

Proceedings of Meetings on Acoustics

Volume 19, 2013

<http://acousticalsociety.org/>



**ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013**

Underwater Acoustics

Session 4pUW: Sparse Process Modeling Techniques for Acoustic Signal Processing

4pUW10. High resolution beamforming using L1 minimization

Ravi Menon* and Peter Gerstoft

***Corresponding author's address: Univ. California San Diego, La Jolla, CA 8588226926, rmenon@ucsd.edu**

We consider the problem of beamforming using fewer snapshots than the number of sensors, using sparse recovery of the signal vector. Given an array of sensors in an environment and signals impinging on the array, it is of practical interest to be able to estimate the direction of arrival and power of the signals using as few snapshots as possible. In a sparse recovery framework, the signal vector is modeled as a sparse vector in the bearing domain. By casting the beamforming operation as an ℓ^1 minimization problem (as opposed to the conventional ℓ^2 minimization), the signal vector can be recovered. The angular resolution of this approach is much higher than the Rayleigh limit, which determines the resolution for the conventional beamformer. The results are demonstrated using simulations.

Published by the Acoustical Society of America through the American Institute of Physics

INTRODUCTION

In this presentation, we will address the problem of estimating the direction of arrival (DoA) of signals using a uniform line array (ULA). DoA estimation plays an important role in mobile telecommunications, radar and underwater acoustic applications. Traditional DoA estimation using conventional beamforming suffers from poor resolution when the signals are closely spaced (in angle) and high-resolution approaches such as MVDR, MUSIC and other subspace based approaches require several observations (snapshots) to be able to estimate reliably. Here, we use a sparse representations based approach to DoA estimation, which has both high resolution and only a single snapshot.

CONVENTIONAL BEAMFORMING

Consider an N element ULA with a spacing to wavelength ratio of $1/2$. Given an array observation vector \mathbf{x} , conventional beamforming is performed as

$$B_\theta = |\mathbf{w}_\theta^H \mathbf{x}|^2 \quad (1)$$

where B_θ is the output and $\mathbf{w}_\theta = [1, e^{i\pi \sin(\theta)}, \dots, e^{i\pi(N-1)\sin(\theta)}]$ is the array steering vector.

Consider an observation vector from two closely spaced signals of equal power (10 dB) at $\theta_1 = 20^\circ$ and $\theta_2 = 21^\circ$, recorded on an $N = 20$ element array. The conventional beamformer output is shown in Fig. 1:

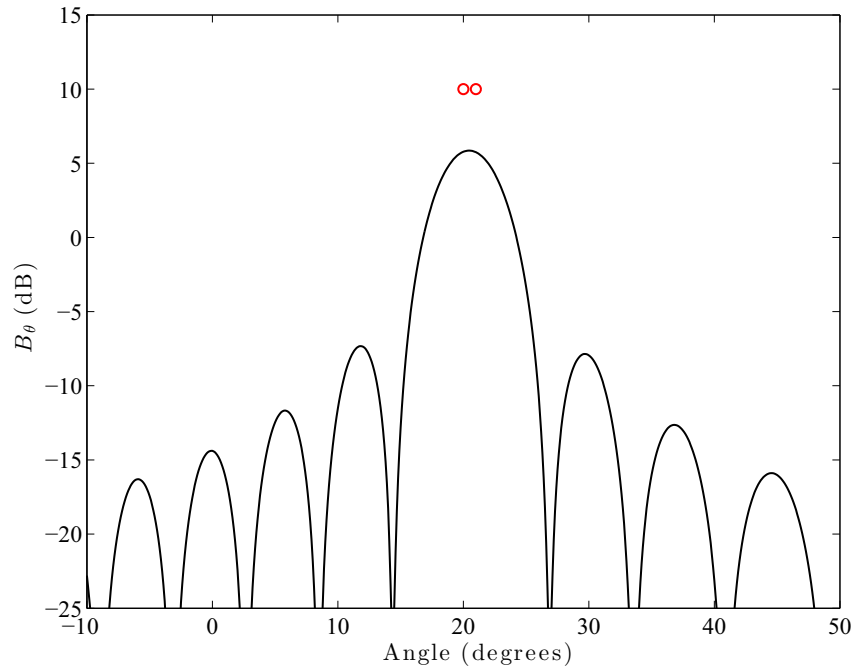


FIGURE 1: Conventional beamformer output for $N = 20$ and two signals at $\theta_1 = 20^\circ$ and $\theta_2 = 21^\circ$ and power 10 dB. Red circles denote the true power and direction.

The beamformer is unable to resolve the two signals, because its resolution is limited by the Rayleigh resolution limit, given by

$$R = \frac{\lambda}{Nd} = \frac{2}{N} \approx 5.73^\circ \quad (2)$$

where λ is the wavelength and d is the spacing between the array elements (here, the ratio d/λ is $1/2$). In other words, the conventional beamformer cannot distinguish between the two signals as their angular separation is lower than the Rayleigh limit.

BEAMFORMING USING ℓ_1 MINIMIZATION

Compressive sensing[1] or ℓ_1 minimization has been used to reconstruct sparse solutions to the beamforming problem[2] if we can assume that the number of signals, k is small compared to the number of array elements (i.e. $k \ll N$). Solving the following optimization problem:

$$\min_{\mathbf{s}} \|\mathbf{s}\|_1 \quad \text{subject to} \quad \mathbf{x} = \mathbf{W}\mathbf{s} \quad (3)$$

where \mathbf{W} is a matrix of possible steering vectors, $\mathbf{W} = [\mathbf{w}_{\theta_1}, \dots, \mathbf{w}_{\theta_M}]$, $M > N$, leads to a sparse solution, because the use of the ℓ_1 norm instead of the ℓ_2 norm (which leads to the conventional beamforming result) promotes sparsity in the solution. The ℓ_1 beamforming output for the same example as before, is shown in Fig. 2 (it is assumed that the true signal direction is contained in the matrix \mathbf{W}):

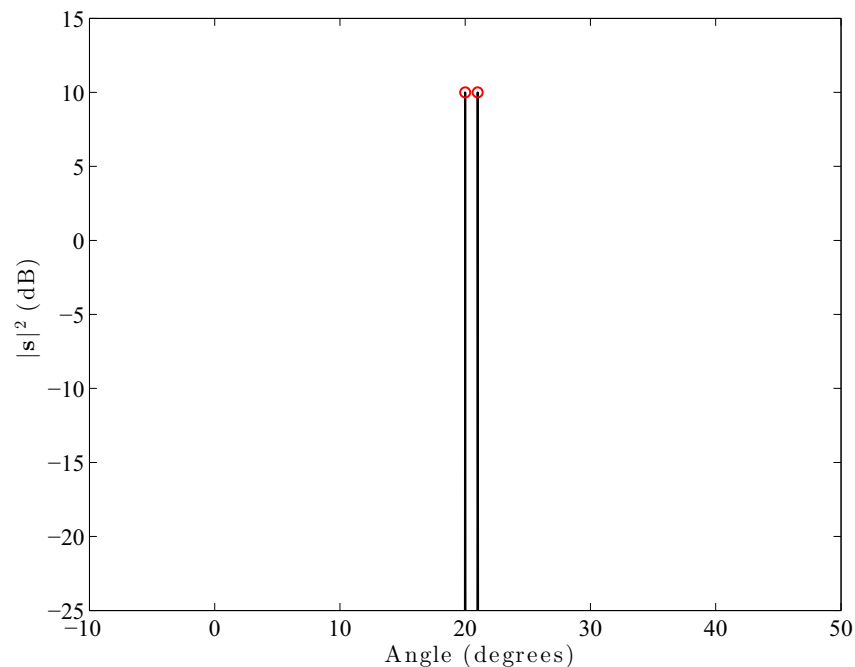


FIGURE 2: Beamforming using ℓ_1 minimization for $N = 20$ and two signals at $\theta_1 = 20^\circ$ and $\theta_2 = 21^\circ$ and power 10 dB. Red circles denote the true power and direction.

As seen from the above figure, the ℓ_1 minimization recovers the DoA of the signals *exactly*. If the matrix \mathbf{W} does not contain the exact direction, the minimization seeks out the column of \mathbf{W} that is closest to the true direction. Quantifying the angular resolution of this approach (in cases when \mathbf{W} contains and does not contain the *exact* direction) as a function of the number of arrays is the focus of current research and will be presented at the meeting at Montréal.

The approach also works with a single observation (in the absence of noise) and can be used in tracking applications. Fig. 3 shows four moving sources of varying power (typical in ocean acoustic applications) and the results from conventional beamforming (left; black lines indicate the true path of the sources) and ℓ_1 minimization (right). As mentioned earlier, the conventional

beamformer loses resolution when the sources get closer to each other, whereas by exploiting sparsity, we can track precisely the locations of the different sources (right).

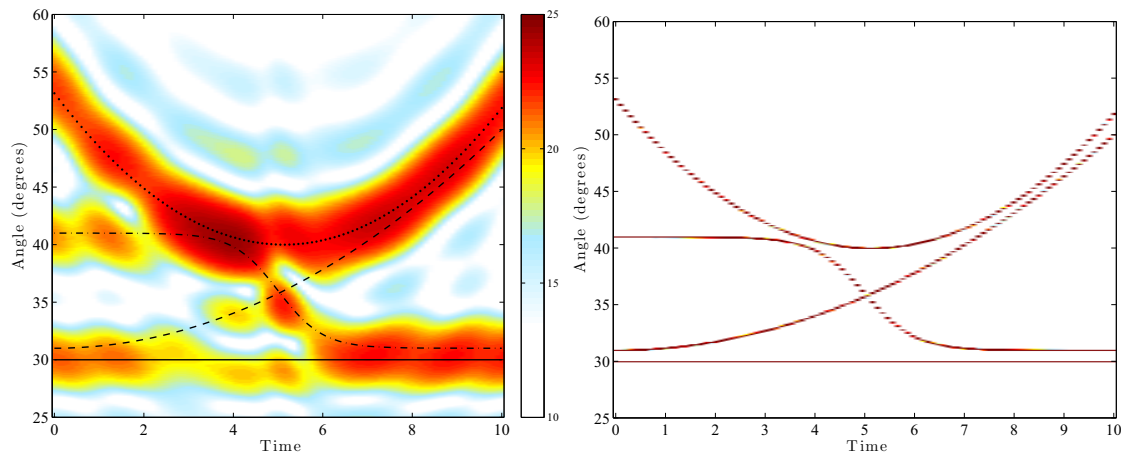


FIGURE 3: Tracking moving sources using conventional beamforming (left) and ℓ_1 minimization beamforming (right). The black lines in the plot on the left indicate the true path of the sources.

REFERENCES

- [1] E. J. Candés, “Compressive sampling”, Proceedings of the International Congress of Mathematicians **3**, 1433–1452.
- [2] G. F. Edelmann and C. F. Gaumond, “Beamforming using compressive sensing”, J. Acoust. Soc. Am. **130**, EL232–EL237 (2011).