ECE 295: Homework 6

May 7, 2015

1 Problem 1

Two sports teams play a game against each other a total of N times. Let α_1 be the skill of Team 1, and α_2 be the skill of Team 2, where skill is defined to the probability that the team would beat a team of *average ability* in a single game. For example, a team with $\alpha = 1$ would win against a team of average ability every time, while a team with $\alpha = 0.5$ would win 50% of the time.

a) If Team 1 (skill = α_1) and Team 2 (skill = α_2) play one game, what is the probability that Team 1 wins? What is the probability that Team 2 wins?

b) Let $\alpha = \alpha_1$ and assume $\alpha_1 + \alpha_2 = 1$ (so $\alpha_2 = 1 - \alpha$). If W is the number of wins for Team 1 in N games, use your result from part a) to write down the likelihood function for W: $P(W|\alpha)$.

c) Assume $\alpha = 0.6$. Using MATLAB's random number generator (or your favorite programming package) simulate the result of the two teams playing one N times, for values of N ranging from 1 to 100 (hint, use part a) to treat each game like a flip of a weighted coin). For a uniform prior on α,

$$p(\alpha) = 1; \quad \alpha : [0, 1] \tag{1.1}$$

make plots that show how the posterior distribution evolves as a function of N through sequential updating. Remember to normalize the posterior distributions so it is a valid pdf.

d) Repeat part c), but with a different value of α. What differences do you notice?

e) Repeat parts c) and d), but with a Gaussian prior with mean of 0.5 and standard deviation of 0.1. What differences do you notice?

f) For $\alpha = 0.5$, what happens if your prior distribution is a"half-sided" Gaussian with a mean of 0 and standard deviation of 0.1 (assume zero probability for negative α). Does the posterior distribution evolve toward the correct value of α?