ECE295, Data Assimilation and Inverse Problems, Spring 2015

1 April, Intro; Linear discrete Inverse problems (Aster Ch 1 and 2) Slides

8 April, SVD (Aster ch 2 and 3) Slides

15 April, Regularization (ch 4)

22 April, Sparse methods (ch 7.2-7.3), radar

29 April, more on Sparse

6 May, Bayesian methods and Monte Carlo methods (ch 11)

13 May, Introduction to sequential Bayesian methods, Kalman Filter (KF)

20 May, Ensemple Kalman Filer (EnKF)

27 May, EnKF, Particle Filter,

3 June, Markov Chain Monte Carlo

Homework:

Just email the code to me (I dont need anything else).

Call the files LastName_ExXX.

Homework is **due 8am on Wednesday.**

8 April: Hw 1: Download the matlab codes for the book (cd_5.3) from this website

15 April: SVD analysis:

SVD homework. You can also try replacing the matrix in the Shaw problem with the beamforming sensing matrix. The sensing matrix is available here .

22 April

Late April: Beamforming

May: Ice-flow from GPS

Compressive sensing in acoustics and seismology
Peter Gerstoft

Sparse/compressive methods has seen a huge growth in the last decade, as many methods are inherently sparse

$$\boxed{\hat{\mathbf{x}} = \arg\min \| \mathbf{x} \|_1 \quad \text{subject to} \ \| \mathbf{Ax} - \mathbf{b} \|_2 < \varepsilon}$$

Yao, Gerstoft, Shearer, Mecklenbräuker (2011), **Compressive sensing of the Tohoku-Oki Mw 9.0 earthquake**: Frequency-dependent Rupture Modes, GRL

Yardim, Gerstoft, Hodgkiss, Traer (2014), **Compressive geoacoustic inversion**, JASA

Menon, Gerstoft (2013), **High resolution beamforming using L1 minimization**, Proc. Acoustics.

Xenaki, Gerstoft, Mosegaard (2014), **Compressed beamforming**, JASA submitted,

Mecklenbräuker, Gerstoft, Panahi, Viberg (2013), Sequential Bayesian Sparse Source Reconstruction using Array Data, IEEE TSP.

Hu, Gerstoft (2014), Sparse Signal Reconstruction for Direction of Arrival Tracking, IEEE TSP subm.,

Yao, Shearer, Gerstoft (2013), Compressive sensing of frequency-dependent seismic radiation from subduction zone megathrust ruptures, PNAS

Structure of talk

- Introduction (13 slides)
- Beamfoming intro (7 slides)
  - Menon, Gerstoft (2013), **High resolution beamforming using L1 minimization**, Proc. Acoustics.
- Fathometer (10 slides)
  - Yardim, Gerstoft, Hodgkiss, Traer (2014), **Compressive geoacoustic inversion**, JASA
- Earthquake (7 slides)
  - Yao, Gerstoft, Shearer, Mecklenbräuker (2011), **Compressive sensing of the Tohoku-Oki Mw 9.0 earthquake**: Frequency-dependent Rupture Modes, GRL
- Beamfoming (7 slides)
  - Xenaki, Gerstoft, Mosegaard (2014), **Compressed beamforming**, JASA submitted,

Citations since 2006

[Compressed sensing]
DL **Donoho** - Information Theory, IEEE Transactions on, 2006 - ieeexplore.ieee.org
Abstract—Suppose is an unknown vector in(a digital image or signal); we plan to measure
general linear fur
transform coding
Cited by 9395 F

[Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information]
EJ Candès, J Romberg, T Tao - Information Theory, IEEE ..., 2006 - ieeexplore.ieee.org
Abstract—This paper considers the model problem of recon-structing an object from incomplete frequency samples. Consider a discrete-time signal and a randomly chosen set of frequencies. Is it possible to reconstruct from the partial knowledge of its Fourier ...
Cited by 6432 Related articles All 38 versions Import into BibTeX Saved More

[Stable signal recovery from incomplete and inaccurate measurements]
EJ Candes, JK Romberg, T Tao - Communications on pure and ..., 2006 - Wiley Online Library
Abstract Suppose we wish to recover a vector x 0∈ ℝ (eg, a digital signal or image) from incomplete and contaminated observations y= A x 0+ e; A is an × matrix with far fewer rows than columns (≪) and e is an error term. Is it possible to recover x 0 accurately ...
Cited by 2879 Related articles All 27 versions Import into BibTeX Save More

[An introduction to compressive sampling]
EJ Candès, MB Wakin - Signal Processing Magazine, IEEE, 2008 - ieeexplore.ieee.org
This article surveys the theory of compressive sampling, also known as compressed sensing or CS, a novel sensing/sampling paradigm that goes against the common wisdom in data acquisition. CS theory asserts that one can recover certain signals and images from far ...
Cited by 2879 Related articles All 46 versions Import into BibTeX Save More

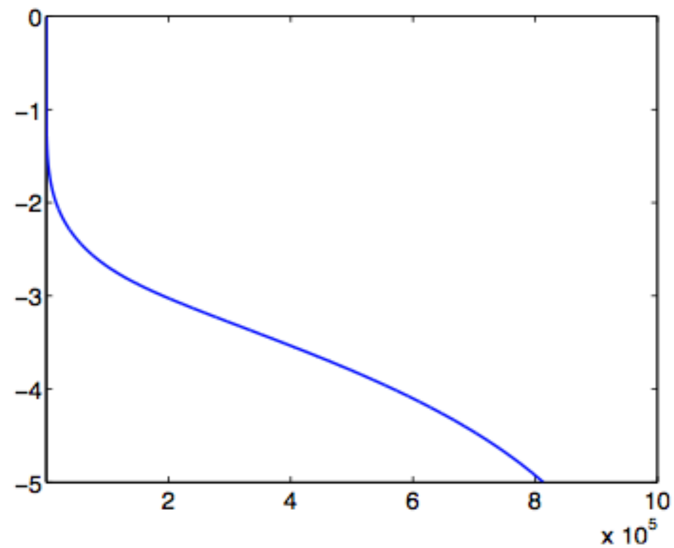
[PDF] **Compressive sensing**
R Baraniuk - IEEE signal processing magazine, 2007 - omni.isr.ist.utl.pt
The Shannon/Nyquist sampling theorem tells us that in order to not lose information when
uniformly sampling
many applications,
Cited by 1794 Re

[Sparse signal reconstruction from limited data using FOCUSS: A re-weighted minimum norm algorithm]
IF Gorodnitsky, BD **Rao** - Signal Processing, IEEE Transactions ..., 1997 - ieeexplore.ieee.org
Abstract—We present a nonparametric algorithm for finding localized energy solutions from limited data. The problem we address is underdetermined, and no prior knowledge of the shape of the region on which the solution is nonzero is assumed. Termed the FOcal ...
Cited by 858 Related articles All 15 versions Import into BibTeX Saved More

Wavelet Approximation

1 megapixel image

25k term approx



$\boldsymbol{B}$-term approx error

Thus images are sparse, but is 8 MP needed when aquirering the picture? => compressive sensing one-pixel camera

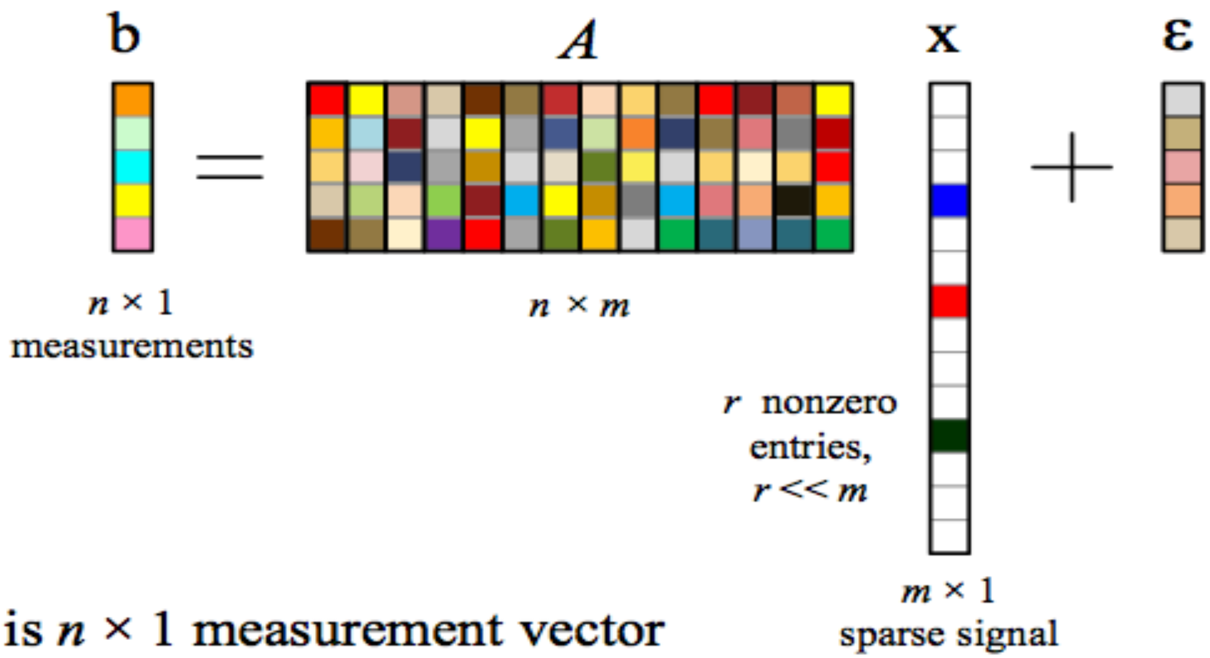
Sparse signals /compressive signals are important

- We don’t need sample at double the Nyquist rate
- Many signals are sparse, but we have solved them under non-sparse assumptions
  - Beamforming
  - Fourier transform
  - Layered structure
- Inverse methods are inherently sparse: We seek the simplest way to describe the data (Occam’s razor…)

But all this requires **new developments**

- Mathematical theory
- New algorithms (interior point solvers, convex optimization)
- Signal processing
- New applications/demonstrations

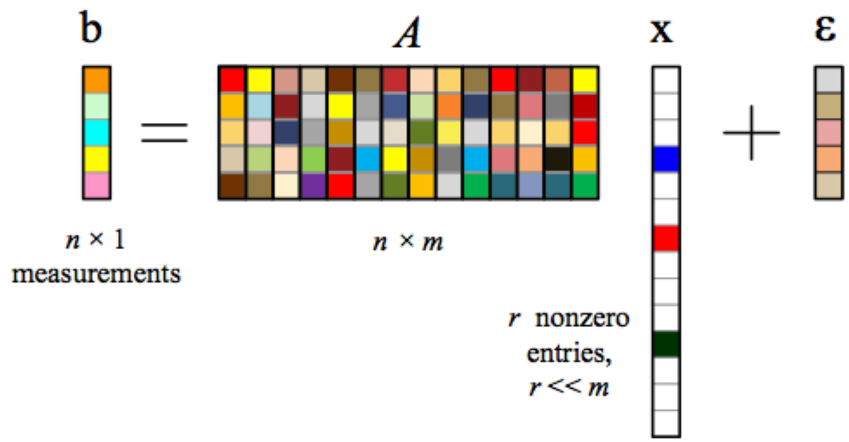
Compressed sensing formulation



- b is $n \times 1$ measurement vector
- A is $n \times m$ measurement/Dictionary matrix, $m >> n$
- x is $m \times 1$ desired vector which is sparse with r nonzero entries
- ε is the measurement noise

- **An underdetermined system of equations has many solutions**
- **Utilizing x is sparse it can often be solved**
- **This depends on the structure of A (RIP!)**

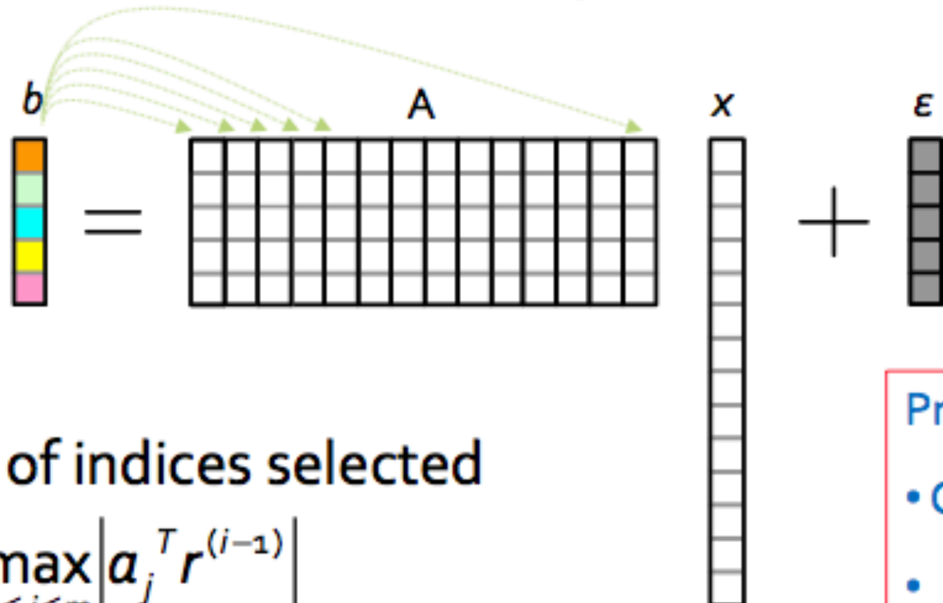
Different applications, but same math



b	**=**	**A**	**x**
Frequency signal	=	DFT matrix	Time-signal
Compressed-Image	=	random matrix	pixel-image
Seismic signals	=	travel-time delay	source-location
Acoustic signals	=	beam weight	source-location
Reflection sequence	=	time delay	layer-reflector

Greedy Search Method: Matching Pursuit

- Select a column that is most aligned with the current residual



  - $r^{(0)} = b$
  - $S^{(i)}$: set of indices selected
  - $l = \underset{1 \le j \le m}{\operatorname{argmax}} \left| a_j^T r^{(i-1)} \right|$

Practical stop criteria:

- Certain # iterations
- $\left\| r^{(i)} \right\|_2$ smaller than threshold

- Remove its contribution from the residual
  - Update $S^{(i)}$: If $l \notin S^{(i-1)}, S^{(i)} = S^{(i-1)} \cup \{l\}$. Or, keep $S^{(i)}$ the same
  - Update $r^{(i)}$: $r^{(i)} = P_{a_l}^{\perp} r^{(i-1)} = r^{(i-1)} - a_l a_l^T r^{(i-1)}$

Matching pursuit

- Performance guarantee?
- More advanced search?

- Easy to implement, as in home work.

l_p-Norms

$$\| x \|_p = \left(\sum_{m=1}^{M} | x_m |^p \right)^{1/p} \quad \text{for} \;\; p > 0$$

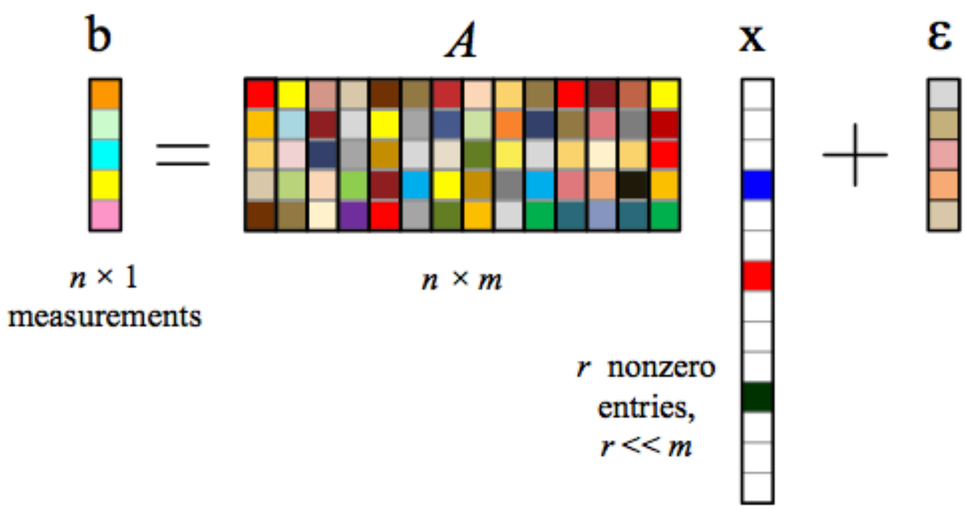
- Classic choices for p are 1, 2, and ∞.
- We will abuse notation and allow also $p = 0$.

Compressive Sensing / Sparse Recovery

- **Alternative viewpoint:** We try to find the sparsest solution which explains our noisy measurements

$$\min_x \| \mathbf{x} \|_0 \qquad \text{subject to} \;\; \| \mathbf{Ax} - \mathbf{b} \|_2 < \varepsilon$$

- Here, the l_0-norm is a shorthand notation for *counting the number of non-zero elements in* x.



Compressive Sensing / Sparse Recovery

- Unfortunately the l_0-norm minimization is not convex and hard to solve.
- We choose to convexify the problem by substituting the l_1-norm in place of the l_0-norm.

$$\min_x \| \mathbf{x} \|_1 \qquad \text{subject to} \quad \| \mathbf{A}\mathbf{x} - \mathbf{b} \|_2 < \varepsilon$$

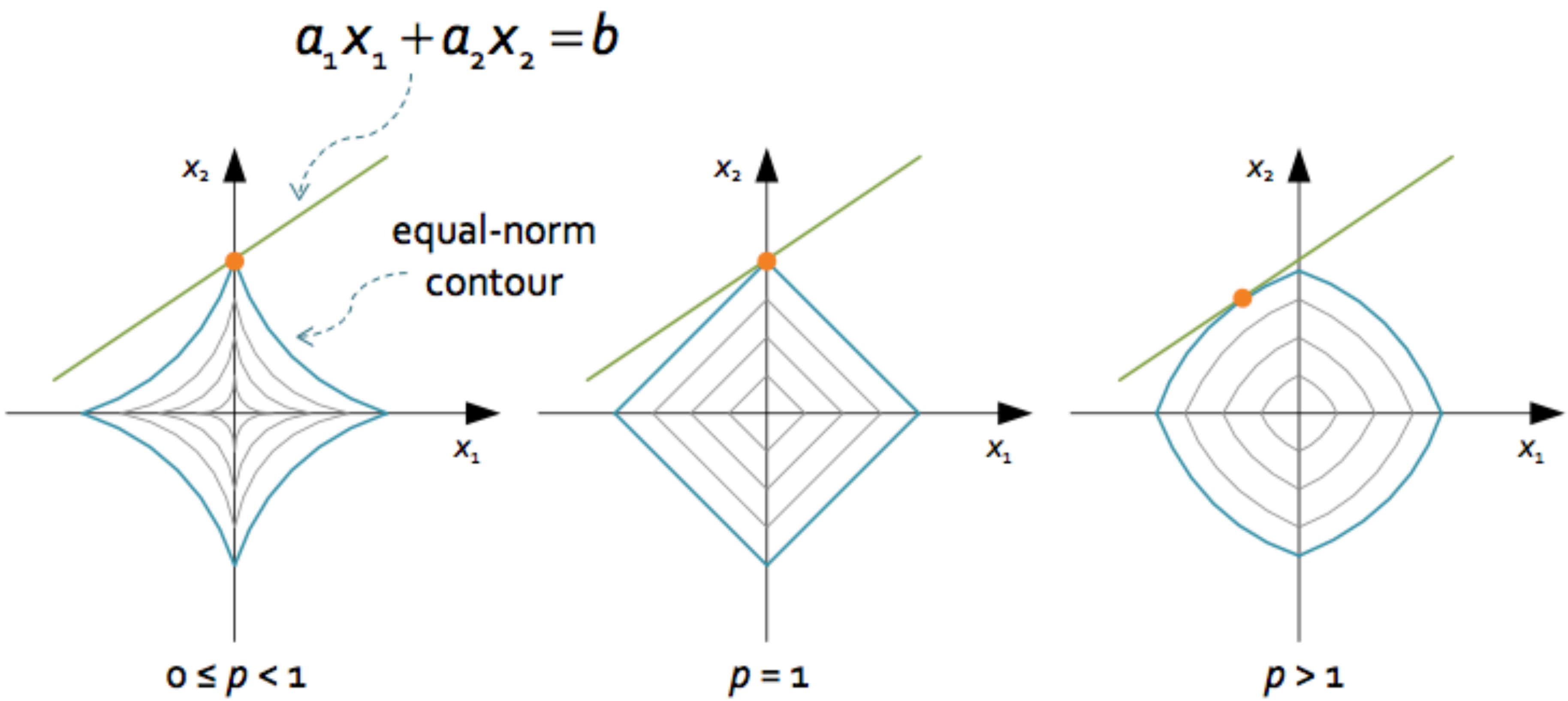
- This can also be formulated as

$$\min_x \| \mathbf{x} \|_1 + \lambda \| \mathbf{A}\mathbf{x} - \mathbf{b} \|_2$$

$$\min_x \| \mathbf{A}\mathbf{x} - \mathbf{b} \|_2 + \lambda' \| \mathbf{x} \|_1$$

$$\min_x \| \mathbf{A}\mathbf{x} - \mathbf{b} \|_2 \qquad \text{subject to} \quad \| \mathbf{x} \|_1 < \delta$$

$$\min \left\| [x_1, x_2]^T \right\|_p^p \quad \text{subject to} \quad a_1x_1 + a_2x_2 = b$$

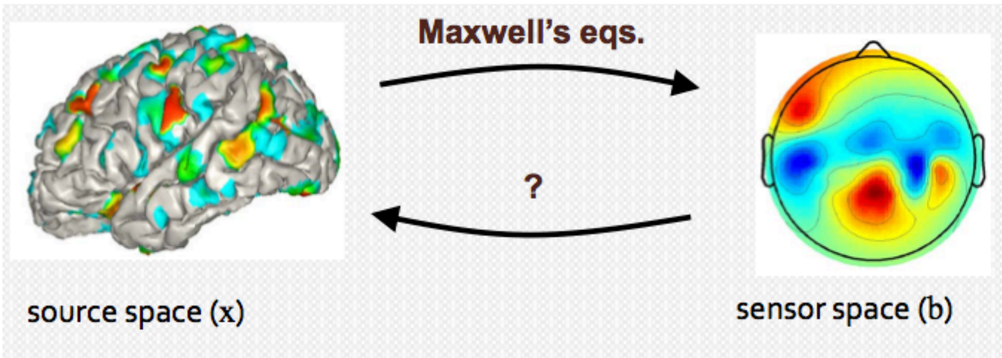


Applications

- MEG/EEG/MRI source location (earthquake location)
- Channel equalization
- Compressive sampling (beyond Nyquist sampling!)
- Compressive camera!

Lots of low hanging fruits

- Beamforming
- Fathometer
- Geoacoustic inversion
- Sequential estimation
- Bayesian
- Grid free methods



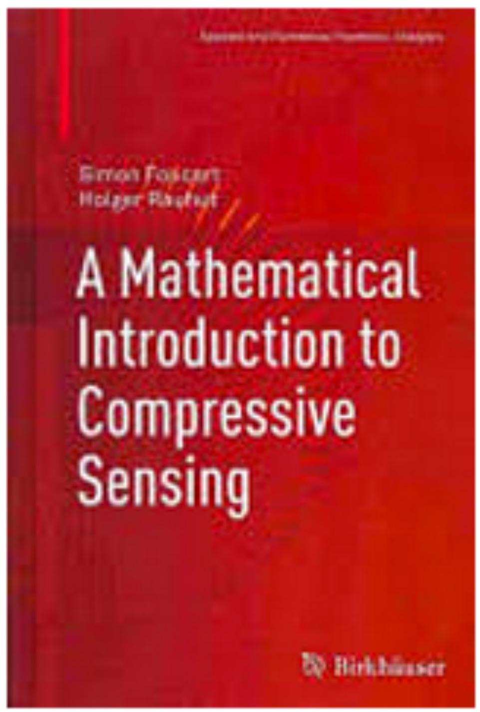
Resources

WEB

- http://nuit-blanche.blogspot.com/

- http://dsp.rice.edu/cs

BOOKS



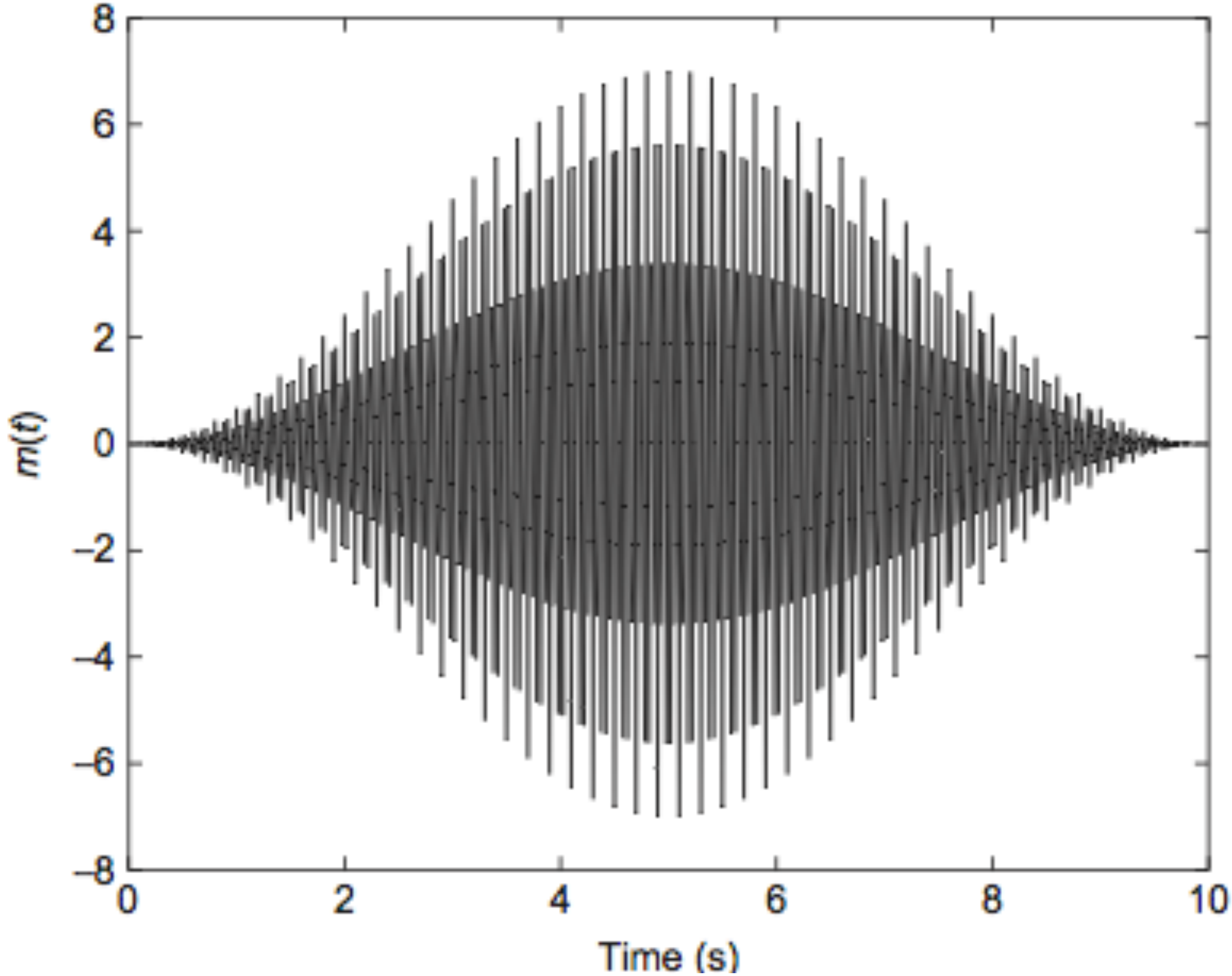
Example 7.4

Consider the recovery of a signal, $\mathbf{m}$, shown in Figure 7.18. This 10-s long time series of $n = 1001$ time points, t_i, is sampled at 100 samples/s and consists of two sine waves at $f_1 = 25$ and $f_2 = 35$ Hz:

$$m_i = h_i \cdot \left(5\ \cos(2\pi f_1 t_i) + 2\ \cos(2\pi f_2 t_i)\right) \quad 1 \leq i \leq n, \tag{7.24}$$

where the signal envelope has also been smoothed with term-by-term multiplication by a Hann taper function,

$$h_i = \frac{1}{2}\ (1 - \cos(2\pi(i-1)/n)) \quad 1 \leq i \leq n. \tag{7.25}$$



8
6
4
2
0
−2
−4
−6
−8
m(t)
0
2
4
6
8
10
Time (s)

L2 solutions

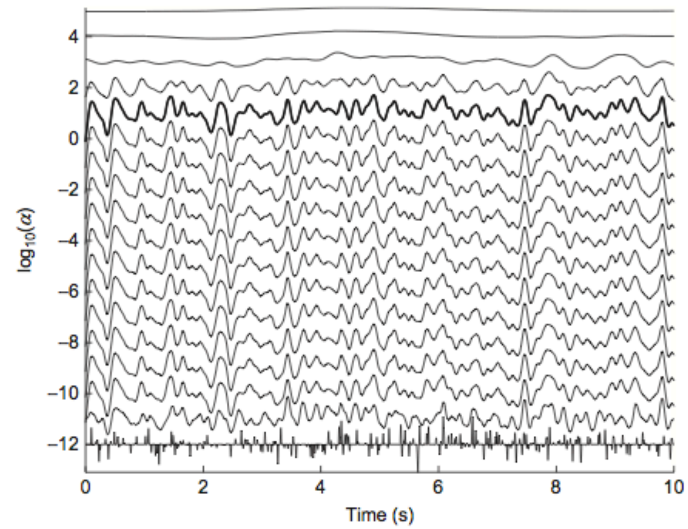


Figure 7.19 Signal recovery using second-order Tikhonov regularization. Solution amplitudes are normalized to improve legibility.

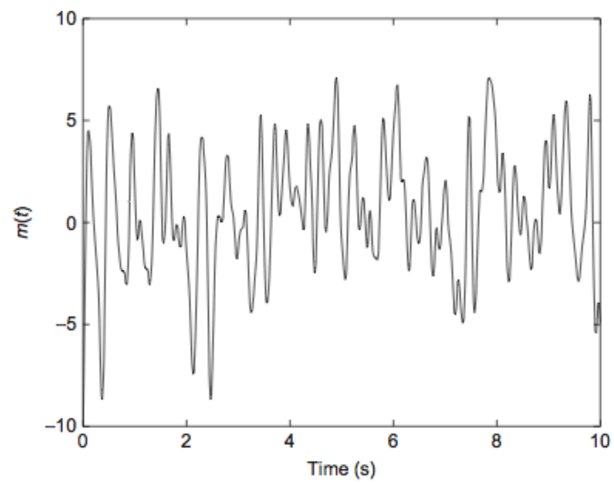


Figure 7.20 A representative solution using second-order Tikhonov regularization that approximately satisfies the discrepancy principle from Figure 7.19 ($\alpha = 10$).

L1 solutions

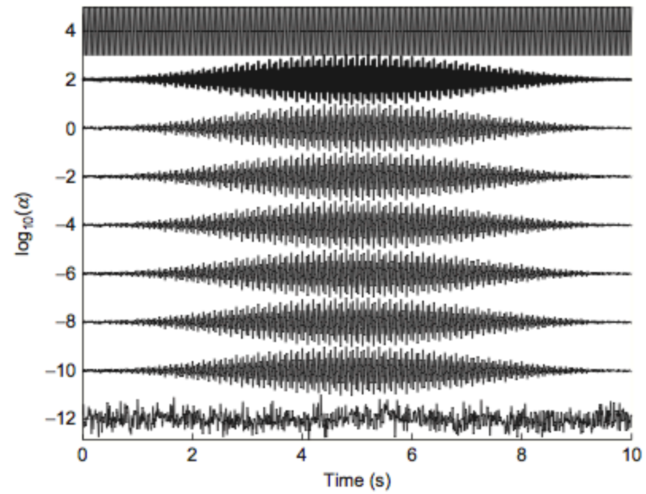


Figure 7.21 Signal recovery using compressive sensing with 100 signal measurements. Solution amplitudes are normalized to improve legibility.

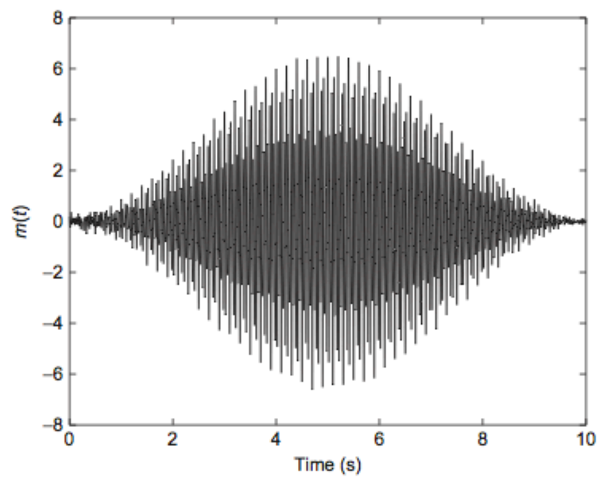
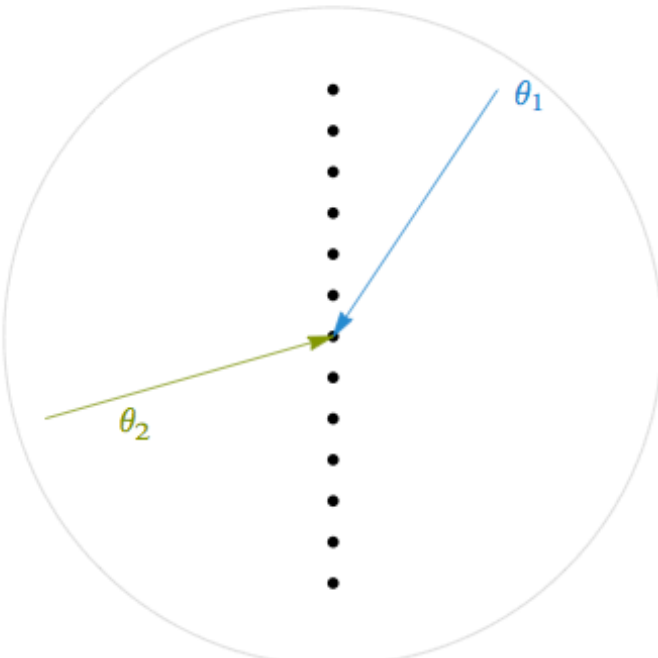


Figure 7.22 A representative solution obtained from Figure 7.21 using compressive sensing with $\alpha = 100$ that approximately satisfies the discrepancy principle.

CS BEAMFORM INTRO

b	=	A	x
Seismic signals	=	travel-time delay	source-location
Acoustic signals	**=**	**beam weight**	**source-location**
Reflection sequence	=	time delay	layer-reflector
Compressed-Image	=	random matrix	pixel-image

Direction of arrival estimation



Plane waves from a source/interferer impinging on an array/antenna

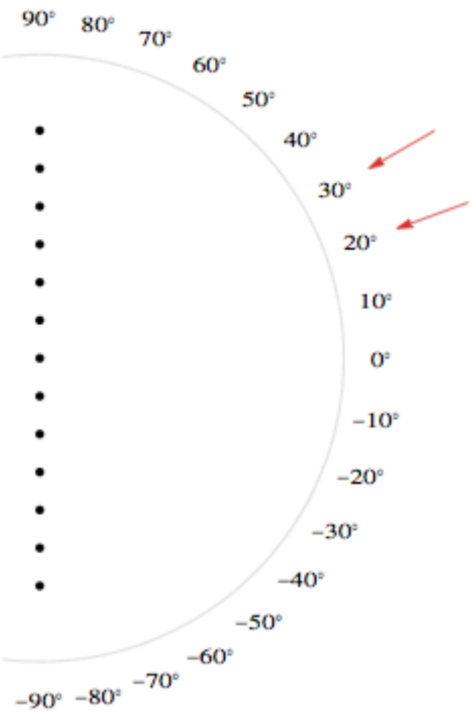
True DOA is sparse in the angle domain

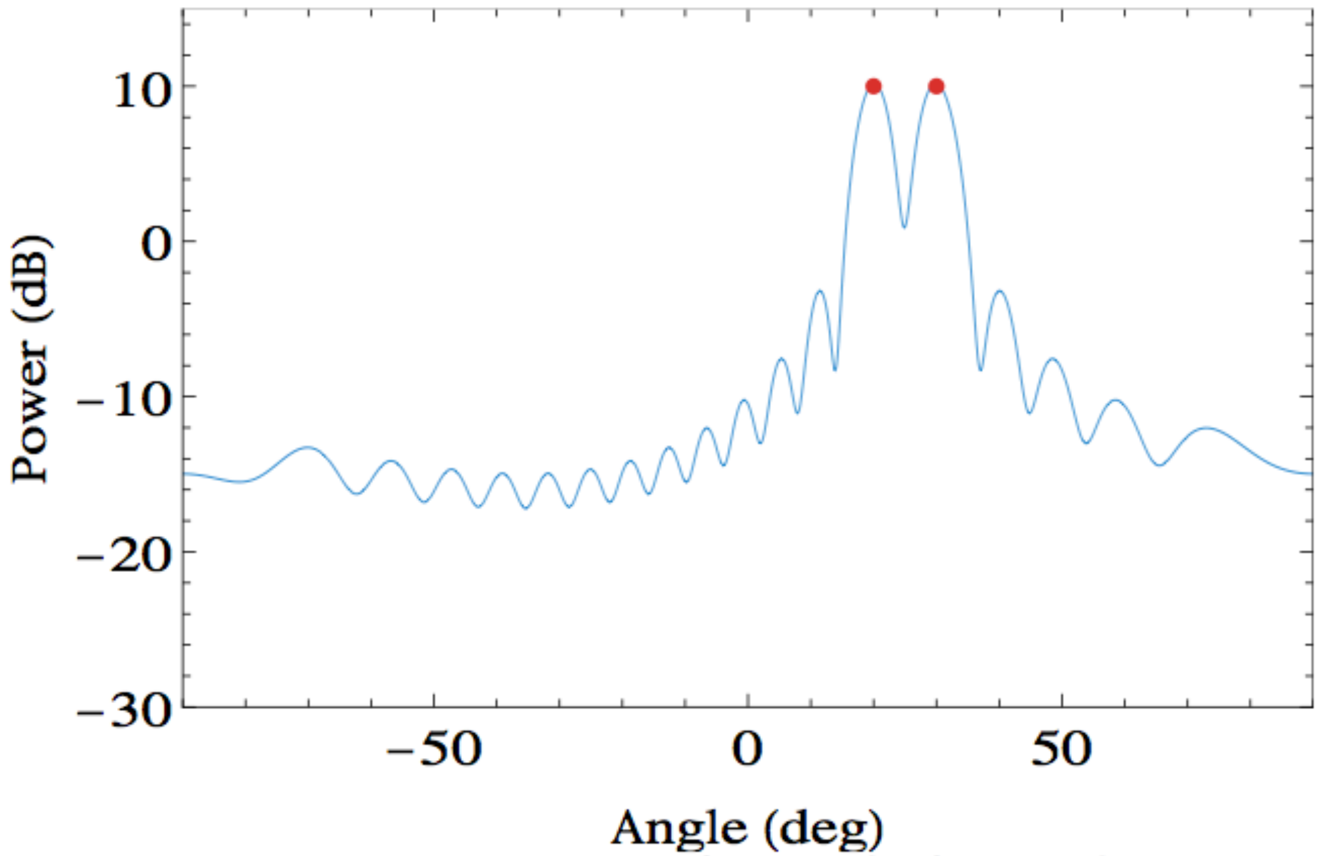
$$\boldsymbol{\Theta} = \{0, \cdots, 0, \theta_1, 0, \cdots, 0, \theta_2, 0, \cdots, 0\}$$

Conventional beamforming

Plane wave weight vector $\mathbf{w}_i = [1, e^{-\imath \sin(\theta_i)}, \cdots, e^{-\imath (N-1) \sin(\theta_i)}]^T$

$$\mathcal{B}(\theta) = |\mathbf{w}^H(\theta)\mathbf{b}|^2$$



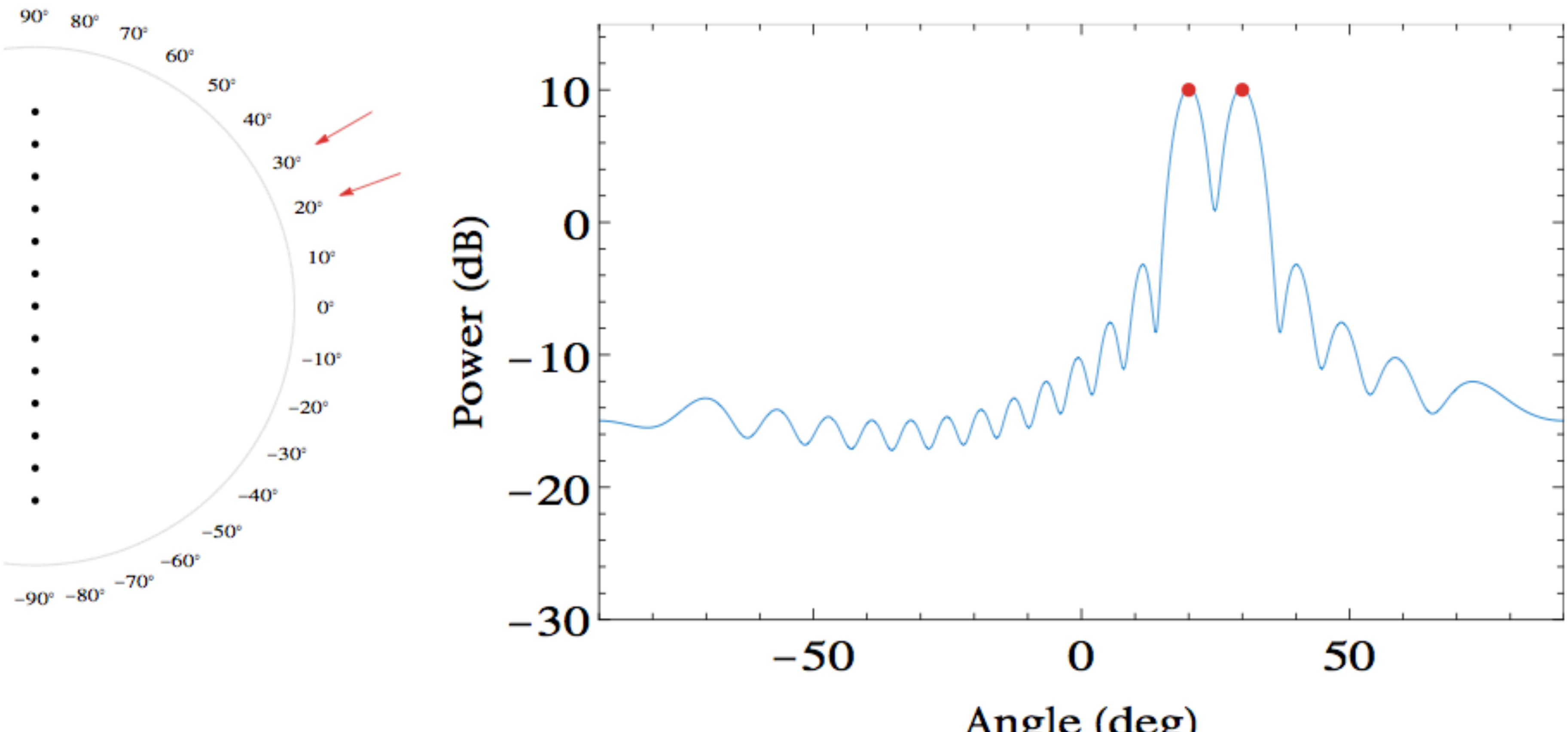


ULA, half-wavelength spacing, $N = 20$ sensors, $\theta_1 = 20°$, $\theta_2 = 30°$,

Conventional beamforming

Equivalent to solving the ℓ_2 problem with $\mathbf{A} = [\mathbf{w}_1, \cdots, \mathbf{w}_M]$, $M > N$.

$$\min \|\mathbf{x}\|_2 \text{ subject to } \mathbf{Ax} = \mathbf{b}$$

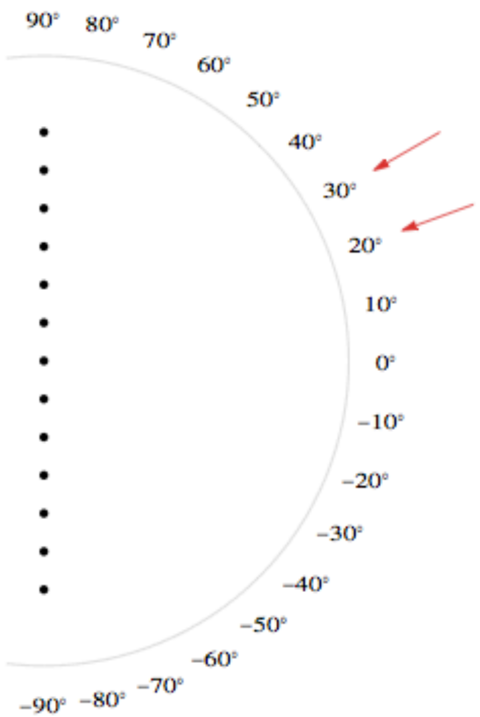


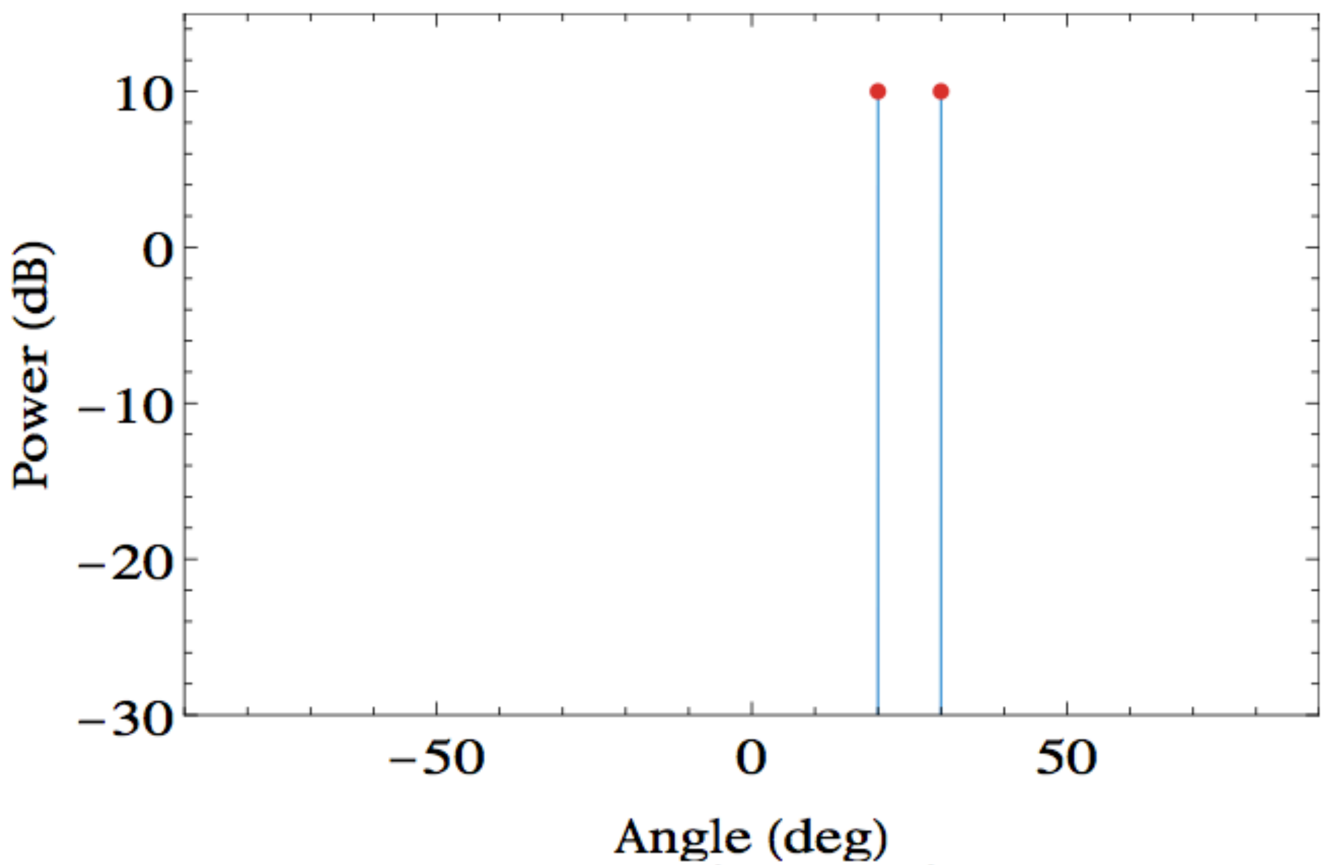
A is an overcomplete dictionary of candidate DOA vectors. Columns span -90° to 90° in steps of 1° ($M = 181$).

ℓ_1 minimization

In contrast ℓ_1 minimization provides a sparse solution with exact recovery:

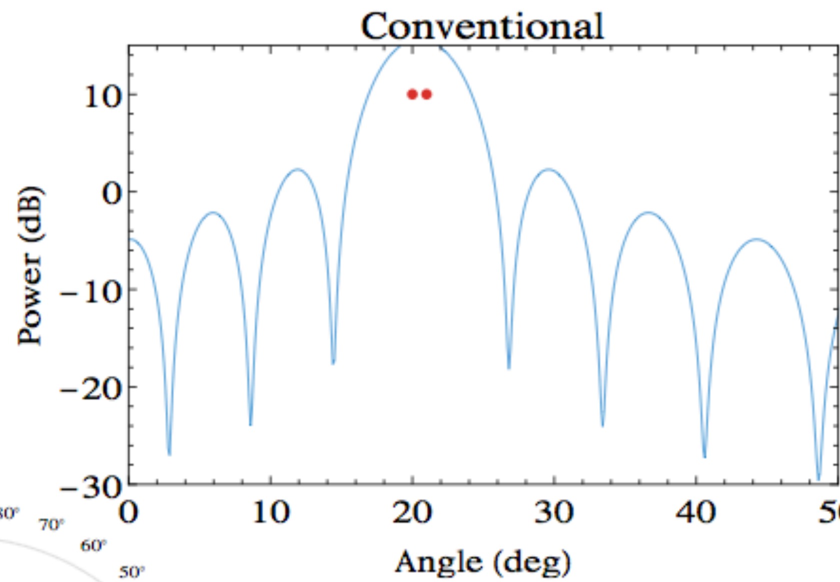
$$\min \|\mathbf{x}\|_1 \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{b}$$

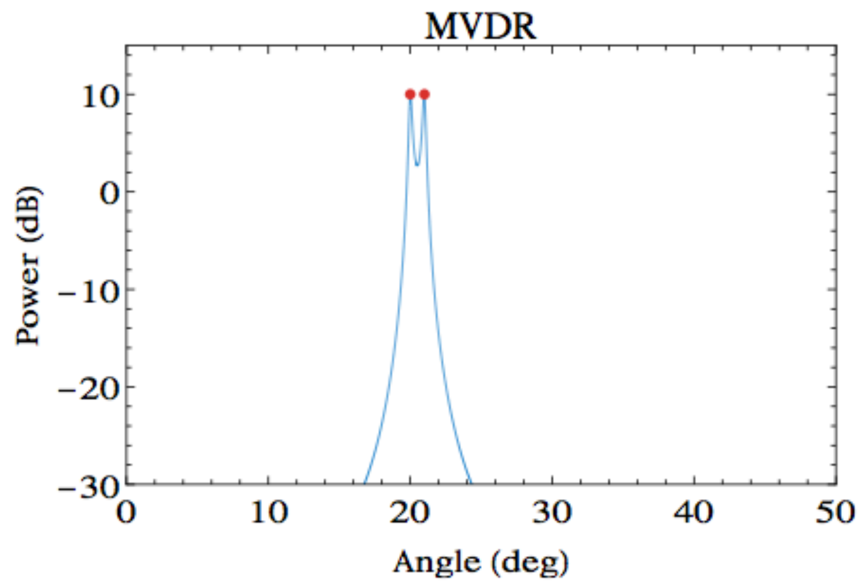


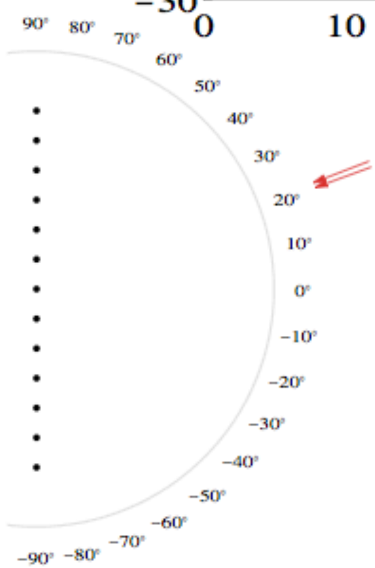


Columns of $\mathbf{A}$ span $-90°$ to $90°$ in steps of $1°$ ($M = 181$).

Resolving closely spaced signals





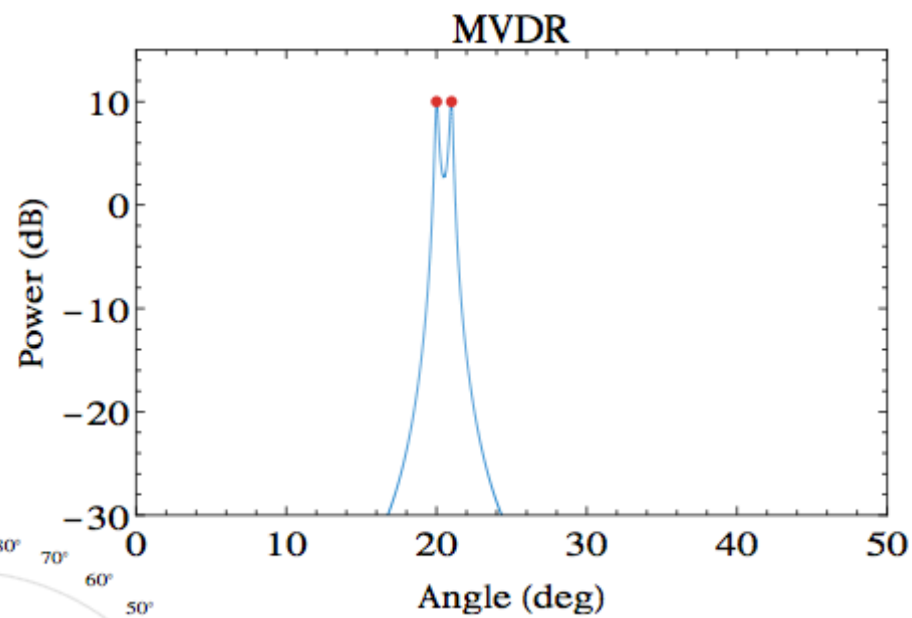


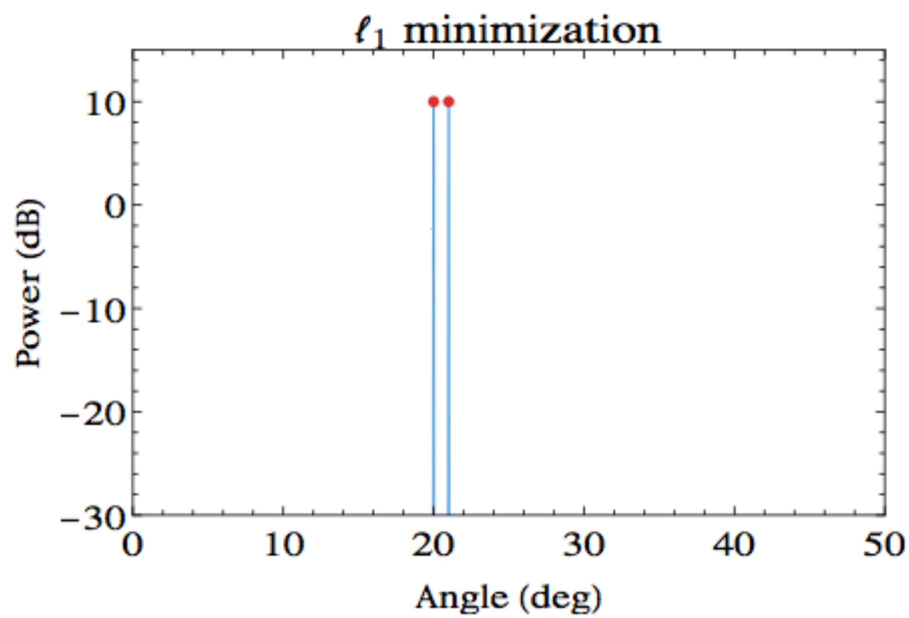
$\theta_1 = 20°$, $\theta_2 = 21°$ (note the change in x-axis)

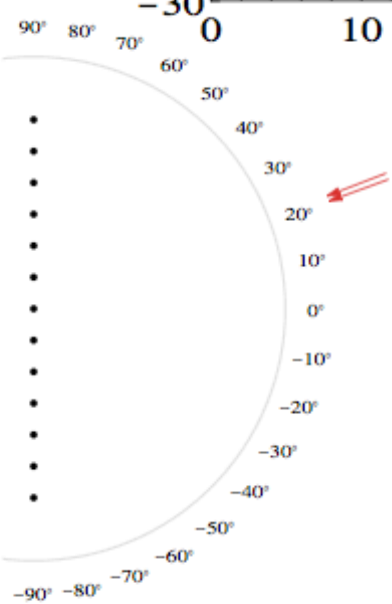
Power overestimated by 6 dB

Resolution of $\sim 5.7°$

Resolving closely spaced signals







$\theta_1 = 20^\circ$, $\theta_2 = 21^\circ$ (note the change in x-axis)