Distributed State and Field Estimation Using a Particle Filter

Florian Xaver*, Christoph Mecklenbräuker*, Peter Gerstoft†, Gerald Matz*
*Institute of Communications & Radio-Frequency Engineering, Vienna University of Technology,
{fxaver,cfm,gmatz}@nt.tuwien.ac.at
†Scripps Institution of Oceanography, University of California, San Diego, gerstoft@ucsd.edu

***Abstract*—This paper addresses distributed tracking of the probability distribution of state parameters of a space-time-variant process described by a linear partial differential equation using a particle filter (PF). We focus on localizing an acoustic source in a given region. The underlying wave equation leads to a high-dimensional estimation problem. We propose a technique for reducing the computational effort of the PF which allows to distribute the signal processing algorithm over the nodes of a wireless sensor network (WSN) without a fusion center.**

I. Background

Approaches to source tracking [1], [2] consider multiple measurements to infer source states, e.g. location and velocity. Numerous Bayesian estimators for state tracking have been proposed. All these methods share a common state-space formulation based on the *state equation*

$$\boldsymbol{x}_{k+1} = g_k(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{w}_k) \ , \tag{1a}$$

which describes the evolution of the d-dimensional state vector $\boldsymbol{x}_k \in \mathbb{R}^d$ over discrete time $k \in \mathbb{N}$. The state vector evolves in response to the known input vector $\boldsymbol{u}_k \in \mathbb{R}^{N_u}$ and the process noise vector $\boldsymbol{w}_k \in \mathbb{R}^d$ with known probability density function (PDF). At every time k a data vector $\boldsymbol{y}_k \in \mathbb{R}^{N_\mathcal{R}}$ is measured by $N_\mathcal{R}$ sensors according to the *measurement equation*,

$$\boldsymbol{y}_k = h_k(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{v}_k) \ . \tag{1b}$$

Here, $\boldsymbol{v}_k \in \mathbb{R}^{N_\mathcal{R}}$ is the measurement or observation noise with known PDF. To estimate the states, both mappings $h_k(\cdot,\cdot,\cdot)$ and $g_k(\cdot,\cdot,\cdot)$, the *a-priori* knowledge of the states in form of the PDF at time $k = 0$, and the noise PDFs have to be known. For linear systems with Gaussian noise, the optimal estimator is the Kalman filter [3]. On the other hand, for non-linear systems with non-Gaussian noise no analytic solution is known. An alternative is given by Monte Carlo methods, e.g. the particle filter (PF, [1], [4], [5], [6], [7]). Since the proposed model is non-linear and the priors are non-Gaussian the PF is used throughout this paper.

An enhanced approach [8] for locating a source involves the underlying physical process. Our paper focuses on an acoustic problem where the sound originating from a source is used to locate it by distributed sensors in an inhomogeneous environment. One possible improvement of the estimation incorporates into the state-space model the structure of the underlying acoustic wave field which is described by the scalar wave equation [9] depending on the location vector $\vec{r}$. There are several numerical methods, e.g. finite difference method (FDM) [10] or finite/spectral element method (FEM, SEM) which discretize partial differential equations (PDE) [11]. The results are distributed systems over space. This implies a simultaneous field and state estimation (SFSE) whereby the state vector implicitly includes source locations and values of the wave field at computational nodes.

In the decentralized version of the model, tracking of an evolving probability distribution is carried out by $N_\mathcal{C}$ adapted individual particle filters acting on sub-vectors of state $\boldsymbol{x}_k$ called clusters. Neighborhood states near the border of each cluster have to be broadcast as well as the weights of the PFs to solve the distributed estimation problem. This implies that our proposed PF neither requires additional assumptions nor includes further approximations.

For that purpose, (1) is decomposed into $N_\mathcal{C}$ sub-state spaces of following form:

$$\boldsymbol{x}_{k+1}^{(m)} = g_k^{(m)}(\boldsymbol{x}_k^{(m)}, [\boldsymbol{x}_k]_{\mathcal{N}^{(m)}}, \boldsymbol{u}_k^{(m)}, \boldsymbol{w}_k^{(m)}) \ ,$$
$$\boldsymbol{y}_k^{(m)} = h_k^{(m)}(\boldsymbol{x}_k^{(m)}, \boldsymbol{u}_k^{(m)}, \boldsymbol{v}_k^{(m)}) \ , m \in \{1, \cdots N_\mathcal{C}\} \ .$$

$(.)^{(m)}$ denotes cluster m. The obvious difference to (1a) is the new input vector $[\boldsymbol{x}_k]_{\mathcal{N}^{(m)}}$ of adjacent states from the neighborhood of cluster m.

This paper provides the following specific contributions to the localization problem of an acoustic source:

- tracking of an evolving probability distribution;
- an estimation problem using the wave equation, transparent boundary conditions and a transient source (the wave equation has twice the states of the diffusion equation [8] and is highly dynamic);
- a source and augmented state-space model;
- simulation results for the decentralized case in comparison to the centralized

II. Spatio-Temporal Field

A. Scalar Wave Equation

In the following the acoustic problem is considered. This is done by a hyperbolic PDE (wave equation), which is defined on an e-D Euclidean space $\mathbb{R}^e$ ([12], [10], [13])

$$\frac{1}{c^2}\partial_t^2 p(\vec{r},t) - \nabla^2 p(\vec{r},t) = s(t)\delta(\vec{r}-\vec{r}_\text{s}),\ \vec{r}\in\Omega\ , \tag{2a}$$

using two different kinds of boundary conditions

$$\frac{1}{c}\partial_t p(\vec{r},t) - \vec{\nabla}p(\vec{r},t)\cdot\vec{n} = 0\ , \quad \vec{r}\in\partial\Omega_1, t\in(0,T)\ , \tag{2b}$$

$$p(\vec{r},t) = 0\ , \quad \vec{r}\in\partial\Omega_2, t\in(0,T)\ , \tag{2c}$$

and initial conditions

$$p(\vec{r},t) = 0\ , \quad \vec{r}\in\Omega, t=0\ , \tag{2d}$$

$$\partial_t p(\vec{r},t) = 0\ , \quad \vec{r}\in\Omega\ , t=0\ . \tag{2e}$$

Here p denotes the pressure, ∂_t is the partial derivative with respect to time, $\vec{\nabla}$ the gradient, ∇^2 the Laplace operator, c is the velocity, $\vec{r}$ the position and $s(t)$ the force or source at location $\vec{r}_\text{s}$ denoted by the Dirac distribution $\delta(\cdot)$. $\Omega\in\mathbb{R}^e$ is the rectangular domain of interest with its boundary $\partial\Omega = \partial\Omega_1\cup\partial\Omega_2$ where $\partial\Omega_1\cap\partial\Omega_2 = 0$. $\partial\Omega_1$ represents a transparent boundary with its outward normal vector $\vec{n}$. $\partial\Omega_2$ is rigid. All physical quantities are normalized.

B. Discretizing the Wave Equation

The following example uses a rectangular area $\Omega = [0,\Delta x N_\text{I}]\times[0,\Delta y N_\text{J}]$. For the PDEs (2a) – (2e) the finite difference method (FDM, [10], [14]) approximates the Laplace operator by

$$\begin{aligned}&\nabla^2 p(\vec{r},t) \approx\\ &\quad\frac{1}{\Delta x^2}\big(p((i-1)\Delta x, j\Delta y, t) + p((i+1)\Delta x, j\Delta y, t) -\\ &\qquad\qquad 2p(i\Delta x, j\Delta y, t)\big) +\\ &\quad\frac{1}{\Delta y^2}\big(p(i\Delta x, (j-1)\Delta y, t) + p(i\Delta x, (j+1)\Delta y, t) -\\ &\qquad\qquad 2p(i\Delta x, j\Delta y, t)\big)\ .\end{aligned}$$

The difference between two sample points is Δx, Δy in space and Δt in time, respectively. Here and in the following, k represents the discrete time variable while $i=[1,N_\text{I}]$ and $j=[1,N_\text{J}]$ are discrete coordinates. The approximation of the time derivative is given by

$$\begin{aligned}&\partial_t p(i\Delta x, j\Delta y, t) \approx\\ &\quad\frac{1}{\Delta t}\big(p(i\Delta x, j\Delta y, (k+1)\Delta t) - p(i\Delta x, j\Delta y, k\Delta t)\big)\ .\end{aligned}$$

The FDM gives with the auxiliary variable $q = \partial_t p$ an equation system. Using the abbreviation $p[i,j,k] := p(i\Delta x, j\Delta y, k\Delta t)$ with the following vector notation

$$\boldsymbol{p}_k := (p[1,1,k],\cdots,p[N_I,N_J,k])^\text{T},\ \text{and}$$
$$\boldsymbol{s}_k := (s[1,1,k],\cdots,s[N_I,N_J,k])^\text{T},$$

the equation system becomes

$$\begin{bmatrix}\boldsymbol{q}_{k+1}\\ \boldsymbol{p}_{k+1}\end{bmatrix} = \underbrace{\left[\begin{array}{c|c}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12}\\ \hline \Delta t\boldsymbol{I} & \boldsymbol{I}\end{array}\right]}_{\boldsymbol{\Phi}_\text{FDM}}\begin{bmatrix}\boldsymbol{q}_k\\ \boldsymbol{p}_k\end{bmatrix} + \Delta t c^2\boldsymbol{I}\begin{bmatrix}\boldsymbol{s}_k\\ \boldsymbol{0}\end{bmatrix}. \tag{3}$$

It has the form of a state-space model. The two sparse sub-matrices $\boldsymbol{\Phi}_{11}$ and $\boldsymbol{\Phi}_{12}$ descend from (2b) and from (2a). Let $\mathcal{L} = \{(i,j)|i=1,\cdots,N_\text{I}; j=1,\cdots,N_\text{J}\}$ be the lattice obtained by the FDM with cardinality $N_\mathcal{L} = N_\text{I}N_\text{J}$. The set of $N_\mathcal{R}$ sensor nodes $\mathcal{R}$ are those tuples (i,j) where sensors are placed, and thus $\mathcal{R}\subset\mathcal{L}$.

III. State-Space Model

A. Source Model

Let $\mathcal{F}$ be the subset of $\mathcal{L}$ where sources exist. The waveform $s[\cdot]$ of the source is assumed to be known whereas the time duration $n[i,j,k]\in\mathbb{N}_0$ between occurrence and time index k is unknown. $n[i,j,k]$ depends on the location in the lattice and the time k. Since the time increases steadily the state transition equation is given by

$$n[i,j,k+1] = n[i,j,k] + 1\ , \quad (i,j)\in\mathcal{F}\ .$$

To exploit the implicit information i and j and to allow more than one source, the following deterministic model is used:

$$\underbrace{\begin{bmatrix}n[1,1,k+1]\\ \vdots\\ n[N_I,N_J,k+1]\end{bmatrix}}_{\boldsymbol{n}_{k+1}} = \underbrace{\begin{bmatrix}n[1,1,k]\\ \vdots\\ n[N_I,N_J,k]\end{bmatrix}}_{\boldsymbol{n}_k} + \underbrace{\begin{bmatrix}\delta_\mathcal{F}(1,1)\\ \vdots\\ \delta_\mathcal{F}(N_I,N_J)\end{bmatrix}}_{\boldsymbol{\delta}_\mathcal{F}}, \tag{4}$$

where $\delta_\mathcal{F}(i,j) = \begin{cases}1, & (i,j)\in\mathcal{F},\\ 0, & (i,j)\notin\mathcal{F},\end{cases}$ is the indicator function which specifies the location of sources (cf. with (2a)).

B. Augmented State-Space Model

In the next step the state vector of Equation (3) is augmented by the source model. With a proper measurement equation, which models the $N_\mathcal{R}$ measurements $\boldsymbol{y}_k\in\mathbb{R}^{N_\mathcal{R}}$ at time k, the following stochastic state-space formulation is given:

$$\begin{aligned}\begin{bmatrix}\boldsymbol{q}_{k+1}\\ \boldsymbol{p}_{k+1}\\ \boldsymbol{n}_{k+1}\end{bmatrix} &= \underbrace{\left[\begin{array}{cc|c}\multicolumn{2}{c|}{\boldsymbol{\Phi}_\text{FDM}} & \begin{matrix}\boldsymbol{0}\\ \boldsymbol{0}\end{matrix}\\ \hline \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{array}\right]}_{\boldsymbol{\Phi}}\underbrace{\begin{bmatrix}\boldsymbol{q}_k\\ \boldsymbol{p}_k\\ \boldsymbol{n}_k\end{bmatrix}}_{\boldsymbol{x}_k} +\\ &\quad\underbrace{\begin{bmatrix}\Delta t c^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}}_{\boldsymbol{\Gamma}}\begin{bmatrix}\boldsymbol{s}[\boldsymbol{n}_k]\\ \boldsymbol{0}\\ \boldsymbol{\delta}_\mathcal{F}\end{bmatrix} + \boldsymbol{w}_k\ ,\end{aligned} \tag{5a}$$

$$\boldsymbol{y}_k = \boldsymbol{C}\begin{bmatrix}\boldsymbol{q}_k\\ \boldsymbol{p}_k\\ \boldsymbol{n}_k\end{bmatrix} + \boldsymbol{v}_k\ , \tag{5b}$$

with an appropriate mapping $\boldsymbol{C}:\mathbb{R}^{3N_\mathcal{L}}\to\mathbb{R}^{N_\mathcal{R}}$. It maps the pressure states $\boldsymbol{p}_k$ at the lattice locations to the measurement vector $\boldsymbol{y}_k$. $\boldsymbol{w}_k$ includes noise from the source(s) whereas $\boldsymbol{v}_k$ summarizes background and measurement noise. The source mapping $\boldsymbol{s}[\boldsymbol{0}] = \boldsymbol{0}$.

IV. Estimation of Wave Field and Source Location

The key idea to reduce the computational effort of an estimation procedure is the use of a proper source model with a special generation of initial particles $\mathcal{X}_0$ described in the following subsection. We assume a standard sampling importance resampling (SIR) particle filter (PF), see [7] for a tutorial or [1]. A PF is a Monte Carlo version of a Bayesian estimator. The states are changing according to a state space model; hence, a PF is suitable for our problem.

A. A-Priori Knowledge

The estimator needs the following priors:

1) the PDFs of transition noise $\boldsymbol{w}_k$ and measurement noise $\boldsymbol{v}_k$ and
2) the initial state PDF.

From the latter, the set of particles $\mathcal{X}_0$ has to be drawn. In the following this is tackled from a different angle.

For simplicity only one source ($|\mathcal{F}| = 1$) is assumed. Using (5a), the set of $N_{\mathcal{X}}$ initial particles is proposed to be

$$\mathcal{X}_0 = \left\{ \boldsymbol{\Phi}^{k'} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{\delta}_{\tilde{\mathcal{L}}} \end{bmatrix} + \sum_{l=0}^{k'-1} \boldsymbol{\Phi}^{k'-1-l} \boldsymbol{\Gamma} \boldsymbol{u}_k \right| \left. \forall (i,j,k') \in \tilde{\mathcal{L}} \right\}. \quad (6)$$

Here, we use the "estimation lattice"

$$\tilde{\mathcal{L}} = \left\{ (i,j,k')_l \sim p_{I,J,K}(i,j,k') \;\middle|\; \{(i,j)\} \subset \mathcal{L} \right\}_{l=1}^{N_{\tilde{\mathcal{L}}}}.$$

with $p_{I,J,K}(i,j,k') = p_{I,J}(i,j) p_K(k')$ denoting the probability mass function (PMF) of source occurrence at location (i,j), k' time instants before estimation.

B. Estimates

According to [1], the approximated posterior PDF of sub-vector $\boldsymbol{n}_k$ is

$$\tilde{f}_{\boldsymbol{N}}(\boldsymbol{n}_k | \boldsymbol{y}_{1:k}) = \sum_{i=1}^{N_{\mathcal{X}}} w_k^{(i)} \delta \left(\boldsymbol{n}_k - \boldsymbol{n}_k^{(i)} \right) = p_{\boldsymbol{N}_k}(\boldsymbol{n}_k | \boldsymbol{y}_{1:k}).$$

Ignoring the start time of the source it follows that

$$\begin{aligned} P\{\text{sources at } \mathcal{F}_k | \boldsymbol{y}_{1:k}\} &= p_{\mathcal{F}_k}(\mathcal{F}_k | \boldsymbol{y}_{1:k}) \\ &= \sum_{i=1}^{N_{\mathcal{X}}} w_k^{(i)} \delta \left(\boldsymbol{\delta}_{\mathcal{F}_k} - Q\left(\boldsymbol{n}_k^{(i)}\right)\right) \end{aligned}$$

where $Q(\cdot) : \mathbb{N}_0^{N_{\mathcal{L}}} \to \{0,1\}^{N_{\mathcal{L}}}$ sets all entries of its argument to 1 which are larger than 0. Assuming only one source and a SIR PF with $w_k^{(i)} = 1/N_{\mathcal{X}}$, the probability

$$\begin{aligned} P\{\text{sources at } (i,j) \text{ at } k | \boldsymbol{y}_{1:k}\} &= p_{I,J}(i,j|\boldsymbol{y}_{1:k}) \\ &= \sum_{i=1}^{N_{\mathcal{X}}} \frac{1}{N_{\mathcal{X}}} \delta \left(\boldsymbol{\delta}_{(i,j),k} - Q\left(\boldsymbol{n}_k^{(i)}\right)\right). \end{aligned} \quad (7)$$

Note that here $\boldsymbol{\delta}_{(i,j),k}$ is a unit vector at time k where the location of the 1 entry corresponds to the location (i,j) of the source.

V. Decentralized computation

A. Clusters

Since every row in the system of equations (5) corresponds to a lattice point, [15] suggests to decompose the state-space model (5) into $N_{\mathcal{M}}$ disjoint subsystems $\mathcal{L}^{(m)} \subset \mathcal{L}$. $N_{\mathcal{R}^{(m)}}$ is the number of sensors in cluster m grouped in set $\mathcal{R}^{(m)}$.

B. Decomposed Augmented State-Space Model

With the set of states $\mathcal{L}^{(m)}$ of cluster m, the decomposed state vector is given by

$$\begin{aligned} \boldsymbol{x}_k^{(m)} &= [\cdots, q_{i,j,k}, \cdots, p_{i,j,k}, \cdots, n_{i,j,k}, \cdots]^{\mathrm{T}} |_{(i,j) \in \mathcal{L}^{(m)}} \\ &=: [\boldsymbol{x}_k]_{\mathcal{L}^{(m)}} . \end{aligned}$$

The decomposed sub-matrices of each cluster follow by

$$\begin{aligned} \boldsymbol{\Phi}^{(m)} &= [\boldsymbol{\Phi}]_{\mathcal{L}^{(m)},\mathcal{L}^{(m)}} , \\ \boldsymbol{\Gamma}^{(m)} &= [\boldsymbol{\Gamma}]_{\mathcal{L}^{(m)},\mathcal{L}^{(m)}} , \\ \boldsymbol{G}^{(m)} &= [\boldsymbol{G}]_{\mathcal{L}^{(m)},\mathcal{L}^{(m)}} , \end{aligned}$$

and $\boldsymbol{C}^{(m)}$ which maps sub-states to the decomposed measurement vector,

$$\boldsymbol{y}_k^{(m)} = [y_{1,k}, \cdots, y_{N_{\mathcal{R}^{(m)}},k}]^{\mathrm{T}}.$$

The sub-state space model is given by

$$\boldsymbol{x}_{k+1}^{(m)} = \boldsymbol{\Phi}^{(m)} \boldsymbol{x}_k^{(m)} + \underbrace{\boldsymbol{\Phi}_{\mathcal{L}^{(m)},\mathcal{N}^{(m)}} [\boldsymbol{x}]_{\mathcal{N}^{(m)}}}_{\text{communication term}} + \boldsymbol{\Gamma}^{(m)} \boldsymbol{u}_k^{(m)} + \boldsymbol{w}_k^{(m)}, \quad (9)$$

and

$$\boldsymbol{y}_k^{(m)} = \boldsymbol{C}^{(m)} \boldsymbol{x}_k^{(m)} + \boldsymbol{v}_k^{(m)}. \quad (10)$$

where the set $\mathcal{N}^{(m)} \subset \mathcal{L} \backslash \mathcal{L}^{(m)}$ collects the neighbor states of $\mathcal{L}^{(m)}$ which have to be signalled.

VI. Particle Filter

A. Weights

The weight of every particle i and cluster m can be calculated decentralized using individual PFs under the restriction of uncorrelated noise between clusters[1]:

$$w_k^{(i,m)} \propto \exp \left(-\frac{1}{2\sigma^2} \left\| \boldsymbol{y}_{k,\text{obs}}^{(m)} - \boldsymbol{C}^{(m)} \boldsymbol{x}_k^{(m)} \right\|_2^2 \right) .$$

These sub-weights have to be signaled between all clusters to calculate the whole weight

$$w_k^{(i)} = w_k^{(i,1)} \cdots w_k^{(i,N_{\mathcal{C}})}$$

on every cluster. It has to be ensured that the re-sample step causes the same result on every cluster by using deterministic re-sampling.

[1]Here, we assume Gaussian noise.

Table I
VECTORS AND VARIABLES WHICH HAVE TO BE BROADCAST BY ANY CLUSTERS TO THEIR NEIGHBORS. MESSAGES ARE ONLY SENT WHEN A SOURCE CROSSES THE BOUNDARY IN A PARTICLE.

cluster 1	$\longleftrightarrow$	cluster 2
$\boldsymbol{q}_k$	*no transmission*	$\boldsymbol{q}_k$
$\boldsymbol{p}_k$	*states at boundary*	$\boldsymbol{p}_k$
$\boldsymbol{n}_k$	*messages*	$\boldsymbol{n}_k$
sub-weights	*transmission*	sub-weights

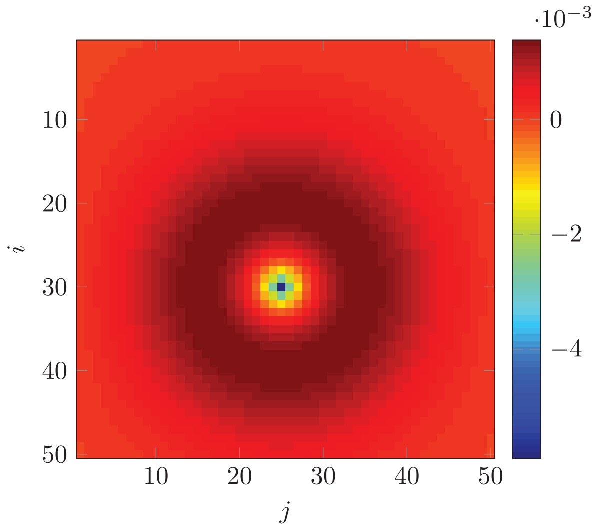

Figure 1. Acoustic field at time $k = 41741$ in the simulations.

B. Communication

An overview of the quantities which have to be transmitted between clusters in case of the distributed PFs are plotted in Table I.

VII. SIMULATIONS

The disposal simulation setup is shown in Figure 2. Settings of the FDM and modified SIR PFs are listed in Table II and Table III, respectively. In the following the plots are generated at time $k = 41741$. The field is shown in Figure 1.

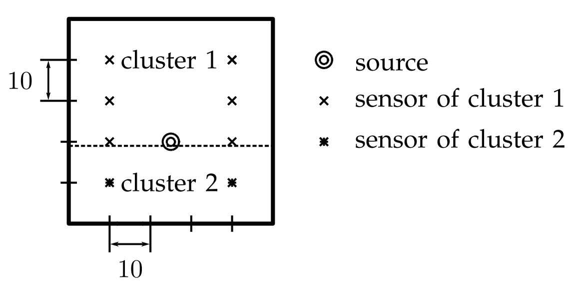

Figure 2. Locations of sensors, source and areas of clusters.

Table II
SETTINGS OF FDM, PDE AND NOISE. THE GAUSSIAN DISTRIBUTION $\mathcal{N}\{\cdot,\cdot\}$ IS DEFINED BY MEAN AND CO-VARIANCE.

Quantity	Value
$\Delta t, \Delta x, \Delta y$	371 ns, 12.24 cm, 12.24 cm
grid	50×50
$s(t)$	Ricker wavelet
c	340 m/s
$f_{\boldsymbol{W}}(\boldsymbol{w})$	$\mathcal{N}\{\boldsymbol{0}, 100\,\mathrm{ps}\boldsymbol{I}\}$
$f_{\boldsymbol{V}}(\boldsymbol{v})$	$\mathcal{N}\{\boldsymbol{0}, 100\,\mathrm{ps}\boldsymbol{I}\}$

Table III
SETTINGS OF SIR PFS. k_{START} IS THE START TIME OF ESTIMATION. $\mathcal{U}\{a,b\}$ REPRESENTS A UNIFORM PDF WITH SUPPORT $[a,b]$.

Quantity	Value
particles	19344
$f_{X,Y,T}(x,y,t)$	$\mathcal{N}\{\boldsymbol{0}, \Delta x^2/4^2, \Delta y^2/4^2, \Delta t^2/4^2\}$
$f_{\boldsymbol{V}}(\boldsymbol{v})$	$\mathcal{N}\{\boldsymbol{0}, 5\,\mathrm{ms}\boldsymbol{I}\}$
$p_N(k')$	$\mathcal{U}\{0, k_{\mathrm{start}}\}$
$p_{I,J}(i,j)$	$\mathcal{U}\{0,50\}\,\mathcal{U}\{0,50\}$

A. Centralized Estimation

In Figure 3 the marginal *a posteriori* PMF (7) obtained with a modified SIR PF is shown.

B. Decentralized Estimation

In Figure 4 both marginal *a posteriori* PMFs from clusters 1 and 2 are shown. Note the difference to Figure 3 which stems from different realizations of the noise. The marginal PMFs tend to PDFs for $\Delta x, \Delta y \to 0$.

The marginal *a posteriori* PMF of the time duration $n[30, 25, 41741]$ between occurrence of the source and estimation time is shown in Figure 5. Note that in the beginning of the estimation the distribution was uniform.

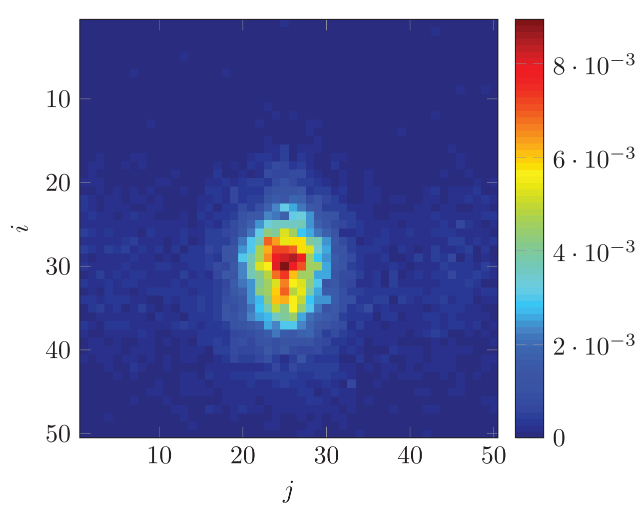

Figure 3. Centralized estimation at time step $k = 41741$. The contour plot shows the source's marginal *a posteriori* PMF depending on the coordinates (i, j). The sensors are arranged as in Figure 2 and the field is plotted in Figure 1.

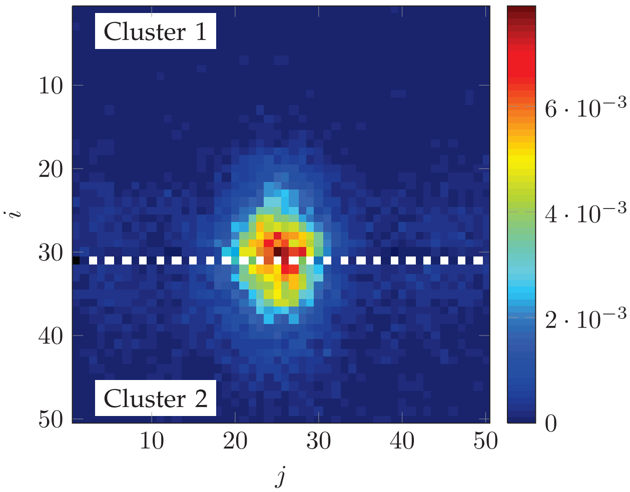

Figure 4. Decentralized estimation at time step $k = 41741$. The contour plot shows the source's marginal *a posteriori* PMF depending on the coordinates (i, j). The sensors are arranged as in Figure 2 and the field is plotted in Figure 1.

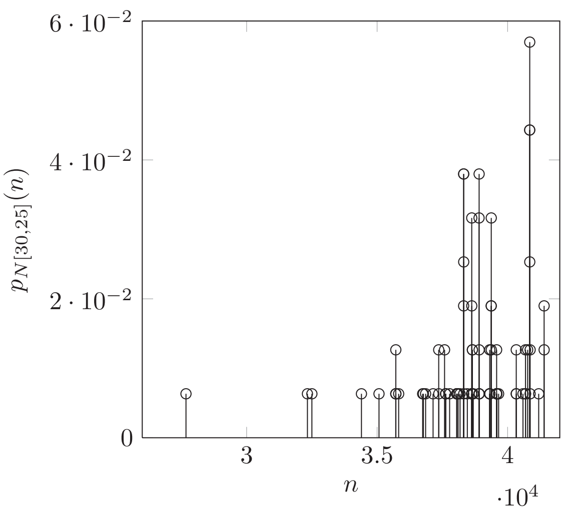

Figure 5. Marginal *a posteriori* PMF of the time duration between occurrence of the source and estimation time 41741 at the position $(30, 25)$ and time $k = 41741$.

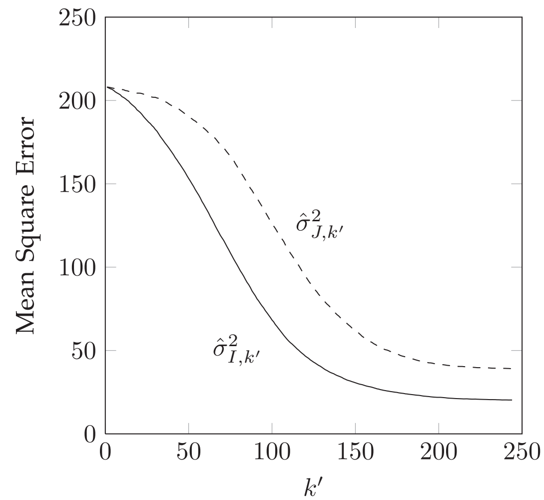

Figure 6. Variances of the two marginal *a posteriori* PMFs depending on i and j over time. With an abuse of notation $k' = k - k_{\text{start}}$ specifies the time duration since the start of estimation at $k_{\text{start}} = 41626$. Sensors along the left and right boundary causes an asymmetric PMF of (i, j), cf. with Figure 4.

Figure 6 shows the variances $\hat{\sigma}^2_{I,k}$ and $\hat{\sigma}^2_{J,k}$ of the marginalized *a posteriori* PMFs over time. The different values of both variances are caused by the array of sensors.

Acknowledgment

This work is funded by Grant ICT08-44 of "Wiener Wissenschafts-, Forschungs- und Technologiefonds" (WWTF).

References

[1] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter: Particle Filters for Tracking Applications*. Artech House, 2004.

[2] O. Hlinka, P. Djuric, and F. Hlawatsch, "Time-space-sequential Distributed Particle Filtering with Low-rate Communications," *invited paper in Proc. 43rd Asilomar Conference on Signals, Systems, and Computers*, 2009.

[3] R. Kalman, "A new approach to linear filtering and prediction problems," *Journal of Basic Engineering*, vol. 82, no. 1, pp. 35–45, 1960.

[4] C. Yardim, Z.-H. Michalopoulou, and P. Gerstoft, "An overview of sequential bayesian filtering in ocean acoustics," *Oceanic Engineering, IEEE Journal of*, 2010.

[5] J. Candy, "Bootstrap particle filtering," *Signal Processing Magazine, IEEE*, vol. 24, no. 4, pp. 73–85, 2007.

[6] Z. Chen, "Bayesian filtering: From Kalman filters to particle filters, and beyond Hamilton," Communications Research Laboratory, McMaster University, Hamilton, Ontario, Canada, Manuscript, 2003.

[7] M. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, Feb 2002.

[8] F. Sawo, "Nonlinear State and Parameter Estimation of Spatially Distributed Systems," Ph.D. dissertation, Universität Karlsruhe, 2009.

[9] DGA, "Akustische Wellen und Felder," Deutsche Gesellschaft für Akustik, Tech. Rep., 2006.

[10] R. Mattheij, S. Rienstra, and J. ten Thije Boonkkamp, *Partial Differential Equations: Modeling, Analysis, Computation*. Society for Industrial Mathematics, 2005.

[11] J. Tromp, D. Komatitsch, and Q. Liu, "Spectral-Element and Adjoint Methods in Seismology," *Communications in Computational Physics*, vol. 3, no. 1, pp. 1–32, 2008.

[12] F. Jensen, W. Kuperman, M. Porter, and H. Schmidt, *Computational Ocean Acoustics*. AIP Series in Modern Acoustics and Signal Processing, 1994.

[13] M. Zhdanov, *Geophysical Inverse Theory and Regularization Problems*. Elsevier Science Ltd, 2002.

[14] A. Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*. Society for Industrial Mathematics, 2005.

[15] F. Sawo, M. Huber, and U. Hanebeck, "Parameter identification and reconstruction for distributed phenomena based on hybrid density filter," in *Information Fusion, 2007 10th International Conference on*, 2007, pp. 1–8.