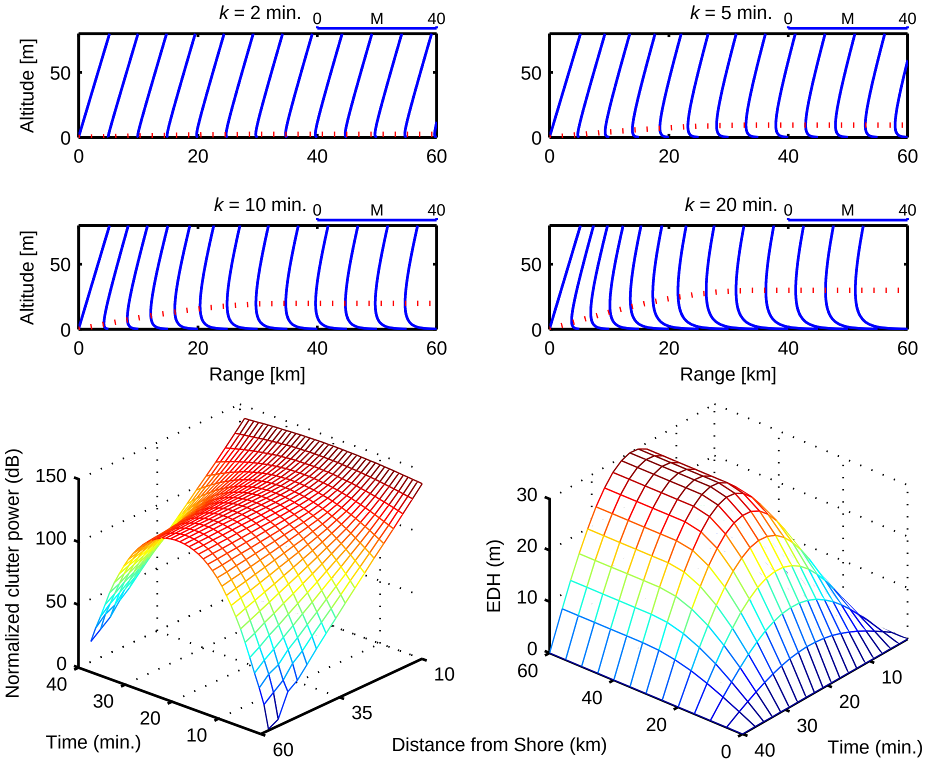

Figure 1: 2-D range-dependent M-profiles at 2, 5, 10, and 20 min. as a function of altitude and distance from the coast. Dotted lines show the evaporation duct height (EDH). Bottom plots include normalized clutter power and EDH as a function of distance and time.

Equation (1) is the state equation for the stochastic environmental model. $\mathbf{F}$ is the linear state transition matrix which will be taken as the identity matrix. The main assumption is that the environment is changing slowly compared to the step index. Although the M-profile is not expected to vary considerably in 1-2 min intervals, sudden fluctuations can occur and the filters will require higher $\mathbf{Q}_k$ to perform adequately in these environments. Evaporative ducts are represented using only the evaporative duct height, which is true when the air and sea temperatures are almost identical with a neutrally buoyant boundary layer. Range-dependence is achieved by defining the duct height at various ranges and interpolating in between using

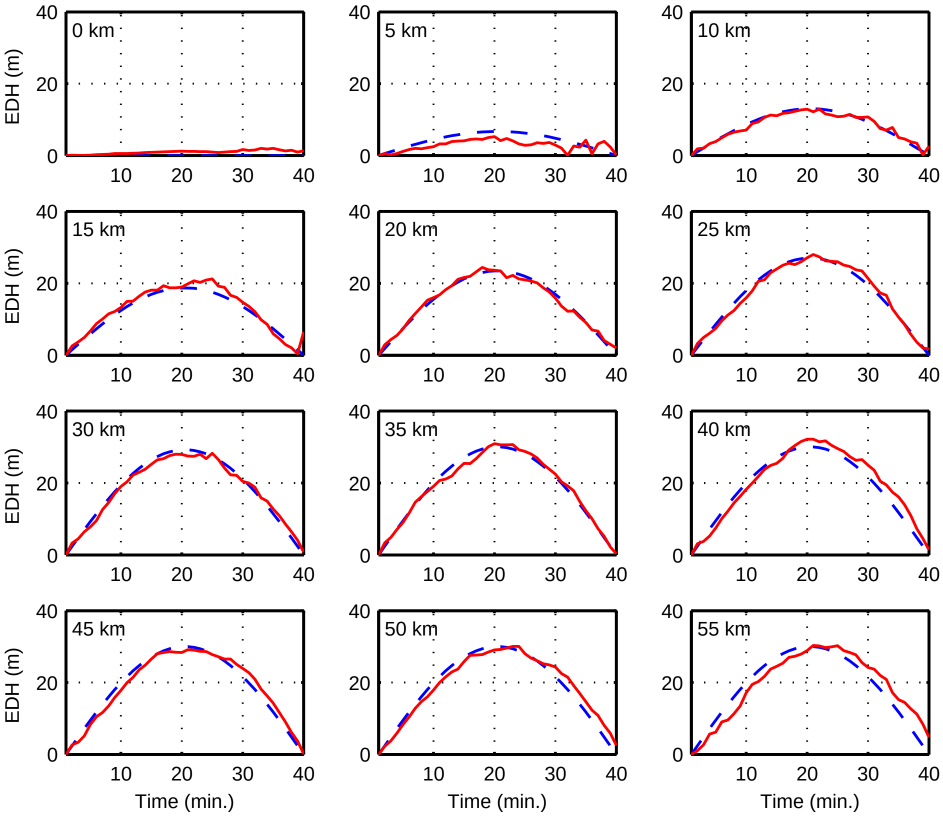

Figure 2: Temporal tracking of a range-dependent coastal evaporation duct. Evolution of evaporation duct heights (EDH) at various distances (given at the upper left of each plot) from the shore. True trajectory (dashed line) and EKF tracking result (solid line).

cubic fit. Hence, the number of state parameters $n_x = n_r$. Afterwards, the 2-D evaporative duct is constructed using the log-linear evaporative duct formula:

$$\mathbf{x}_k = [h_d(r_1)\ h_d(r_2)\ \ \dots\ \ h_d(r_{n_r})]^{\mathrm{T}} \tag{3}$$

$$M(z,r) = M_0 + c_o \left(z - \tilde{h}_d \log \frac{z + z_o}{z_o} \right), \tag{4}$$

where $\tilde{h}_d$ represents the cubic fitted duct height at range r, the constant c_o and the roughness factor z_o are taken as 0.13 and 1.5×10^{-4}, respectively.

Equation (2) is the measurement equation and it relates the environment given by $\mathbf{x}_k$ to the radar clutter power $\mathbf{y}_k$ in dB through a nonlinear $\mathbf{h}(\cdot)$. First the field is propagated in range using the following recursive split-step FFT PE formula [4]

$$\begin{aligned} u_k(z, r + \triangle r) = \exp\left[i k_o \triangle r M(\mathbf{x}_k) 10^{-6} \right] \times \\ \mathfrak{F}^{-1} \left\{ \exp\left[i \triangle r \left(\sqrt{k_o^2 - k_z^2} - k_o \right) \right] \mathfrak{F}\left\{ u_k(z,r) \right\} \right\} \end{aligned} \tag{5}$$

where $u_k(z,r)$ is the vertical electromagnetic field at range r at step index k, k_o and k_z are the wavenumber and its vertical component, $\triangle r$ is the range increment in PE,

$\mathfrak{F}$ and $\mathfrak{F}^{-1}$ are the Fourier and inverse Fourier transforms and $M(\mathbf{x}_k)$ is the 2-D M-profile $M(z,r)$ computed above. Following [1], the clutter power $\mathbf{P_c}$ for low grazing angles can be calculated using $\mathbf{P_c} = c\mathbf{L}^{-2}(\mathbf{x}_k)r\sigma^o$, where c accounts for the constant terms in the radar equation, $\mathbf{L}(\mathbf{x}_k)$ is the one way propagation loss obtained from the electromagnetic field $u_k(z,r)$ calculated at the effective scattering height and σ^o is the normalized sea surface RCS. Eqn.(2) is obtained in dB with $\mathbf{y}_k = 10\log(\mathbf{P_c})$, $\mathbf{w}_k = 10\log(\sigma^o)$, and $\mathbf{h}(\mathbf{x}_k) = -20\log\mathbf{L}(\mathbf{x}_k) + 10\log(cr)$ where the measurement noise $\mathbf{w}_k$ is additive Gaussian since σ^o is the sea surface RCS taken as a log-normal pdf.

The results of a 40 minute temporal tracking is given in Fig. 1–2. It uses 12 parameters (evaporation duct height (EDH) at every 5 km) and simulates the formation and dissipation of a strong evaporation duct in the open sea with a decreasing EDH towards the coast. EKF successfully tracks all 12 parameters. Since the clutter is mildly nonlinear for most of the evaporation duct problems as shown in the figure, EKF works well for the evaporative case and higher order filters such as unscented Kalman and particle filters are not needed. This method, however, fails for highly non-linear surface-based ducts [5, 6], which will necessitate particle filters to work.

It can easily be observed from Fig. 2 that the most poorly determined tracks are EDH near the coast and at 55 km range (the final profile). The coastal regions are poorly determined because these regions have shallow EDH usually below the radar height itself. Since the radar is above the duct these regions have almost no effect on the clutter and hence are poorly determined. Similarly the last profile only can affect the clutter return at ranges beyond itself so it is also poorly estimated.

References

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